# QFT, supersymmetry and localization

Alexander Tabler

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS

December 20, 2018

# 1) QFT, 2) supersymmetry and 3) localization

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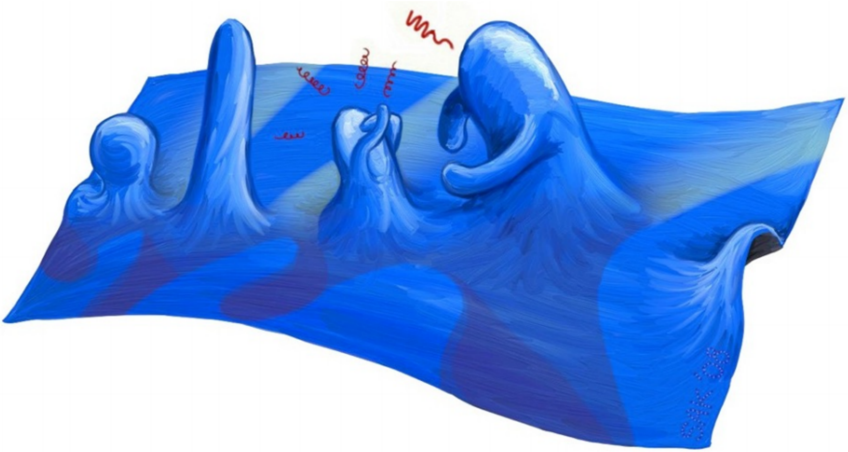
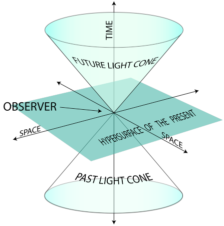
# What *is* quantum field theory?



[1]

[1] from damtp.cam.ac.uk/user/tong

# What *is* quantum field theory?

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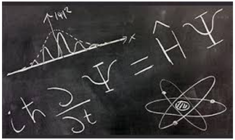

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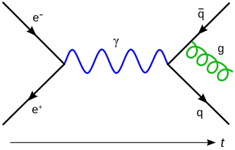



- Special relativity (classical field theory) $+$ quantum mechanics
- Modern language/framework for particles
- (Much more!)

## Example: Standard Model

*Amazingly* successful: agreement to within $10^{-12}$![a]

[a] gabrielse.physics.harvard.edu/gabrielse/overviews/TestQED/TestQED

# But what *is* quantum field theory?

**Aim**

Compute transition amplitudes!

The *action*: (compare to Lagrangian mechanics!)

$$S = \int_{\mathbb{R}^4} \mathrm{d}^4 x \, \mathcal{L}[\phi(x), \partial\phi(x)], \implies \text{Euler-Lagrange: } \frac{\partial \mathcal{L}}{\partial \phi} - \partial \frac{\partial \mathcal{L}}{\partial(\partial\phi)} = 0$$

The *path integral*:

$$\mathcal{Z} = \int \mathcal{D}\phi \; e^{-\frac{i}{\hbar} S[\phi]}, \quad \mathcal{D}\phi \text{``} = \prod_x \int \mathrm{d}\phi(x)\text{''} \quad \text{(makes the ``quantum'' in QFT)}$$

The *correlation functions*:

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle = \int \mathcal{D}\phi \; e^{-\frac{i}{\hbar} S[\phi]} \mathcal{O}(x_1) \cdots \mathcal{O}(x_n)$$

The *transition amplitudes*: $A \sim | \langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle |^2$!

# But what *is* quantum field theory?

Example: Standard model

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\overline{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \\
\partial_\mu\bar{X}^+X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^-X^0 - \\
\partial_\mu\bar{X}^0X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+X^+ - \\
\partial_\mu\bar{X}^-X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+X^+ - \\
\partial_\mu\bar{X}^-X^-) - \tfrac{1}{2}gM\left(\bar{X}^+X^+H + \bar{X}^-X^-H + \tfrac{1}{c_w^2}\bar{X}^0X^0H\right) + \tfrac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + igMs_w\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0\right) .
\end{gathered}
$$

...easy, right?

# Any problems?

# Any problems?

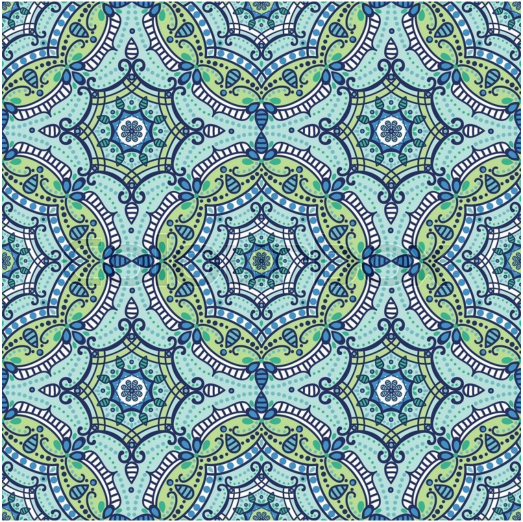
Yes.

- Exact solutions... *usually* impossible.
- Perturbation theory? Just *very hard*! Leads to Feynman diagrams
- (Non-perturbative effects?)
- No gravity!
- Path integral not well-defined: infinite dimensional

$$\mathcal{D}\phi = \prod_x \int \mathrm{d}\phi(x) \sim \int \mathrm{d}^\infty \phi$$

- In fact, *no general, precise definition of QFT*... but still useful!

# Symmetry — what is it? why do we care?



# Symmetry — what is it? why do we care?

A change in description, but *same observables*

- Exists! ... almost
- Makes life much easier
- e.g. in QFT classifies particles
- Tied to beautiful mathematics

# Symmetry — Description?

## Example: rotations

$x \to x' = Rx,$
$S \to S' = \int_{\mathbb{R}^4} \mathrm{d}^4x \, \mathcal{L}[\phi(x'), \partial'\phi(x')] = \int_{\mathbb{R}^4} \mathrm{d}^4x' \, \mathcal{L}[\phi(x'), \partial'\phi(x')] = S$

This is general: symmetries $\Leftrightarrow \delta S = 0$
In QM: symmetries $\longrightarrow$ operators + commutation relations.

## Example: rotations in QM

$[\hat{L}_i, \hat{L}_j] = i\hbar\epsilon_{ijk}\hat{L}_k$

Abstractly: symmetries $\Leftrightarrow$ Lie algebras

## Example: Poincaré algebra

Rotations $\hat{L}_i$ + boosts $\hat{K}_i$ + translations $\hat{P}_0, \hat{P}_i$ = Poincaré algebra

# Supersymmetry

Theory with bosons (even spin) and fermions (odd spin)
$\mathcal{L} = \mathcal{L}[\phi, \psi, \ldots]$.
Poincaré maps $|\text{Boson}\rangle \to |\text{Boson}\rangle$, $|\text{Fermion}\rangle \to |\text{Fermion}\rangle$

# Supersymmetry

Theory with bosons (even spin) and fermions (odd spin)
$\mathcal{L} = \mathcal{L}[\phi, \psi, \ldots]$.
Poincaré maps $|\text{Boson}\rangle \to |\text{Boson}\rangle$, $|\text{Fermion}\rangle \to |\text{Fermion}\rangle$
Extension: *Supersymmetry* Need Lie *super*-algebras!

$$\{Q, \bar{Q}\} = P$$

$Q, \bar{Q}$ fermionic, $Q^2 = 0 = \bar{Q}^2$.

$$Q\,|\text{Boson}\rangle = |\text{Fermion}\rangle\,, \quad Q\,|\text{Fermion}\rangle = |\text{Boson}\rangle$$

Quite natural from theoretical perspective!

# Supersymmetry — Why?

- Super rigid
- #bosons=#fermions
- Cancellations in perturbation theory

# Supersymmetry — Why?

- Super rigid
- #bosons=#fermions
- Cancellations in perturbation theory
- *Exact results*

# Once again, goals:

Recall, want to compute:

$$\langle O \rangle = \int \mathcal{D}\phi \mathcal{D}\psi \; \mathcal{O} e^{-S[\phi,\psi]}$$

This was

- Infinite dimensional
- Exact results impossible for most theories
- Very hard even in perturbation theory

# Can supersymmetry help?

[2] Details in last slide

# Can supersymmetry help?

Yes!

For certain theories $S[\phi, \psi]$ and certain observables $\mathcal{O}$:

1. The infinite integral *"localizes"* to a finite dimensional integral[2]
2. The result is exact!

This is highly non-trivial; small glimpse of a full QFT...

[2] Details in last slide

# This is *active* research



## On supersymmetric Wilson loops in two dimensions

**Rodolfo Panerai,**[a] **Matteo Poggi,**[b,c] **and Domenico Seminara**[d]

[a] *Centre for Research in String Theory, School of Physics and Astronomy, Queen Mary University of London, Mile End Road, E1 4NS London, United Kingdom*

[b] *International School of Advanced Studies (SISSA) and INFN Sezione di Trieste, via Bonomea 265, 34136 Trieste, Italy*

[c] *School of Physics, Korea Institute for Advanced Study (KIAS), 85 Hoegiro, Dongdaemun-Gu, Seoul 02455, Republic of Korea*

[d] *Dipartimento di Fisica, Università di Firenze and INFN Sezione di Firenze, via G. Sansone 1, 50019 Sesto Fiorentino, Italy*

*E-mail:* r.panerai@qmul.ac.uk, mpoggi@kias.re.kr, seminara@fi.infn.it

Abstract: We classify bosonic $\mathcal{N} = (2, 2)$ supersymmetric Wilson loops on arbitrary backgrounds with vector-like R-symmetry. These can be defined on any smooth contour and come in two forms which are universal across all backgrounds, one annihilated by a left-moving supercharge and one annihilated by a right-moving supercharge. We show that these Wilson loops, thanks to their cohomological properties, are all invariant under smooth deformations of their contour. At genus zero they can be always mapped to local operators and computed exactly with supersymmetric localisation.

# This is *active* research

**On supersymmetric Wilson loops in two dimensions**

## Supersymmetric localization in two dimensions

**Francesco Benini[1,2] and Bruno Le Floch[3]**

[1] *International School for Advanced Studies (SISSA), via Bonomea 265, 34136 Trieste, Italy*
[2] *Blackett Laboratory, Imperial College London, London SW7 2AZ, United Kingdom*
[3] *Princeton Center for Theoretical Science, Princeton University, Princeton, NJ 08544, USA*
`fbenini@sissa.it`, `blefloch@princeton.edu`



**Abstract**

This is an introductory review to localization techniques in supersymmetric two-dimensional gauge theories. In particular we describe how to construct Lagrangians of $\mathcal{N}=(2,2)$ theories on curved spaces, and how to compute their partition functions and certain correlators on the sphere, the hemisphere and other curved backgrounds. We also describe how to evaluate the partition function of $\mathcal{N}=(0,2)$ theories on the torus, known as the elliptic genus. Finally we summarize some of the applications, in particular to probe mirror symmetry and other non-perturbative dualities.

This is a contribution to the review volume "Localization techniques in quantum field theories" (eds. V. Pestun and M. Zabzine) which contains 17 Chapters available at [1]

# This is *active* research

**On supersymmetric Wilson loo**

## Supersymmetric localization

**Francesco Benini[1,2] and Bru**

[1] *International School for Advanced Studies (SISSA), v*
[2] *Blackett Laboratory, Imperial College London, Lo*
[3] *Princeton Center for Theoretical Science, Princeton U*
fbenini@sissa.it, blefloch@p

2955v4 [hep-th] 15 Oct 2016

arX

**Abstract**

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This is a contribution to the review volume "Loca theories" (eds. V. Pestun and M. Zabzine) which cont

## Janus interface in two-dimensional supersymmetric gauge theories

Kanato Goto and Takuya Okuda

*University of Tokyo, Komaba, Tokyo 153-8902, Japan*

v:1810.03247v1 [hep-th] 8 Oct 2018

**Abstract**

We study the Janus interface, a domain wall characterized by spatially varying couplings, in two-dimensional $\mathcal{N} = (2,2)$ supersymmetric gauge theories on the two-sphere. When the variations of the couplings are small enough, SUSY localization in the Janus background gives an analytic continuation of the sphere partition function. This directly demonstrates that the interface entropy is proportional to the quantity known as Calabi's diastasis, as originally shown by Bachas et.al. When the variations are not small, we propose that an analytic continuation of the sphere partition function coincides with the Janus partition function. We give a prescription for performing such analytic continuation and computing monodromies. We also point out that the Janus partition function for the equivariant A-twist is precisely the generating function of A-model correlation functions.

# This is *active* research

arXiv:1812.06473v1 [hep-th] 16 Dec 2018

## Twisting with a Flip (the Art of Pestunization)

**Guido Festuccia[a], Jian Qiu[a,b], Jacob Winding[c], Maxim Zabzine[a]**

[a] *Department of Physics and Astronomy, Uppsala University, Box 516, SE-75120 Uppsala, Sweden*

[b] *Mathematics Institute, Uppsala University, Box 480, SE-75106 Uppsala, Sweden*

[c] *School of Physics, Korea Institute for Advanced Study, Seoul 130-722, Korea*

**Abstract**

We construct $\mathcal{N}=2$ supersymmetric Yang-Mills theory on 4D manifolds with a Killing vector field with isolated fixed points. It turns out that for every fixed point one can allocate either instanton or anti-instanton contributions to the partition function, and that this is compatible with supersymmetry. The equivariant Donaldson-Witten theory is a special case of our construction. We present a unified treatment of Pestun's calculation on $S^4$ and equivariant Donaldson-Witten theory by generalizing the notion of self-duality on manifolds with a vector field. We conjecture the full partition function for a $\mathcal{N}=2$ theory on any 4D manifold with a Killing vector. Using this new notion of self-duality to localize a supersymmetric theory is what we call "Pestunization".

2955v4 [hep-th] 15 Oct 2016

...e in two-dimensional
...etric gauge theories

...to and Takuya Okuda

... Komaba, Tokyo 153-8902, Japan

**Abstract**

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# Summary

- QFT is the modern language of theoretical physics;
- It's exceptionally hard; $\mathcal{Z}, \langle \mathcal{O} \rangle$ are impossible or very hard to compute
- Symmetries help simplify problems; supersymmetry is another symmetry
- For certain SUSY theories and certain observables, we can compute results *exactly*
- This has been enormously successful in the literature

Thank you

# Some details on localization

If

1. $Q \cdot S[\phi, \psi] = 0$ "theory is supersymmetric",
2. $Q \cdot \mathcal{O} = 0$ "$\mathcal{O}$ is $Q$-closed"

then, for any fermionic $V$:

$$\mathcal{Z} = \int \mathcal{D}\phi\mathcal{D}\psi\ \mathcal{O}e^{-S[\phi,\psi]} = \int \mathcal{D}\phi\mathcal{D}\psi\ \mathcal{O}e^{-S[\phi,\psi]-tQ\int V}$$

$t$ parameter, $V[\ldots]$ *any* fermionic functional.

**Proof:** Fact: $Q$ is a differential operator:

$Q(AB) = (QA)B + (-1)^{|A|}A(QB)$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int \mathcal{D}\phi\mathcal{D}\psi\ \mathcal{O}e^{-S[\phi,\psi]-tQ\int V} &= \int \mathcal{D}\phi\mathcal{D}\psi\ \mathcal{O}(Q\int V)e^{-S[\phi,\psi]-tQ\int V} \\ &= \int \mathcal{D}\phi\mathcal{D}\psi\ Q\left[\mathcal{O}\int V e^{-S[\phi,\psi]-tQ\int V}\right] \\ &= 0\end{aligned}$$

# Some details on localization

So,

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\phi \mathcal{D}\psi \; \mathcal{O} e^{-S[\phi,\psi]} = \int \mathcal{D}\phi \mathcal{D}\psi \; \mathcal{O} e^{-S[\phi,\psi] - tQ\int V}$$

and for $t \to \infty$:

$$\lim_{t\to\infty} \langle \mathcal{O} \rangle = \int \mathcal{D}\phi \mathcal{D}\psi \; \mathcal{O} e^{-S[\phi,\psi] - tQ\int V} = \int_{\{QV=0\}} [\text{stuff we can compute}]$$

Integral localizes on $\{QV = 0\}$!