Supersymmetric boundaries and defects

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Overview

Field theory recap

Action functional, e.g. in 2 dimensions (τ, σ):

$$S[\phi_i, \phi_i', \dot{\phi}_i] = \int d\sigma d\tau \; \mathcal{L}(\phi_i, \phi_i', \dot{\phi}_i), \quad \dot{\phi}_i \equiv \partial_\tau \phi_i, \; \phi_i' \equiv \partial_\sigma \phi_i$$

Euler-Lagrange equations of motion:

$$\frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}_i}\right) + \frac{\partial}{\partial \sigma}\left(\frac{\partial \mathcal{L}}{\partial \phi_i'}\right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0$$

Symmetries: Noether's theorem

Classical field theory:

$$\text{Continuous symmetries} \xrightarrow{\textit{Noether}} \text{Conserved quantities: "Charges"}$$

Quantum field theory: Charges become operators

$\rightsquigarrow$ Lie algebra (i.e. a set of commutation relations that "close")

Examples

Example 1: Complex scalar field

$$\mathcal{L} = |\dot{\phi}|^2 - |\phi'|^2 - m^2|\phi|^2, \quad \text{EOM: } \ddot{\phi} - \phi'' + m^2\phi = 0$$

Symmetry: $\phi \to e^{i\epsilon}\phi = (1 + i\epsilon)\phi$
Conserved charge: $Q = \int d\sigma \, (\dot{\phi}^*\phi - \phi^*\dot{\phi})$

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Example 2: Spacetime symmetries

- Time translation $\to$ Energy
- Space translation $\to$ Momentum
- Rotation (spacetime) $\to$ Angular momentum

In quantum theory: Poincaré algebra (Momenta, Angular momenta, Boosts)

Examples

Example 3: Supersymmetry

$$\mathcal{L}(\phi, \psi, \text{derivatives})$$

Symmetric under supersymmetry transformation. Schematically:

$$\phi \to \phi + \epsilon\psi, \quad \psi \to \psi + \epsilon\phi$$

$\Rightarrow$ Conserved charges: Supercharges
In quantum theory: Supersymmetry algebra (Poincare + supercharges)

$$\{Q, \bar{Q}\} = P$$

Boundaries

We saw: symmetry of spacetime $\rightarrow$ conserved quantity of the theory.
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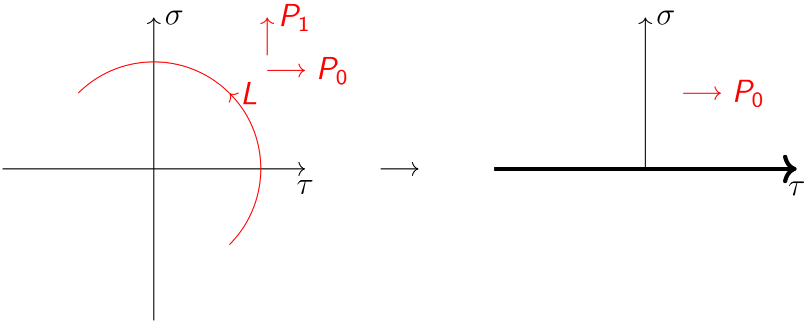

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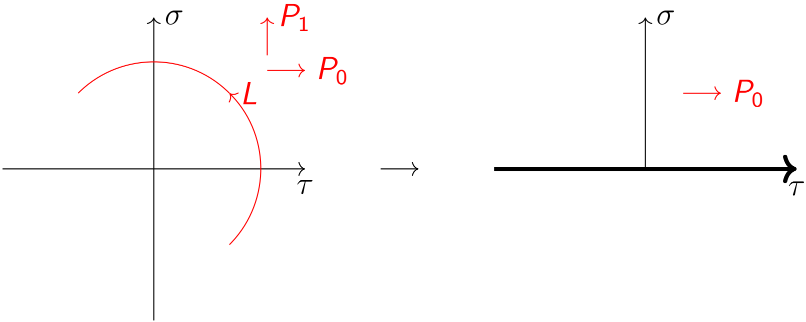

$\Rightarrow$ Some conserved quantities are no longer conserved!

Boundaries in SUSY

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Some supercharges not conserved anymore either (since some P is not conserved).
But we still want to have supersymmetric theory!

- Nice mathematical description
- We can compute things via localization
- We like it

Best we can do: keep some of them (at most half)

Matrix factorizations

Preserve a *specific* half of supersymmetry: Condition on the superpotential.
Superpotential: a holomorphic function of the scalar fields $W(\phi_i)$.

Condition to preserve SUSY

$$W = \sum_i E_i J_i$$

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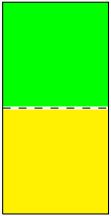
Example: $W = \phi^d$

$W = \phi^d = \phi^L \cdot \phi^{d-L}$: Not unique!

Every factorization of this type defines a (generalized) boundary condition.

Defects



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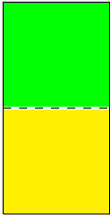
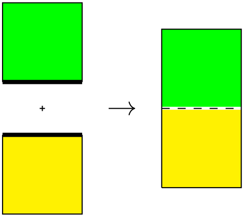
Defects



- What are defects: Lines separating different theories on the same surface
- Another way to view them: theories with boundaries glued together



Defects

- Defects also break some (super)symmetry, like boundaries
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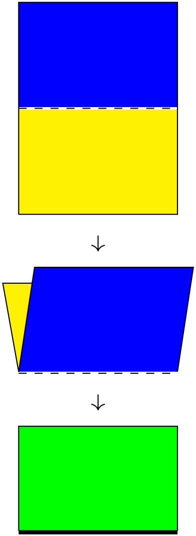
- Preserve (some) half of supersymmetry: Factorization of difference of superpotentials
- Folding trick: Defect between $\mathcal{C}_1$ and $\mathcal{C}_2$ $\Leftrightarrow$ boundary of $\mathcal{C}_1 \otimes \overline{\mathcal{C}}_2$
- SUSY preserving defect= Factorization of $W_1 - W_2$



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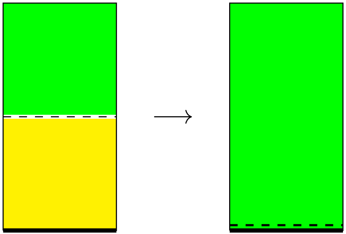

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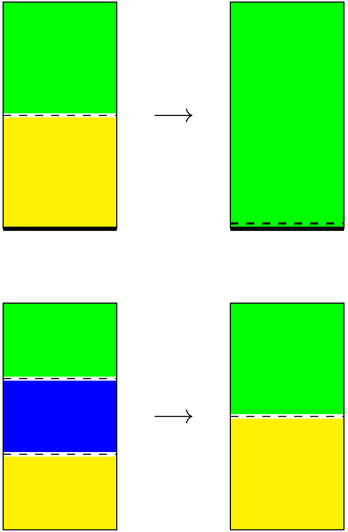

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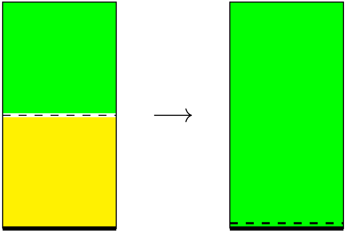
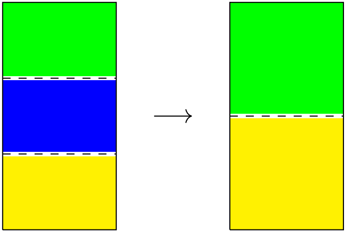
- Merge defects together: $(W_1 - W_2) + (W_2 - W_3) = W_1 - W_3$
- Multiplicative structure: Defects are "operators" acting on boundaries: complete algebraic description.





Outlook

- Symmetries $\xrightarrow{Noether}$ Conserved charges $\xrightarrow{QFT}$ Operators
- Boundaries break some symmetries
- Special type of boundaries described by factorization of superpotential
- Defects: generalized operators acting on boundaries

The End

Thank you for your attention!