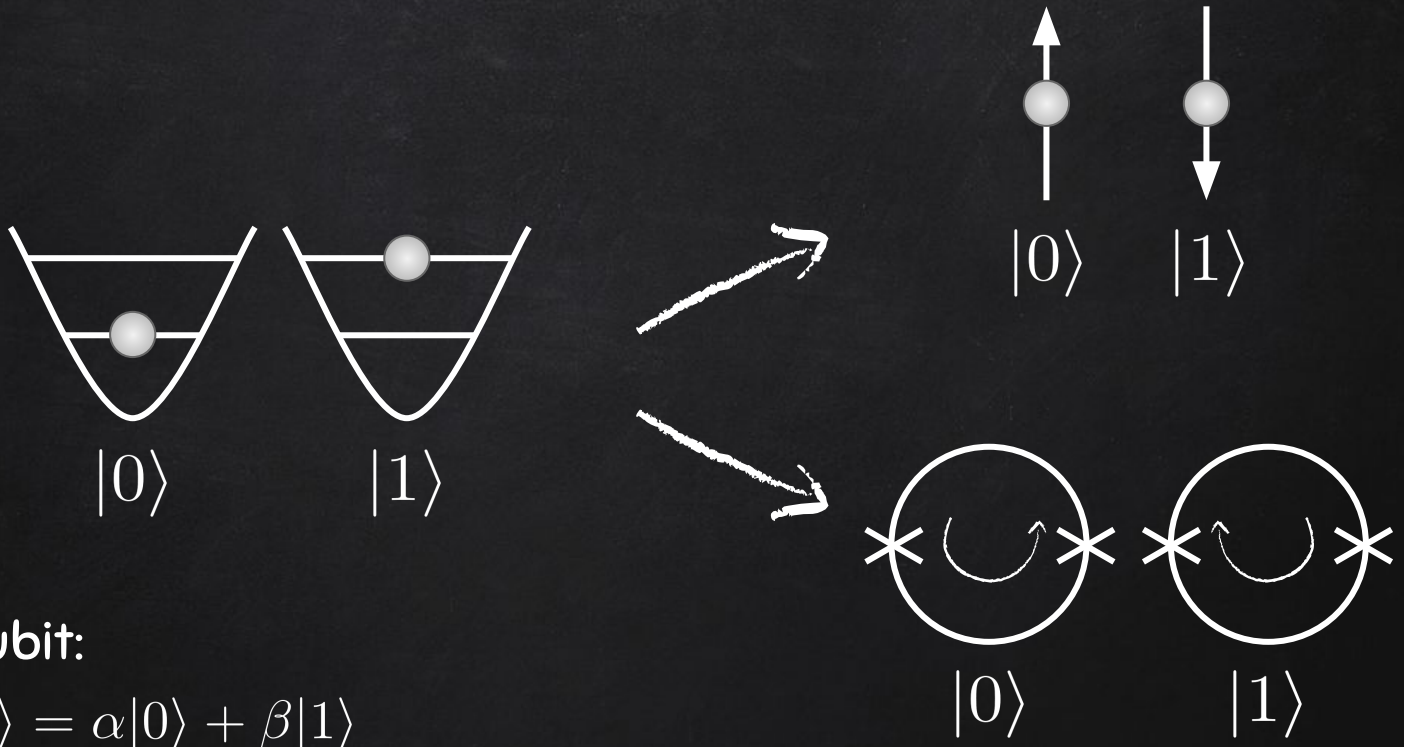
Many-Body Localization Phase for Decoherence Protected Quantum memory

Evangelos Varvelis

Supervisor: David DiVincenzo

Institute for Quantum Information, RWTH Aachen

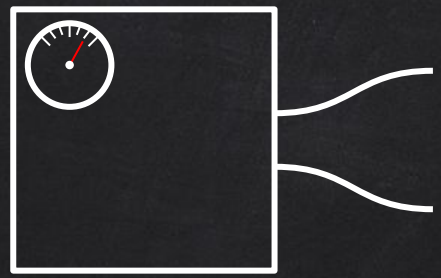
Quantum Information



Qubit:

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Pure VS Mixed States



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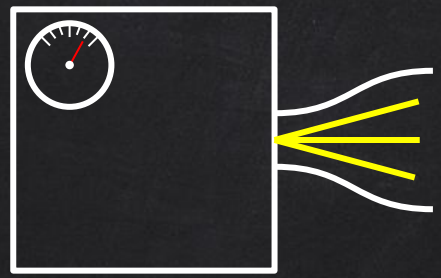
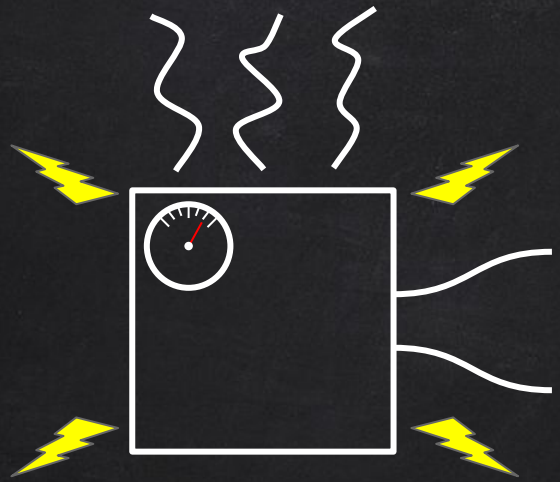


$|0\rangle$ 100% of the times

Pure VS Mixed States



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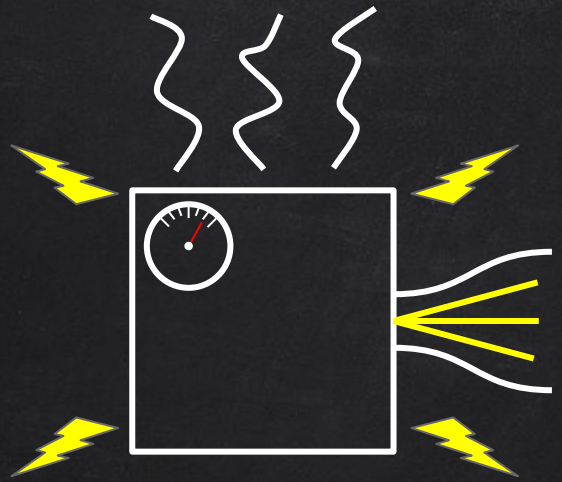
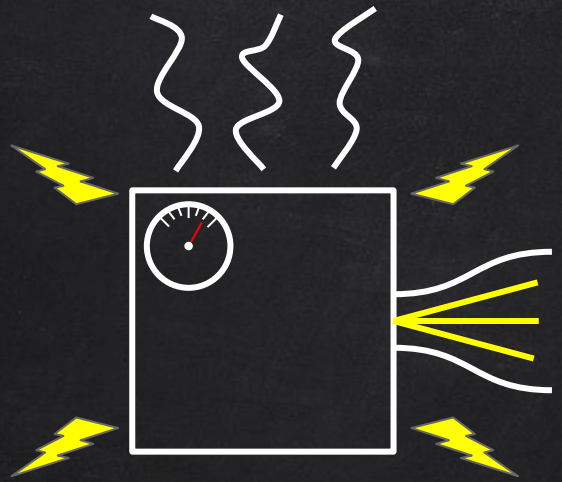


$|0\rangle$ 50% of the times

$|1\rangle$ 50% of the times

Pure VS Mixed States

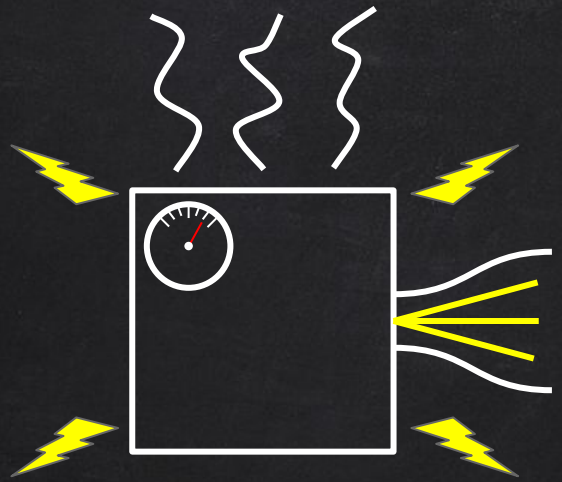


$|0\rangle$ 50% of the times

$|1\rangle$ 50% of the times

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} ???$$

Pure VS Mixed States



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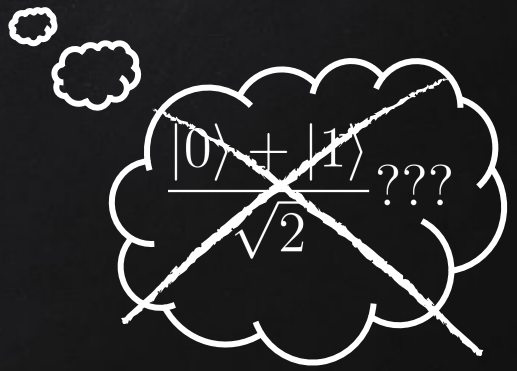

Pure VS Mixed States

Classical Probabilities

- ✘ 50% of the times $|0\rangle$
- ✘ 50% of the times $|1\rangle$

Quantum Probabilities

- ✘ 50% of the times $|0\rangle$
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Pure VS Mixed States

Classical Probabilities

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- ✘ 50% of the times $|1\rangle$
- ✘ Energy on average:

$$\langle H \rangle = \frac{1}{2}\langle 0|H|0\rangle + \frac{1}{2}\langle 1|H|1\rangle$$

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Pure VS Mixed States

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Pure VS Mixed States

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Interference

$$+\frac{1}{2}\langle 0|H|1\rangle + \frac{1}{2}\langle 1|H|0\rangle$$

Density operator

- $|\psi\rangle \rightarrow \rho = |\psi\rangle\langle\psi|$

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- Business as usual: $\langle\psi|\mathcal{O}|\psi\rangle = \mathrm{Tr}\left(\langle\psi|\mathcal{O}|\psi\rangle\right) = \mathrm{Tr}\left(|\psi\rangle\langle\psi|\mathcal{O}\right) = \mathrm{Tr}\left(\rho\mathcal{O}\right)$

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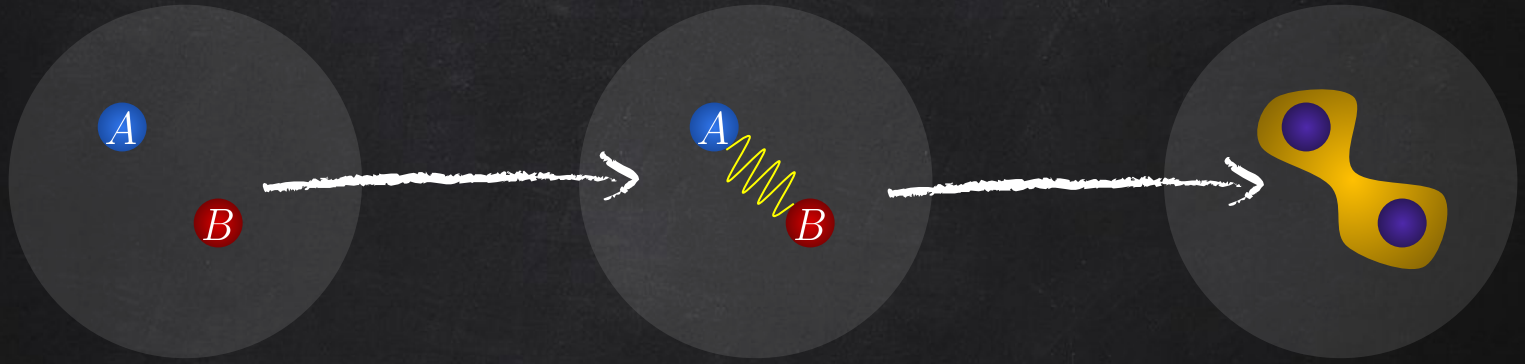
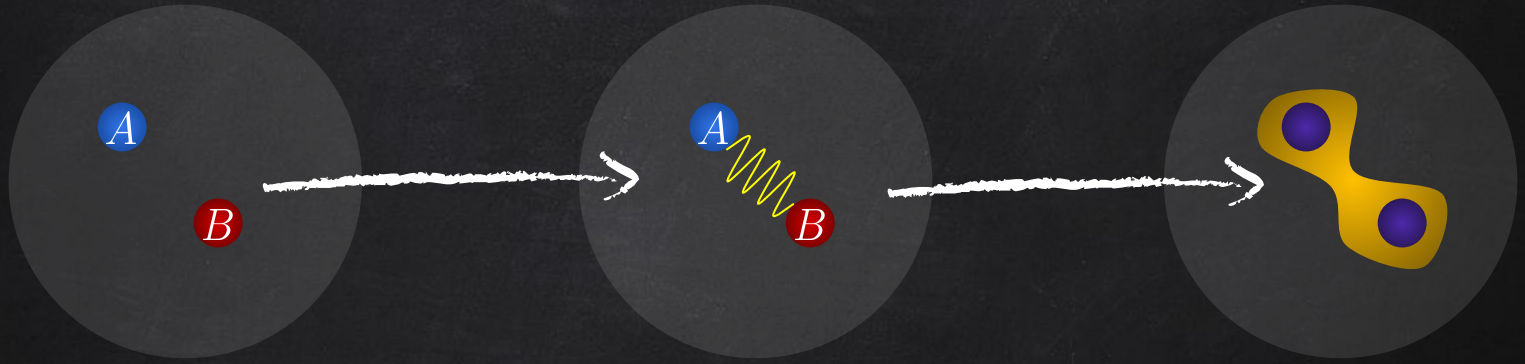
- ✘ Partial trace: $|\psi_1\rangle \otimes |\psi_2\rangle \to \mathrm{Tr}_2\rho = |\psi_1\rangle\langle\psi_1| = \rho_1$
- ✘ Distinguish pure $\mathrm{Tr}\rho^2 = \mathrm{Tr}\rho = 1$ from mixed states $\mathrm{Tr}\rho^2 < \mathrm{Tr}\rho = 1$

Entanglement



$\rho_A \otimes \rho_B$ Interaction ρ_{pair}

Entanglement

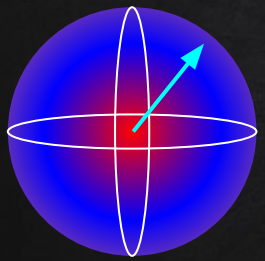


$$\rho_1 = \mathrm{Tr}_2 \rho_{\text{pair}} \qquad \rho_2 = \mathrm{Tr}_1 \rho_{\text{pair}}$$

$$\mathrm{Tr}\rho_1^2 < 1 \qquad \mathrm{Tr}\rho_2^2 < 1$$

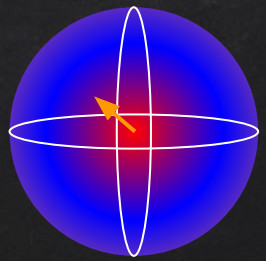
A Measure of Entanglement

$$|\vec{n}| = 1 \qquad \rho = \frac{I + \vec{n} \cdot \vec{\sigma}}{2} \qquad |\vec{n}| < 1$$



Pure

(On surface)



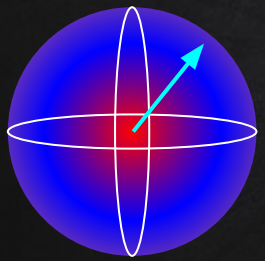
Mixed

(Inner Sphere)

A Measure of Entanglement

Entanglement Entropy: $S = -\mathrm{Tr}\left(\rho_A \log_2 \rho_A\right) = -\mathrm{Tr}\left(\rho_B \log_2 \rho_B\right)$

$|\vec{n}| = 1$ $\rho = \frac{I + \vec{n} \cdot \vec{\sigma}}{2}$ $|\vec{n}| < 1$



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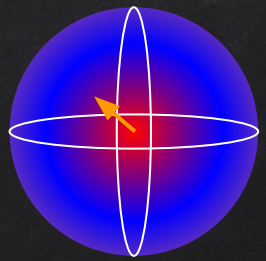

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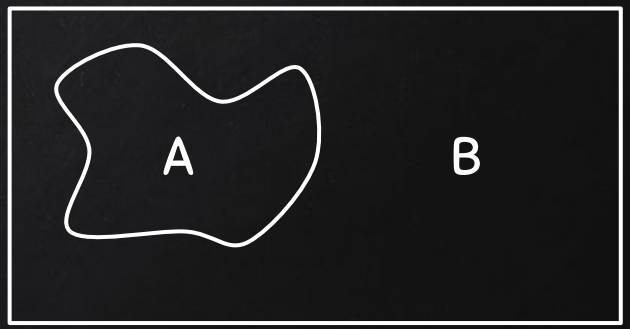
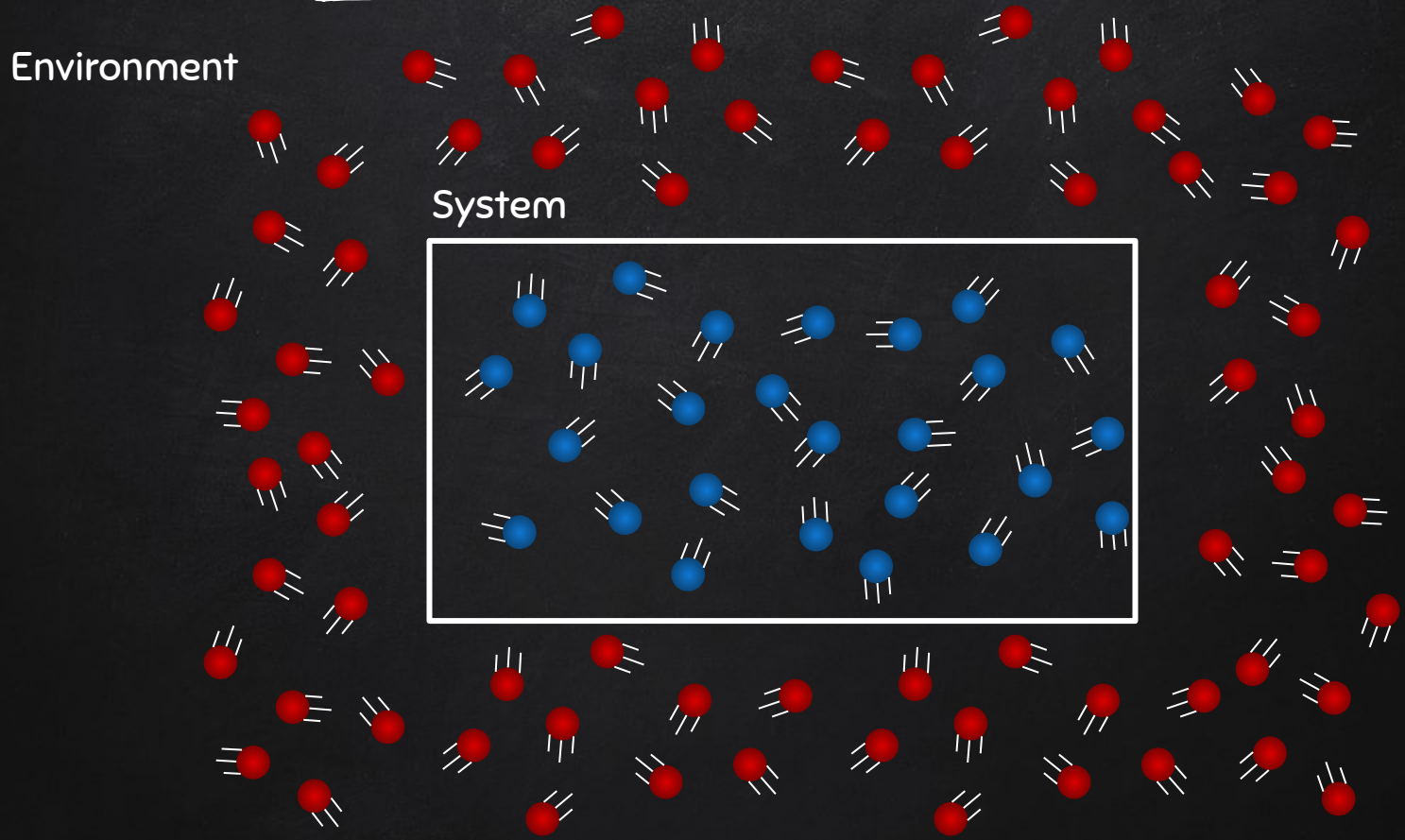


The Enemy: Decoherence



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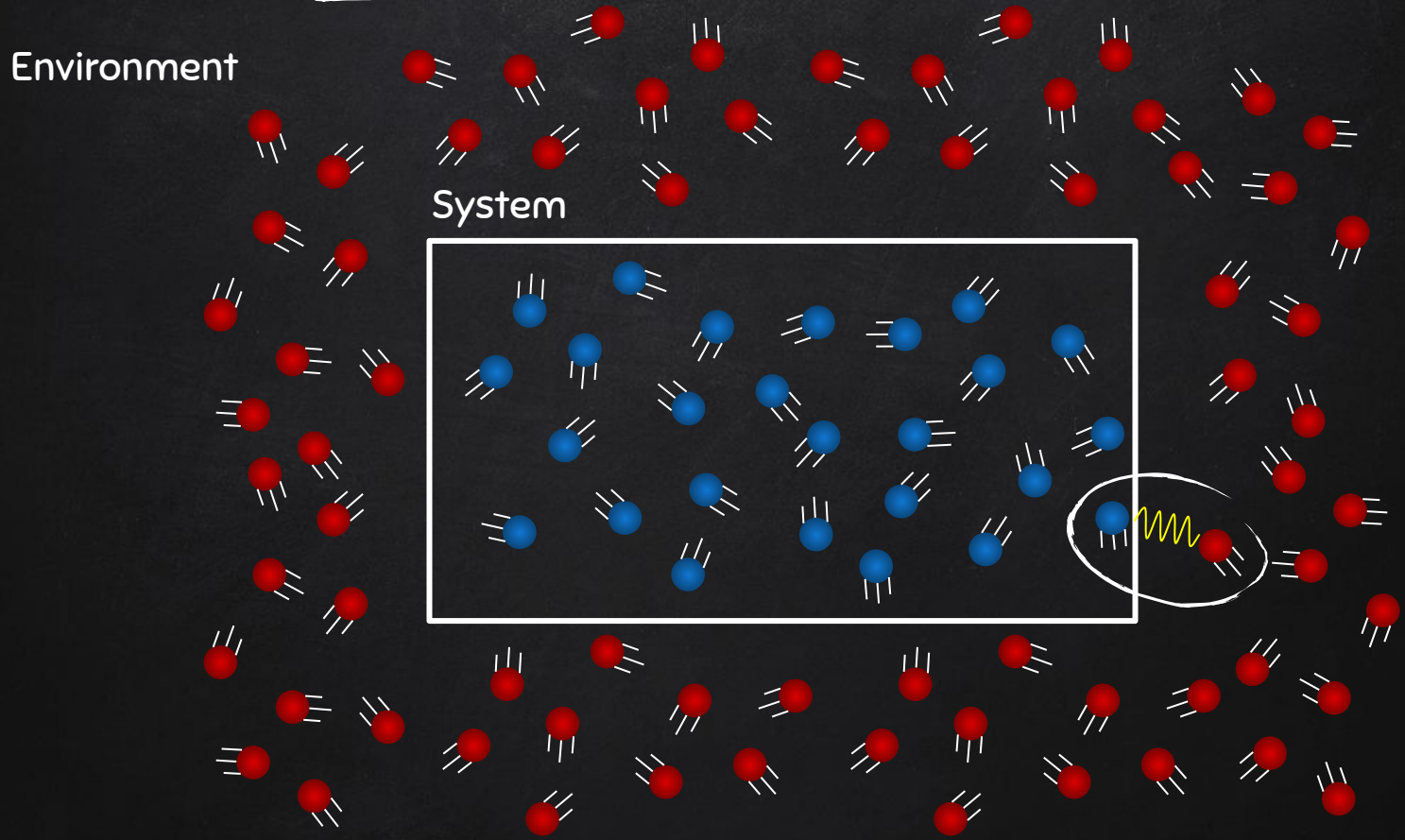

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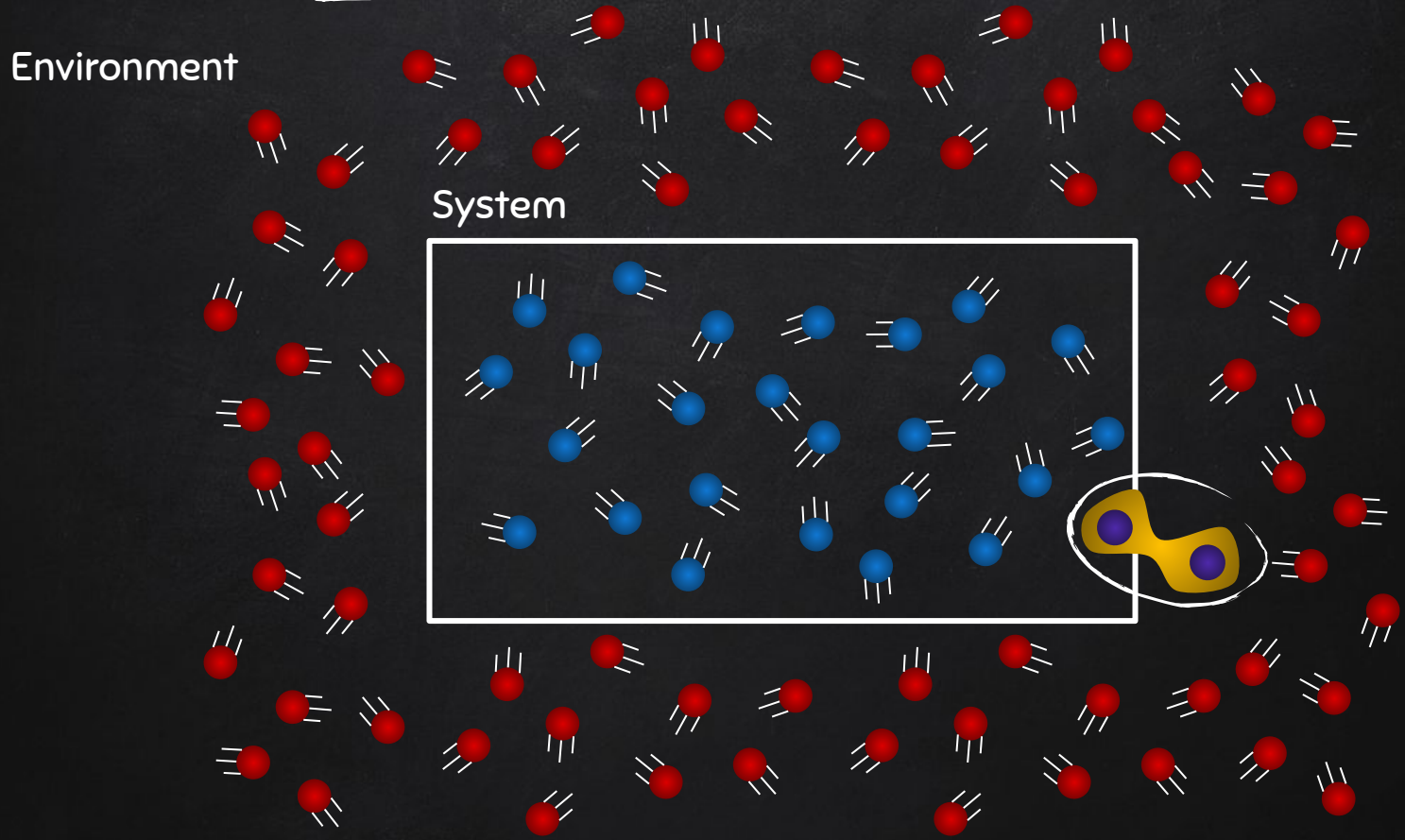

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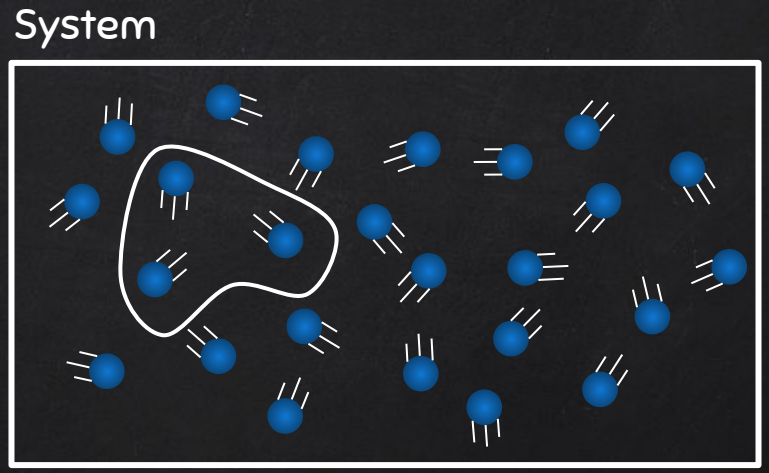

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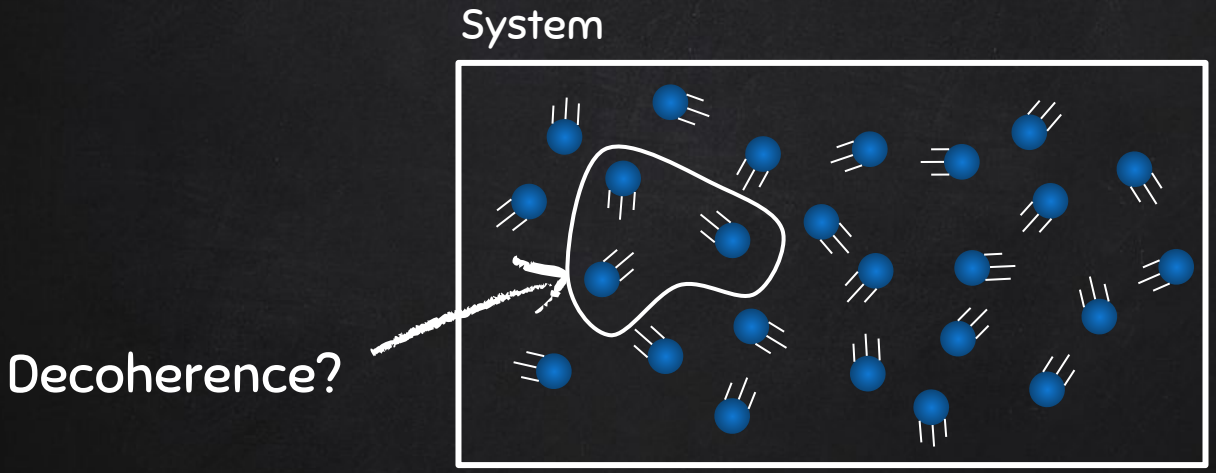

Ergodic Phase

Ergodicity Hypothesis: *"Over a long period of time, all microstates of the system are accessed with equal probability"*

$$\langle \mathcal{O}_A \rangle_\infty = \lim_{t\to\infty} \lim_{L\to\infty} \frac{1}{t} \int_0^t d\tau \mathrm{Tr}\left(\rho_A(t)\mathcal{O}_A\right) = \sum_{n\in \mathrm{Eig}(A)} p_n \mathrm{Tr}\left(\rho_n \mathcal{O}_A\right) = \sum_{n\in \mathrm{Eig}(A)} \frac{p_n}{\mathcal{Z}} \mathrm{Tr}\left(e^{-\beta H}\mathcal{O}_A\right)$$

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Thermal System: *"Different parts of the ergodic system exchange energy and particles"* → Conducting

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Thermal System: *"Different parts of the ergodic system exchange energy and particles"* ⟶ Conducting

Eigenstate Thermalization Hypothesis: *"If a system thermalizes for any initial configuration then all eigenstates are thermal"*

$$\rho_{\mathrm{eig}}(t) = \rho_{\mathrm{eig}}(0)$$

Ergodic Phase: Phenomenology

- ✘ ETH Ansatz: $\langle n|\mathcal{O}|m\rangle = \mathcal{O}_{mc}(\bar{E})\delta_{nm} + e^{-S_{th}(\bar{E})/2} R_{nm} f(\bar{E}, \omega)$

$\bar{E} = (E_n + E_m)/2$
$\omega = E_n - E_m$

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- ✘ Level Spacing Statistics: $\Delta E \propto \frac{e^{-S_{th}(E)/2}}{\Delta} \propto \frac{e^{-S_{th}(E)/2}}{e^{-S_{th}(E)}} = e^{S_{th}(E)/2} \propto e^{V/2}$

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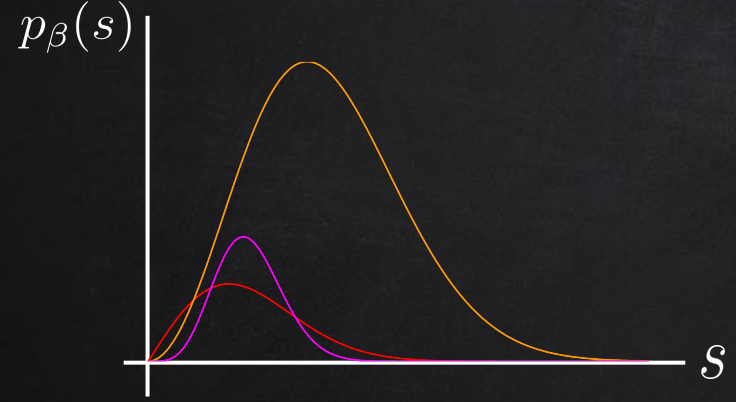

$$p_\beta(s) \propto s^\beta e^{-a_\beta s^2}$$

GOE $\beta = 1$
GUE $\beta = 2$
GSE $\beta = 4$

→ Quantum Chaos

$$\lim_{s_n \to 0} p(s_n) = 0$$

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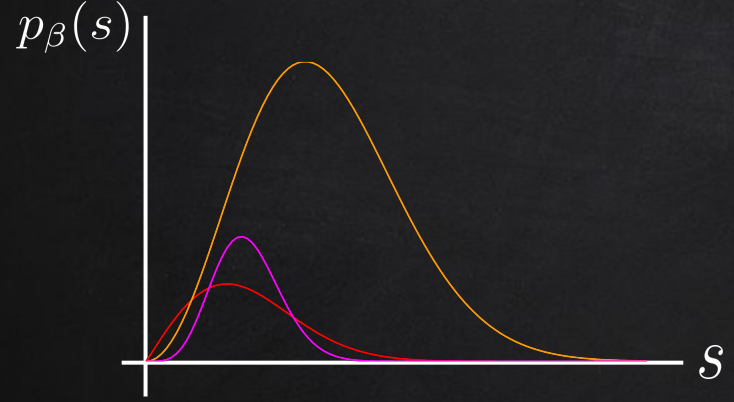

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GOE $\beta = 1$
GUE $\beta = 2$
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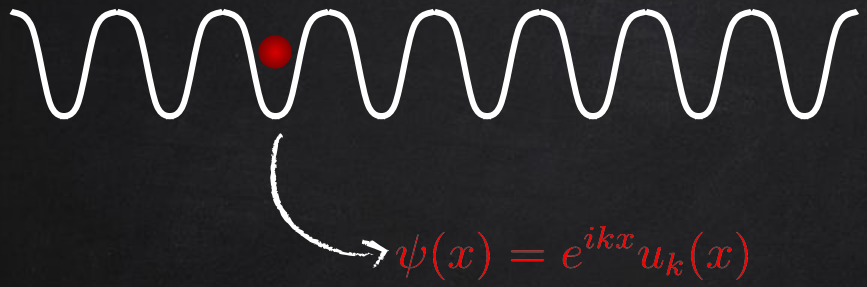
- ✘ Entanglement Volume Law: $S_{\text{ent}}(A) = S_{\text{th}}(A) \propto V_A$
- ✘ Entanglement Ballistic Spread: $S_{\text{ent}}(t) \propto t$

Anderson Localization

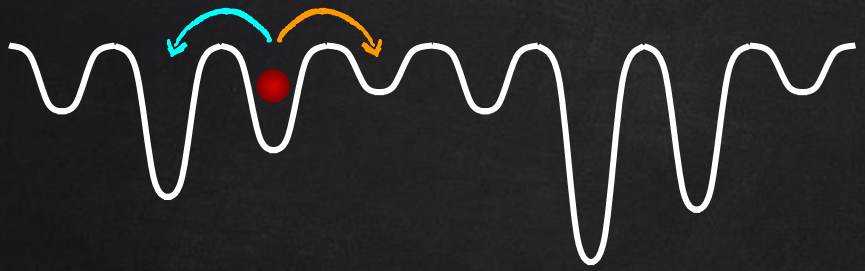
Anderson Localization



Bloch Theorem: Wavefunction extended everywhere

Anderson Localization

Introduce disorder → Periodicity breaks

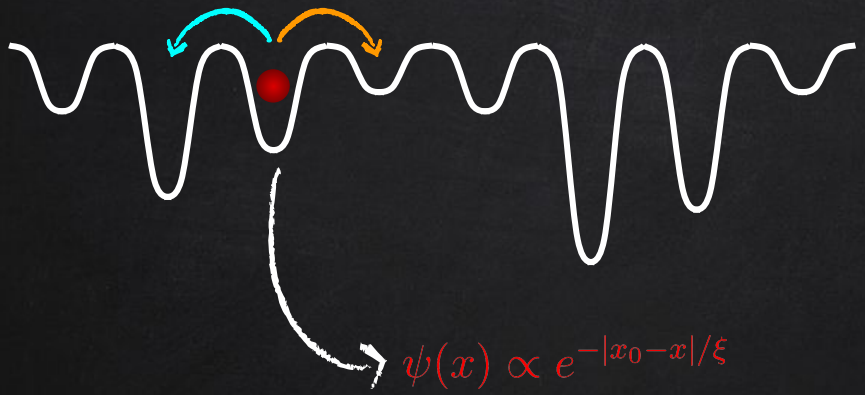


$$H = \sum_{i \in \mathbb{Z}^d} w_i c_i^\dagger c_i + \sum_{i \in \mathbb{Z}^d} t c_i^\dagger c_{i+1} + \sum_{i \in \mathbb{Z}^d} t c_{i+1}^\dagger c_i$$

$$w_i \in \left[-\frac{W}{2}, \frac{W}{2} \right]$$

Anderson Localization

Introduce disorder → Periodicity breaks



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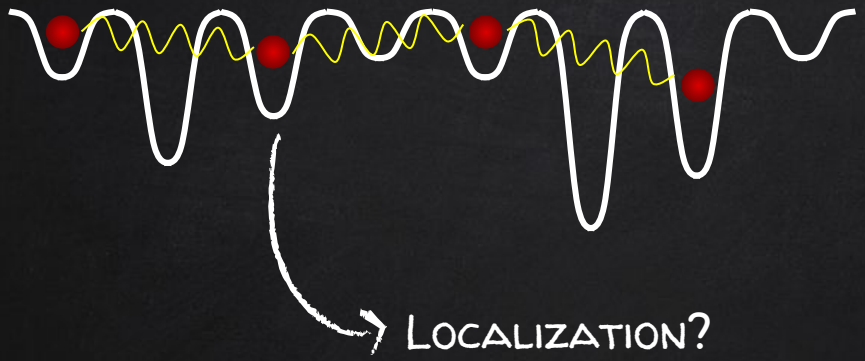
$$w_i \in \left[-\frac{W}{2}, \frac{W}{2}\right]$$

Wavefunction localized around initial state forever!!!

Many-Body Localization

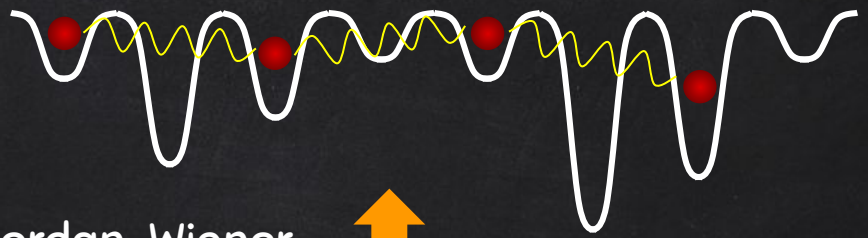


$$H = \sum_{i\in\mathbb{Z}^d} w_i c_i^\dagger c_i + \sum_{i\in\mathbb{Z}^d} t c_i^\dagger c_{i+1} + \sum_{i\in\mathbb{Z}^d} t c_{i+1}^\dagger c_i + \sum_{i\in\mathbb{Z}^d} V\left(c_i^\dagger c_i - \frac{1}{2}\right)\left(c_{i+1}^\dagger c_{i+1} - \frac{1}{2}\right)$$

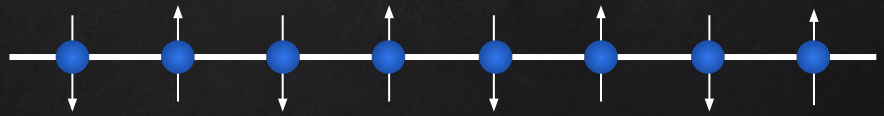
$$w_i \in \left[-\frac{W}{2}, \frac{W}{2}\right]$$

Many-Body Localization

$$H_{SYK} = -\frac{1}{4!}\sum_{i,j,k,\ell} J_{ijk\ell}\psi_i\psi_j\psi_k\psi_\ell$$



Jordan-Wigner Transformation

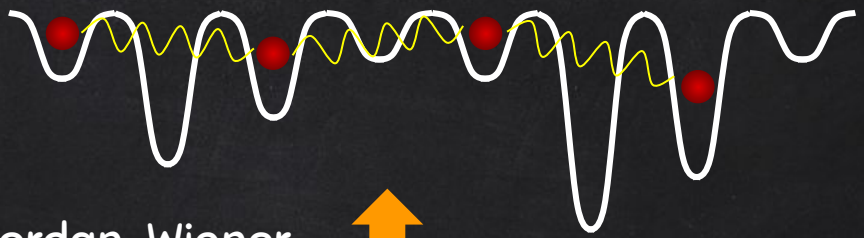


$$H_I = -\sum_i w_i\sigma_i^z - \sum_i \gamma_i\sigma_i^x - \sum_{\langle i,j\rangle} J_{ij}\sigma_i^z\sigma_j^z$$

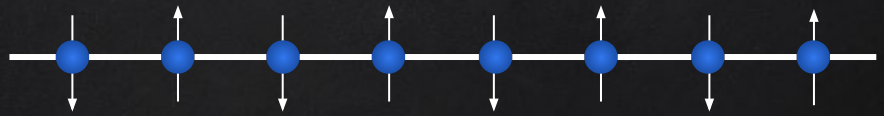
$$H_H = -\sum_i w_i\sigma_i^z - \sum_{\langle i,j\rangle} J_{ij}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y + \Delta_{ij}\sigma_i^z\sigma_j^z\right)$$

Many-Body Localization

$$H_{SYK} = -\frac{1}{4!}\sum_{i,j,k,\ell} J_{ijk\ell}\psi_i\psi_j\psi_k\psi_\ell$$



Jordan-Wigner Transformation



$$H_I = -\sum_i w_i\sigma_i^z - \sum_i \gamma_i\sigma_i^x - \sum_{\langle i,j\rangle} J_{ij}\sigma_i^z\sigma_j^z$$

$$H_H = -\sum_i w_i\sigma_i^z - \sum_{\langle i,j\rangle} J_{ij}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y + \Delta_{ij}\sigma_i^z\sigma_j^z\right)$$

Local Integrals of Motion

$$H_I = -\sum_i w_i \sigma_i^z - \sum_i \gamma_i \sigma_i^x - \sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z$$

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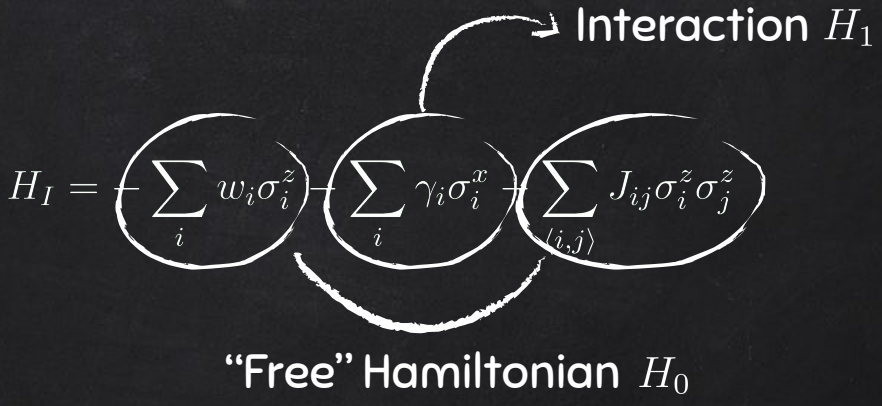
"Free" Hamiltonian H_0

Local Integrals of Motion



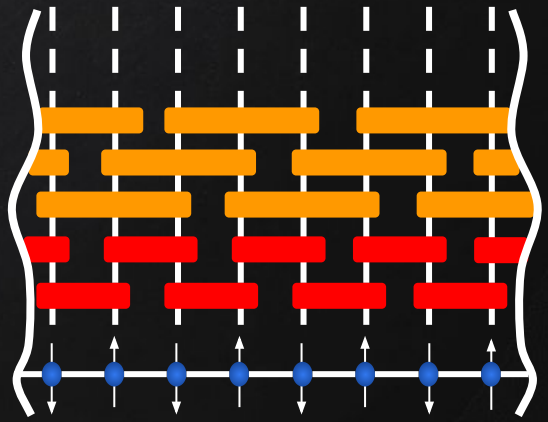
Local Integrals of Motion

Interaction H_1

$$H_I = \sum_i w_i \sigma_i^z + \sum_i \gamma_i \sigma_i^x + \sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z$$

"Free" Hamiltonian H_0

$$U = \prod_i \cdots U^{(3)}_{i,i+1,i+2} U^{(2)}_{i,i+1}$$



Quasilocal Transformation: $U H_0 U^\dagger = H_0 + H_1 = H_I$

$$[\sigma_i^z, H_0] = 0$$

Local Integrals of Motion

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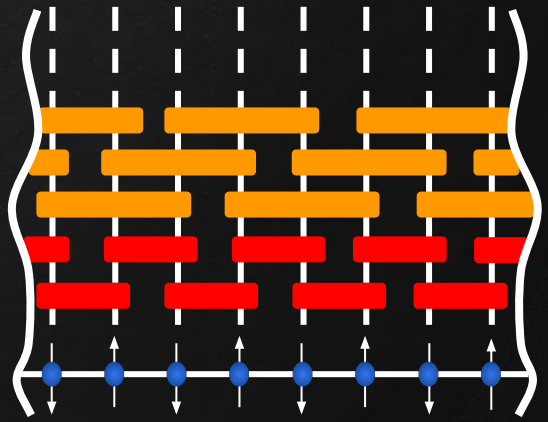

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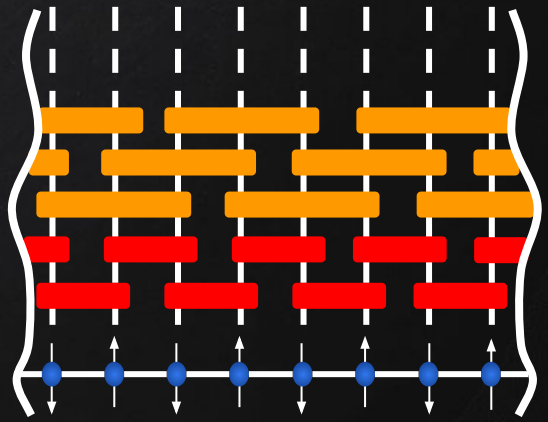

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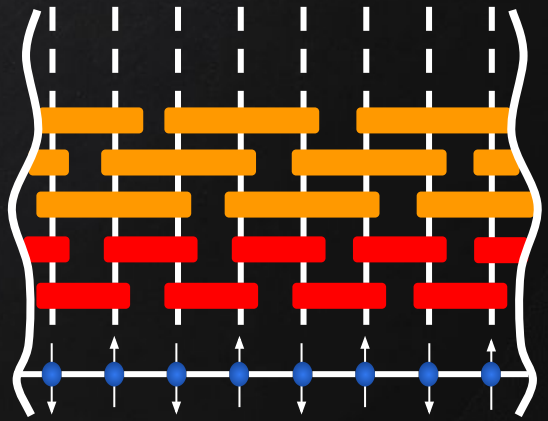

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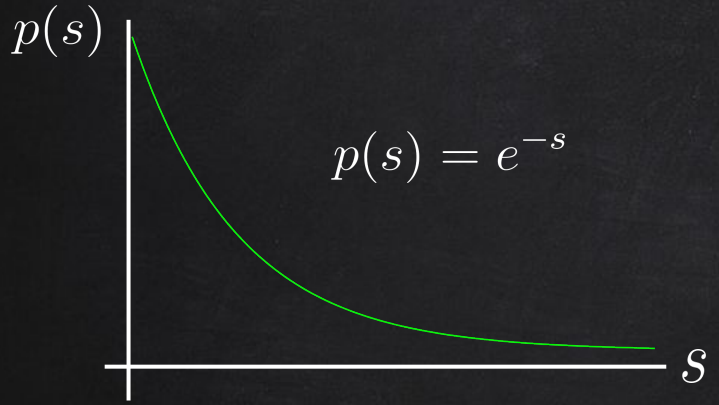
Quantum Integrable

MBL Phase : Phenomenology

- ✘ Level Spacing Statistics: $\lim_{s\to 0} p(s) = 1$



$s_n = E_{n+1} - E_n$

- ✘ Entanglement Area Law: $S_{\text{ent}}(A) \propto \partial A \neq S_{\text{th}}(A)$

- ✘ Entanglement Logarithmic Spread: $S_{\text{ent}}(t) \propto \log(t)$

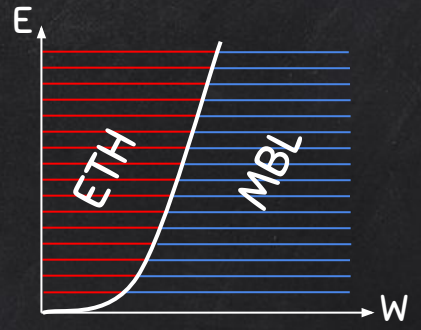
MBL – ETH Transition

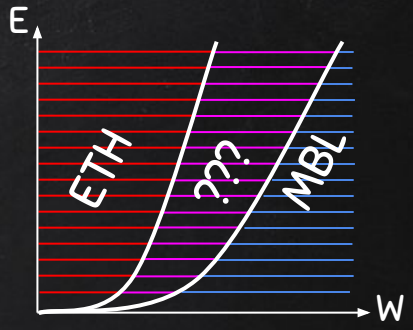
- ✗ Proper Phases of Matter
- ✗ Dynamical Phases
- ✗ Many-Body Eigenstate Transition

MBL – ETH Transition

MBL – ETH Transition

- ✘ MBL in 1D: Proved!
- ✘ MBL in 2D: Maybe, maybe...
- ✘ MBL in 3D: Most likely not...





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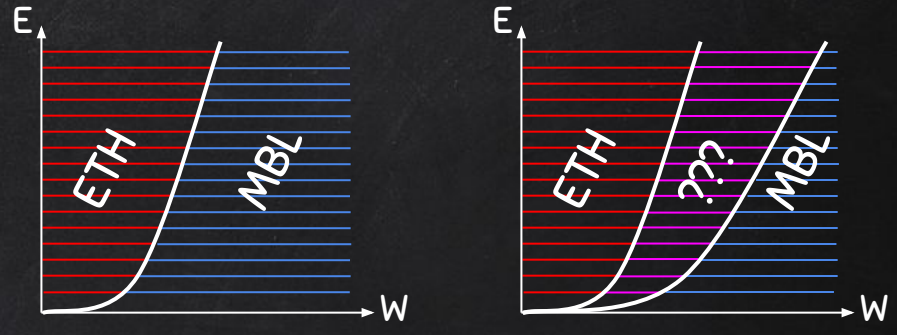

Numerics

- ✘ Plagued by exponentially growing memory requirements with system size
- ✘ Need $L > 50$ to avoid finite system effects on the transition

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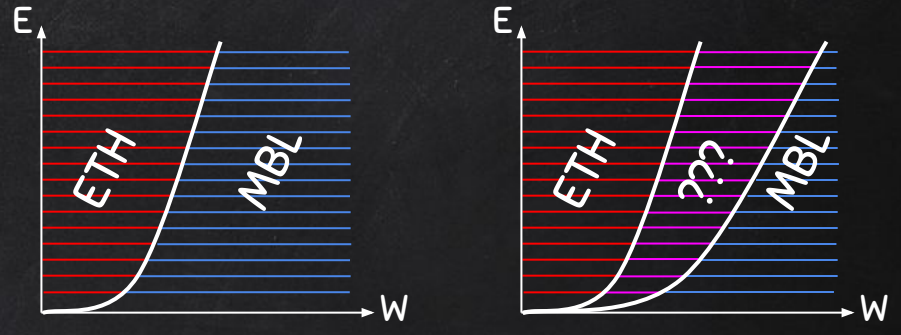

Numerics

- ✘ Plagued by exponentially growing memory requirements with system size
- ✘ Need L > 50 to avoid finite system effects on the transition

Analytics

- ✘ Renormalization Group methods : Non-existent
- ✘ SYK Model: Remarkable analytic results (...but we need Majoranas to make it work)

Future Goals

- ✘ Numerical Simulations with $L > 50$ (DMRG? Tensor Networks?)
- ✘ Analytical methods (RG? SYK?)
- ✘ 2D with $L \times \log_2 L$ (Toric Code)

THANKS!

Any questions?

Credits

Special thanks to all the people who made and released these awesome resources for free:

Presentation template by SlidesCarnival