Massive gravity, bimetric theory and supersymmetry

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Overview

1. **Massive spin–2 fields**
2. **Ghost–free bimetric theory**
3. **Relation to String Theory**

Massless spin–2

1. Minkowski background:
   - action: $\mathcal{L}_{\rm lin} = \frac{1}{2} h^{\mu\nu} \varepsilon_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma}$
   - Lichnerowicz operator:

$$\begin{aligned} \varepsilon_{\mu\nu}{}^{\rho\sigma} \quad = \quad & \tfrac{1}{2}\big(\eta_\mu{}^\rho \eta_\nu{}^\sigma \partial^2 - \eta_\nu{}^\sigma \partial_\mu \partial^\rho - \eta_\mu{}^\sigma \partial_\nu \partial^\rho \\ & + \eta_{\mu\nu} \partial^\sigma \partial^\rho + \eta^{\rho\sigma} \partial_\mu \partial_\nu - \eta_{\mu\nu} \eta^{\rho\sigma} \partial^2 \big) \end{aligned}$$

   - gauge symmetry: $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

2. curved background:
   - action: $S_{\rm GR} = M_{\rm Pl}^2 \int d^4x \sqrt{-g} R$
   - gauge symmetry: diffs

2 propagating d.o.f.

no ghosts

Massive spin–2

1. Minkowski background:
   - action: $\mathcal{L}_{\rm FP} = \frac{1}{2} h^{\mu\nu} \varepsilon_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma} - \frac{m_{\rm FP}^2}{2} (h^{\mu\nu} h_{\mu\nu} - h^2)$

     Fierz, Pauli 1939

   - no gauge symmetry; alternative: Stückelberg

2. curved background:
   - $g^{\mu\nu} g_{\mu\nu}$ is no mass term!
   - action: $S_{\rm dRGT} = M_{\rm Pl}^2 \int d^4x \sqrt{-g} \Big(R + \frac{m^2}{2} \sum_{n=2}^{4} \alpha_n e_n(\mathcal{K}) \Big)$,

     where $\mathcal{K} = 1 - \sqrt{g^{-1} \eta}$

     de Rham, Gabadadze, Tolley 2010

5 propagating d.o.f.

no ghosts; else: Boulware–Deser 1972

Two dynamical metrics

- action:

$$S = m_g^2 \int d^4x \sqrt{g} R(g) + m_f^2 \int d^4x \sqrt{f} R(f)$$
$$- m^4 \int d^4x \sqrt{g} \sum_{n=0}^{n=4} \beta_n e_n(\sqrt{f^{-1} g})$$

Hassan, Rosen, Schmidt-May, von Strauss '11-'12

- elementary symmetric polynomials $e_n(S) \quad , \quad S^2 = g^{-1} f$

$$e_1(S) = \mathrm{Tr}[S] \quad , \quad e_2(S) = \tfrac{1}{2}\big((\mathrm{Tr}[S])^2 - \mathrm{Tr}[S^2]\big)$$
$$e_3(S) = \tfrac{1}{6}\big((\mathrm{Tr}[S])^3 - 3\,\mathrm{Tr}[S^2]\,\mathrm{Tr}[S] + 2\,\mathrm{Tr}[S^3]\big)$$

Spectrum

- massless $\delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}$ and massive $\delta f_{\mu\nu} - \delta g_{\mu\nu}$ eigenstates around proportional backgrounds solution, $\alpha \equiv m_f/m_g$

- the Fierz–Pauli mass of the massive spin–2 mode is

$$m_{\rm FP} = \sqrt{\beta_1 + 2\beta_2 + \beta_3}\, m_{\rm Pl} \quad , \quad m_{\rm Pl}^2 = m_g^2(1+\alpha^2)$$

- Heavy spin–2 dark matter?
  Babichev, Marzola, Raidal, Schmidt–May, Urbani, Veermäe, von Strauss '16

Limits

- What happens when $m_{\rm FP} \to 0$?
  - $\mathcal{L} \supset \kappa\, \phi T^{\mu}_{\mu} \Rightarrow$ massless limit of (linear) massive gravity is *not* linearized GR!

    van Dam, Veltman 1970 // Zakharov 1970

  - solution: nonlinearities

    Vainshtein 1972

- GR limit of bimetric theory:

$$\alpha \to 0 \quad , \quad m_g = \text{const.} \quad , \quad T^{f}_{\mu\nu} = 0$$

Multigravity

- center and chain Hassan-Rosen couplings
- no loops allowed!
- generalization:

$$\mathcal{S}_V = -\sum_{i,j,k,l} \int T_{ijkl}\, \varepsilon_{abcd}\, (E_i)^a \wedge (E_j)^b \wedge (E_k)^c \wedge (E_l)^d$$

Hinterbichler and Rosen '12

- see also CM, Rudolph, Schmidt–May '18

Massive spin–2 in other contexts

- Kaluza–Klein spin–2 modes

- massive spin–2 as a resonance

Dvali, Gabadadze, Porrati 2000

- massless and massive spin–2 from product CFTs

$$\partial_\mu T_1^{\mu\nu} = 0 \quad , \quad \partial_\mu T_2^{\mu\nu} \neq 0$$

Kiritsis '06

Aharony, Clark, Karch '06

Supersymmetry and massive spin–2

- Massive supermultiplets $(2, 3/2, 3/2, 1)$ around Minkowski
  $\rightarrow$ global susy trsfs

  Zinoviev '02, Ondo and Tolley '16

- massive supergravity

  Gregoire, Schwartz, Shamdi '04

- conformal supergravity and its relation with string theory

$$\sqrt{g}\left(m_g^2 R + \frac{a}{2}W^2\right) \quad \rightarrow \quad \int d^2\Theta\, \mathcal{E}\left(2\mathcal{R} + 4\tau\mathcal{W}^2\right)$$

Ferrara, Kehagias, Lüst '18

Ongoing work

- superfield formulation of supersymmetric bimetric theory
  - "standard" sugra techniques don't seem to work
  - explicit breaking to diagonal GCTs, LLTs and local susy trsfs

    CM, Schmidt-May

- spin–2 amplitudes in superstrings vs. bimetric theory amplitudes

  Lüst, CM, Mazloumi, Stieberger