Asymptotics and holographic duality

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Sixties – the dawn of GR renaissance

Key experimental and theoretical breakthroughs

- Gravitational redshift (Pound & Rebka 1959)
- Quasars (Hong-Yee Chiu 1964)
- Shapiro effect (1964)
- CMB (Penzas & Wilson1965)
- Black-hole physics (Penrose paper on singularity formation: 1965)
- ...

1962 Bondi, van der Burg, Metzner and Sachs

Asymptotic symmetry group of a *strict* asymptotically flat spacetime in four dimensions

$$\text{Lorentz} \ltimes \text{Supertranslations} \longrightarrow 6 + \infty \equiv \text{BMS}_4$$

rather than Poincaré $\equiv$ Lorentz $\ltimes$ Translations $\longrightarrow 6 + 4$

Recognized as a valuable tool (Ashtekar, Komar, Penrose and many others) [see Brussels school for modern perspective and developments]

- Classical: solution space, conserved charges, algebra etc.
- Quantum: BMS_4-invariant S matrix (massless particles)

Anti de Sitter

Maximally symmetric Einstein spacetime with negative curvature (cosmological constant)

Einstein spacetimes palette of asymptotic symmetries

e.g. for *strict* AdS_n asymptotics

$$SO(n-1,2) \longrightarrow {}^{n(n+1)}/_{2} \equiv \text{conformal group in } n-1 \text{ dim}$$

→ symmetry of the *boundary field theory* → CFT

AdS/CFT holographic correspondence

Type IIB string theory in the *bulk* and $N = 4$ super-Yang–Mills on the *boundary* – AdS_5 soon extended to arbitrary dimension

Intriguing & timely question

What about asymptotically flat spacetime holography?

$n-1$-dim boundary theory invariant under BMS_n

Clues from solving classical equations

"Phenomenological" holography

- Keep only gravitational dynamics in the bulk
- Interpret the solution space from a boundary perspective

Gravitational bulk dynamics encoded in boundary data $\rightarrow$ holographic interpretation

Central questions on "flat holography"

- Where/which is the boundary?
- How is BMS symmetry implemented on the boundary?
- What are the boundary degrees of freedom?

Subject of the talk

Asymptotic symmetries in dimension n

Basic field in gravity: the curvature

potential: metric $G_{AB} \to$ diffeomorphism invariance

The gauge-fixing approach

- Fix a gauge: n conditions
- Determine the residual diffeomorphisms
- Set "reasonable" fall-offs/boundary conditions

The *asymptotic symmetry group* (ASG) is the subset of residual diffeomorphisms compatible with the adopted fall-offs

The electromagnetic paradigm

Basic field: $F_{AB} = \partial_A A_B - \partial_B A_A$

potential: $A_B \to$ gauge invariance $A_B \to A_B + \partial_B \lambda$

The gauge-fixing approach

- Fix a gauge e.g. Lorenz gauge: $\nabla_B A^B = 0$
- Determine the residual diffeomorphisms: $\Box \lambda = 0$

The *residual symmetry group* is the set of harmonic functions

Einstein spacetimes with $\Lambda < 0$ in n dimensions

The Newman–Unti gauge $(r, t, x^i, i = 1, \dots, n-2)$

- Gauge conditions: $G_{rr} = 0$, $G_{rt} = -1$, $G_{ri} = 0$

$$ds^2 = \frac{V}{r}dt^2 - 2dtdr + G_{ij}\left(dx^i - U^i dt\right)\left(dx^j - U^j dt\right)$$

V, G_{ij}, U^i functions of *all* coordinates

- Residual diffeomorphisms: $\omega(t, \mathbf{x})$, $f(t, \mathbf{x})$, $Y^i(t, \mathbf{x})$
- Several options for fall-offs/boundary conditions
  - mildest boundary conditions $\to$ Weyl, supertranslations and superrotations ($\ni$ singular) – ∞-dim
  - Dirichlet boundary conditions e.g. locked $G_{ij} \simeq r^2 S^{n-2} \to \omega = 0$, f restricted, Y^i conformal $\to$ $SO(n-1, 2)$ conformal group in $n-1$ dimensions – finite (AdS/CFT paradigm)

Incomplete Newman–Unti gauge fixing [Ciambelli, Marteau, Petropoulos, Ruzziconi '20; Ciambelli, Marteau, Petkou, Petropoulos, Rivera, Ruzziconi, Siampos]

- Gauge conditions: $G_{rr} = 0$, $G_{rt} = -1$, $G_{ri} \neq 0$
- Mild boundary conditions $\rightarrow$ Weyl, supertranslations, superrotations plus local $SO(n-2,1)$ – ∞-dim

Einstein spacetimes reconstructed

Solution space with incomplete Newman–Unti gauge and mild boundary conditions

- $\frac{n(n-1)+2}{2}$ Einstein's equations $\rightarrow n^2 - 3$ functions of $(t, \mathbf{x})$ $\rightarrow$ boundary data $[\mu, \nu = 0, 1, \ldots, n-2]$
  - $g_{\mu\nu}$ symmetric $\leftarrow \frac{n(n-1)}{2}$
    boundary metric
  - $T_{\mu\nu}$ symmetric and traceless $\leftarrow \frac{n(n-1)}{2} - 1$
    conformal boundary energy–momentum tensor
  - $u^\mu \leftarrow n - 2$
    boundary normalized vector field
- remaining $n - 1$ Einstein's equations

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

$\rightarrow$ map to a Weyl-covariant relativistic fluid with velocity u^μ – trigger for fluid/gravity holographic correspondence [Bhattacharyya, Hubeny, Minwalla, Rangamani '07; Haack, Yarom '08; etc.]

Relativistic hydrodynamics [Eckart '40; Landau and Lifshitz '60]

- Ignoring matter current and chemical potential
- On arbitrary (non-flat) geometry $g_{\mu\nu}$ of dim $d+1$

$\nabla_\mu T^{\mu\nu} = 0$ plus Gibbs–Duhem & equation of state (conformal)

- $\|u\|^2 = -k^2 \quad h^{\mu\nu} = g^{\mu\nu} + \frac{u^\mu u^\nu}{k^2}$

$$T^{\mu\nu} = \varepsilon \frac{u^\mu u^\nu}{k^2} + p h^{\mu\nu} + \tau^{\mu\nu} + \frac{u^\mu q^\mu}{k^2} + \frac{u^\nu q^\mu}{k^2}$$

- energy density $\varepsilon = \frac{1}{k^2} T_{\mu\nu} u^\mu u^\nu$ thermodynamic pressure p
- heat current and viscous stress tensor $q^\mu, \ \tau^{\mu\nu}$ – transverse
- fluid velocity u^μ – arbitrary [Eckart '40; Landau and Lifshitz '60]

Properties here

Infinite-dim bulk ASG ≡ boundary-fluid invariance

Weyl, sT ⋊ sR ≡ boundary diffeos & local $SO(n-2,1)$ ≡ hydrodynamic-frame invariance

Choosing a hydrodynamic frame discards the local $SO(n-2,1)$ invariance and completes the bulk gauge fixing (e.g. $\mathbf{u} = -k^2 \mathrm{d}t$)

Imposing the velocity & Dirichlet boundary conditions

$g_{\mu\nu} = \eta_{\mu\nu}$

ASG ≡ conformal group in $n-1$ dim – $SO(n-1,2)$

The seed for a holographic dictionary

In $n = 4$ dimensions with complete gauge fixing

General solution with $\Lambda = -3k^2$: $6 + 5$ arbitrary boundary data

- $g_{\mu\nu}$ $\left(\mathrm{d}s^2 = -k^2\left(\Omega\mathrm{d}t - b_i\mathrm{d}x^i\right)^2 + a_{ij}\mathrm{d}x^i\mathrm{d}x^j\right)$
- $T_{\mu\nu} \to \{\varepsilon = 2p, q^\mu, \tau^{\mu\nu}\}$ with $\tau^\mu{}_\mu = 0$ & $\nabla_\mu T^{\mu\nu} = 0$

$$
\begin{aligned}
\mathrm{d}s^2_{\text{Einstein}} \quad = \quad & 2\frac{\mathbf{u}}{k^2}(\mathrm{d}r + r\mathbf{A}) + r^2\mathrm{d}s^2 - 2\frac{r}{k^2}\sigma_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \frac{\mathrm{S}}{k^4} \\
& + \frac{8\pi G}{k^4 r}\left[\varepsilon\mathbf{u}^2 + \frac{4\mathbf{u}}{3}\left(\mathbf{q} - \frac{1}{8\pi G} * \mathbf{c}\right)\right. \\
& \left. + \frac{2k^2}{3}\left(\boldsymbol{\tau} + \frac{1}{8\pi G k^2} * \boldsymbol{c}\right)\right] + \mathrm{O}\left(1/r^2\right)
\end{aligned}
$$

$S_{\mu\nu} = 2u_{(\mu}\mathscr{D}_\lambda\left(\sigma_{\nu)}{}^\lambda + \omega_{\nu)}{}^\lambda\right) - \frac{\mathscr{R}}{2}u_\mu u_\nu + 2\omega_{(\mu}{}^\lambda\sigma_{\nu)\lambda} + (\sigma^2 + k^4\gamma^2)\,h_{\mu\nu}$

$C_{\mu\nu} \to \{c, c^\mu, c^{\mu\nu}\}$ with $c^\mu{}_\mu = 0$ & $\nabla_\mu C^{\mu\nu} = 0$ (Cotton)

Holography is more

Genuine duality

between

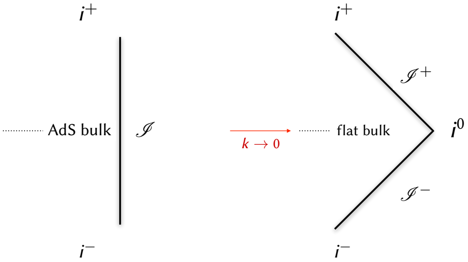
- bulk gravitational theory with AdS_n asymptotics
- boundary CFT – quantum theory with $SO(n-1,2)$ symmetry

enables the computation of correlation functions with $g_{\mu\nu}$ a source and $T_{\mu\nu}$ a vev

The asymptotic structure

From AdS$_n$ to flat$_n$ asymptotics

$$\Lambda = -\frac{(n-1)(n-2)}{2}k^2 \to 0$$



Emergence of Carrollian geometry in $n-1$ dimensions

- $\mathrm{d}s^2 = -k^2\left(\Omega \mathrm{d}t - b_i \mathrm{d}x^i\right)^2 + a_{ij}\mathrm{d}x^i\mathrm{d}x^j \to a_{ij}\mathrm{d}x^i\mathrm{d}x^j = \mathrm{d}\ell^2$
- *Carrollian diffeomorphisms* $t' = t'(t, \mathbf{x}) \quad \mathbf{x}' = \mathbf{x}'(\mathbf{x})$

Ricci-flat in incomplete Newman–Unti gauge

First hint in $n = 4$ $\lim_{k\to 0} \mathrm{d}s^2_{\text{Einstein}}$ [Ciambelli et al. '18]

$\mathrm{d}s^2_{\text{Ricci-flat}}$ described in terms of *Carrollian boundary data*

- Carrollian geometry (6)
  - degenerate metric $\mathrm{d}\ell^2 = a_{ij}\mathrm{d}x^i\mathrm{d}x^j$ (3)
  - Ehresmann connection $\Omega\mathrm{d}t - b_i\mathrm{d}x^i$ (3)
- Carrollian fluid (5)
  - energy ε (1)
  - momenta – heat current π_i (2) and stress tensor E_{ij} (2)
- Carrollian-fluid "velocity" (2)

Full solution space [Brussels school; Ciambelli et al. '21]

infinite number of further Carrollian data obeying Carrollian dynamics – at every $O\left({}^{1}/_{r^n}\right)$

Ricci-flat spacetimes up to $O\left(1/r^2\right)$

$$
\begin{aligned}
\mathrm{d}s^2_{\text{flat}} \;=\;& 2\boldsymbol{\mu}\left(\mathrm{d}r + r\varphi_a\boldsymbol{\mu}^a - r\frac{\theta}{2}\boldsymbol{\mu} + *\boldsymbol{\mu}^b\hat{\mathscr{D}}_b * \varpi - \frac{1}{2}\boldsymbol{\mu}^a\hat{\mathscr{D}}_b\mathscr{C}^b{}_a\right)\\
&+\left(\rho^2 + \frac{\mathscr{C}_{cd}\mathscr{C}^{cd}}{8}\right)\mathrm{d}\ell^2 + \mathscr{C}_{ab}\left(r\boldsymbol{\mu}^a\boldsymbol{\mu}^b - *\varpi * \boldsymbol{\mu}^a\boldsymbol{\mu}^b\right)\\
&+\frac{1}{r}\left[\left(8\pi G\varepsilon - \hat{\mathscr{K}}\right)\boldsymbol{\mu}^2 + \frac{32\pi G}{3}\left(\pi_a - \frac{1}{8\pi G} * \psi_a\right)\boldsymbol{\mu}\boldsymbol{\mu}^a\right.\\
&\left.-\frac{16\pi G}{3}E_{ab}\boldsymbol{\mu}^a\boldsymbol{\mu}^b\right] + O\left(1/r^2\right)
\end{aligned}
\qquad
\begin{cases}
\boldsymbol{\mu} = \lim_{k\to 0}\frac{\mathbf{u}}{k^2}\\
\rho^2 = r^2 + \star\varpi^2\\
\mathscr{N}_{ab} = \hat{\mathscr{D}}_{\boldsymbol{v}}\mathscr{C}_{ab}
\end{cases}
$$

Bulk ASG matches the boundary invariances

- Weyl $\omega(t,\mathbf{x})$
- sT $f(t,\mathbf{x}) \rtimes$ sR $Y^i(\mathbf{x}) \equiv$ Carrollian diffeos
- Carrollian hydrodynamic-frame transformations

for Dirichlet ($\mathrm{d}\ell^2 \simeq S^2$ & zero Ehresmann) $\to$ $\mathrm{BMS}_4 \equiv \mathrm{CCarroll}_3$

Hints for flat holography

Fundamental feature

bulk reconstruction $\rightarrow$ infinite number of boundary data
$\Rightarrow$ possibly *non-holographic* bulk/boundary duality

The 3-dim dual field theory on the Carrollian bry.

- must be invariant under $\text{CCarroll}_3 \equiv \text{BMS}_4 \equiv \text{sT} \rtimes SL(2, \mathbb{C})$
- expected to be *non-local*

progress requires reinterpreting gravitational processes from a boundary perspective

WHAT ABOUT FLAT$_4$/CFT$_2$ *CELESTIAL* HOLOGRAPHY? [HARVARD SCHOOL]

FRAMEWORK
- $\mathscr{S}_2 \equiv$ spatial section of the Carrollian bry.
- 2-dim energy–momentum tensor $\sim \int \mathrm{d}t \mathscr{N}_{ab}$

ACHIEVEMENTS mapping of some massless bulk *S*-matrix elements to CFT$_2$ correlators

FEATURES
- non-local, non-AdS-like en.–mom. tensor
- limited to $SL(2,\mathbb{C})$ invariance – vs. BMS$_4$
- ignores the *deep chthonian* degrees of freedom

kinematic book-keeping device for *radiation S*-matrix

Facts & quotable

Flat bulk is mapped onto BMS/Carrollian boundary dynamics

- supports a *Carrollian-fluid/flat-gravity* sector
- requires infinite sets of *deep* boundary data
- suggests flat-holographic duals are *non-local* field theories

Worth investigating

- pursue the quest of BMS-invariant field theories [Le Bellac, Lévy-Leblond '73; Souriau '85; Duval et al. '14; Bagchi et al. '20; Henneaux, Salgado–Rebolledo '21]
- circumscribe the precise role of celestial CFT

Carrollian geometry [Duval et al. '14; Bekaert et al. '16; Ciambelli et al. '19]

Basic ingredients in $d+1$ dimensions

- degenerate metric: $\mathrm{d}\ell^2 = a_{ij}(t,\mathbf{x})\mathrm{d}x^i\mathrm{d}x^j \quad i,j = 1,\ldots,d$
- Ehresmann connection: $\boldsymbol{e} = \Omega\mathrm{d}t - b_i\mathrm{d}x^i$

General covariance

Carrollian diffeomorphisms: $t' = t'(t,\mathbf{x}) \quad \mathbf{x}' = \mathbf{x}'(\mathbf{x})$

Example: zero-c limit of Minkowski spacetime [Lévy-Leblond '65]

- $\mathrm{d}\ell^2 = \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j \quad \boldsymbol{e} = \mathrm{d}t$
- isometries: Carroll group $\begin{cases} t' = t + B_i x^i + t_0, \\ x'^k = R^k_i x^i + x^k_0 \end{cases}$

Property

$\mathrm{CCarrroll}_{d+1} \equiv \mathrm{BMS}_{d+2}$

Equations of motion [Eckart '40; Landau and Lifshitz '60]

- Ignoring matter current and chemical potential
- On arbitrary (non-flat) geometry $g_{\mu\nu}$ of dim $d+1$

$\nabla_\mu T^{\mu\nu} = 0$ plus Gibbs–Duhem & equation of state (conformal)

$$T^{\mu\nu} = \varepsilon \frac{u^\mu u^\nu}{k^2} + p h^{\mu\nu} + \tau^{\mu\nu} + \frac{u^\mu q^\nu}{k^2} + \frac{u^\nu q^\mu}{k^2}$$

- $\|\mathrm{u}\|^2 = -k^2 \quad h^{\mu\nu} = g^{\mu\nu} + \frac{u^\mu u^\nu}{k^2}$
- energy density $\varepsilon = \frac{1}{k^2} T_{\mu\nu} u^\mu u^\nu$ thermodynamic pressure p
- heat current and viscous stress tensor $q^\mu,\ \tau^{\mu\nu}$ – transverse normally expressed as u^μ- and T-derivative expansions with transport coefficients

General covariance and Weyl invariance

Fluid equations covariant – diffeomorphism invariance

Diffeomorphisms are generated by vector fields ($i, j = 1, \ldots, d$)

$$\xi = f\partial_t + Y^i\partial_i$$

$f(t, \mathbf{x})$ and $Y^i(t, \mathbf{x})$ $d + 1$ functions of time and space

$$\delta_\xi = -\mathscr{L}_\xi$$

Conformal (Weyl-covariant) fluids: fluid equations invariant under arbitrary rescaling of the metric

$$\delta_\omega g_{\mu\nu} = -2\omega g_{\mu\nu} \quad \delta_\omega \boldsymbol{u}^\mu = \omega \boldsymbol{u}^\mu$$

$\omega(t, \mathbf{x})$ arbitrary function of time and space

$$\delta_\omega = w\omega$$

The hydrodynamic-frame invariance

Landau–Lifshitz's following 1940 Eckart's statements

[Theoretical Physics vol. 6 §136]

u^μ is not physical/measurable – a book-keeping device

Translation: gauge invariance

Arbitrary *local* Lorentz transformations of u^μ can be compensated by appropriate modifications of $T, \varepsilon, p, q^\mu, \tau^{\mu\nu}$ such that $T^{\mu\nu}$ *and* the entropy current S^μ remain invariant

Note: These are *not* Lorentz isometries (generally absent) but tangent-space *local* transformations generated by Z^i (d boosts), S_{ij} antisymmetric ($^{d(d-1)}/_2$ rotations)

In summary

Conformal-fluid symmetries on arbitrary backgrounds

∞-dim generated by $\{\omega(t,\mathbf{x}), f(t,\mathbf{x}), Y^i(t,\mathbf{x}), Z^i(t,\mathbf{x}), S_{ij}(t,\mathbf{x})\}$