Quantum integrability: Perturbations and quasi-integrals

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What it the aim of theoretical physics?

- To find out the underlying fundamental physical laws
- To describe phenomena based on the fundamental laws and to make classifications; new phenomena can be predicted theoretically
- To identify the limitations of a law (if we assume that it is not the absolute law of Nature); finding disagreement between theory and experiment

Understanding=Solvability

- For the last two points, central point is about solving non-trivial cases; the problem boils down to solving a (set of) differential equation(s)
- Numerical solutions can be extremely helpful; there are drawbacks: limitations of a method, limited computational power and memory
- Finding examples of exactly solvable non-trivial models: Exceptional cases;

→ **Integrability**

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Integrability

- A topic that is studied by physicsts and mathematicians
- A set of techniques and methods of treating certain special problems whose solutions we can sketch of

Exact solvability means the reduction of the initial problem into a simpler form. For example, instead of a differential equation, we have to solve an integral or a set of algebraic equations

Integrability in classical mechanics

- Hamiltonian system: 2m-dimensional phase space $\mathcal{M}$, a Hamiltonian function $\mathcal{H}$. **Liouville integrable**:

i. Phase space differentiable everywhere

ii. m independent integrals of motion $\{\mathcal{Q}_k,\mathcal{H}\}=0$

iii. In involution: $\{\mathcal{Q}_k,\mathcal{Q}_m\}=0$

- The e.o.m can be solved by quadratures
- If the level set is compact and connected, it is diffeomorphic to a T^m: Invariant tori

From mechanical systems to field theories

- With the previous, it is not possible to specify the integrals, let alone to deal with a field theory, i.e. m $\rightarrow \infty$
- If you can find two matrices L,M (Lax pair) that depend on phase space variables:

$$\frac{dL}{dt} = [M, L] \longrightarrow F_k = Tr(L^k)$$

- Based on this, generalization on (1+1)-field theories is possible. The solution of the problem can be taken by Inverse Scattering Method (ISM)

Quantum integrability

- Although the obvious upgrading {,}→[,] to create something quantum, this is not enough for a definition; Scalar VS Operator, Phase Space VS Hilbert Space
- By and large, not a unanimous definition for quantum integrability; One to rule them all
- However, we can identify non-trivial quantum models that are exactly solvable and with many independent integrals of motion. →1D field theories, lattices..

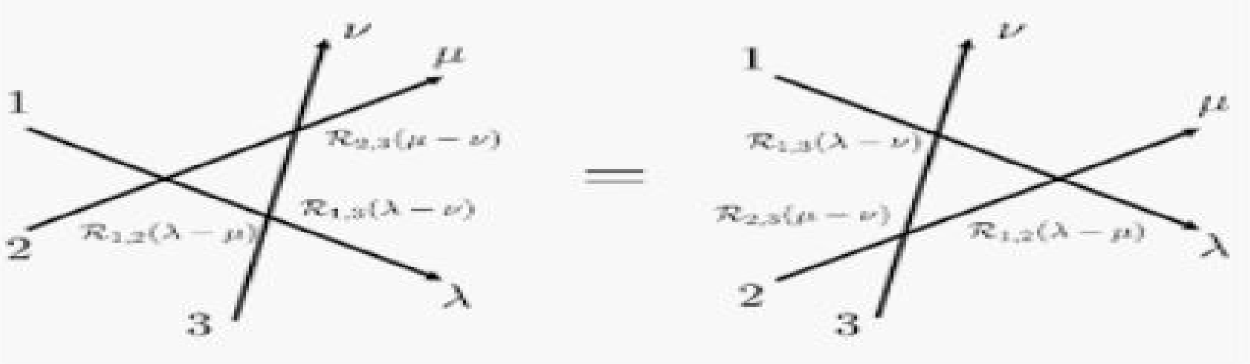
Methods for quantum integrable models

- Bethe Ansatz:

A general way for writing the eigenstates. After that: solve algebraic equations. For non-integrable, equations without solution

- Quantum ISM, Algebraic Bethe Ansatz

Identify matrices with specific properties that can generate the “integrability conditions”.



Phenomenology of (non) integrable models

- The fundamental phenomenological difference between an integrable and a generic system, is that the latter is chaotic and ergodic. At a quantum level, this is translated in thermalization (for generic systems) for long time scales
- Another criterion that are believed to classify an integrable from a non-integrable: level spacing

The question

- Integrable models are single points in the parameter space of all the possible models.
- Based on the previous discussion, we understand that the behaviour between the two kinds of systems is extremely different
- What is the behaviour of a nearly integrable system $\mathcal{H}=\mathcal{H}_{int}+\varepsilon h$? Does it thermalize as a generic system or is there some “integrability remnant”?

The answer in classical physics

- In the first half of 20th century it was believed that even a tiny bit addition in the $\mathcal{H}_{int}$ ruins integrability: Ergodic hypothesis.
- Kolmogorov, Arnold and Moser proved that this was not the case. The majority of the invariant tori are just slightly deformed ($\mathcal{O}(\varepsilon^{1/2})$ are only lost) (KAM theorem).
- Moreover, Nekhoroshev estimates ensures that ALL initial conditions are sufficiently near to the integrable ones for exponentially long times

A quantum KAM???

- It is more than obvious to extend it to the quantum realm
- However, the derivation does not seem easy at all:

i. The KAM theorem derived by using phase space arguments

ii. Fundemental differences of quantum and classical theories

Phenomenology VS qKAM

- The derivation of a quantum KAM seems difficult (maybe impossible)
- However, studies on weakly perturbed integrable quantum models show that the system doesn't thermalize immediately; for intermediate times it relaxes to a different phase, the so-called **pre-thermalization**
- Thus there must be something that prevents immediate thermalization

Slow operators

- Given that the conservation laws give the late time behaviour, we may assume the existence of slow operators, i.e. operators that are quasi-conserved
- The unperturbed system has a set of {Qn} whose time derivative will be proportional to the perturbation strength
- Based on these, can we modify them properly and their commutativity with Hamiltonian be smaller?

Model

- We begin with the XXX model+NNN

$$H = \sum_{j \in \Lambda} \left(\vec{\sigma}_j \cdot \vec{\sigma}_{j+1} + \lambda \vec{\sigma}_j \cdot \vec{\sigma}_{j+2} \right)$$

- Using the symmetries, we may narrow down the possible corrections (up to a maximum support) weighted by some scalar quantities that are arbitrary
- By minimizing the commutator, we may fix them
- Is the final result a proper quasi-integral?

The results

- For the first non-trivial integral, it is possible to add corrections so that it commutes with Hamiltonian proportional to λ^2
- For the second one, the commutativity can be improved by some power a, $1<a<2$
- For all next integrals, the power remains essentially 1

Conclusion

- A formal derivation of non-ergodicity theorem for nearly integrable quantum models might be impossible
- Through indirect studies on particular quantum systems, we see that there exists some non-trivial memory of integrability in weakly perturbed systems
- This behaviour can be attributed to quasi-integrals

THANKS FOR YOUR ATTENTION!