Classical and Quantum Symmetries

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Overview

What is a symmetry

- A symmetry of a physical system is any set of transformations that leave some properties of that system invariant, e.g. reflection symmetry.

Source: wild.maths.org

- The symmetry principle puts constraints on the allowable dynamical laws we use to describe nature.
- Symmetries imply conservation laws. The energy conservation is a consequence of the time translation symmetry.

Classical symmetries

- In a classical theory, the equations that describe the behaviour of a physical system are derived from the action.
- A classical symmetry is defined as any transformation that leaves the action invariant.
- Translation symmetry

$$S = \int dt \ \frac{1}{2} m (\frac{dx(t)}{dt})^2$$

$$x' \to x + \alpha \Rightarrow dx' = dx \Rightarrow S' = S \Rightarrow \delta S = 0$$

$$m\frac{dx}{dt} = \text{constant}$$

Quantum symmetries

- In quantum theories, we determine probabilities for the time evolution of physical systems by computing quantities known as correlation functions

$$|\langle \mathsf{A}||\mathsf{B}\rangle|^2$$

- Very often these computations turn out to be infinite!
- We suitably obtain the quantum observables with a procedure called renormalization.
- A quantum symmetry, is a classical symmetry of the action that is preserved by the renormalization procedure.

Regularization

- Suppose we have an action S_{cl} and we compute an observable quantity E

$$E = \int_a^{\infty} x dx = \infty$$

- Clearly it makes no sense for an observable quantity to be infinite. Can we save somehow our theory?
- We introduce a parameter called regulator which encodes the infinities of our model.

$$E = \lim_{L \to \infty} \int_a^{L} x dx = \lim_{L \to \infty} \left(\frac{L^2}{2} - \frac{a^2}{2} \right)$$

Renormalization

- We modify the theory by 'adding' a new term to the classical action

$$S_{ren} = S_{cl} + S_{ct}$$

and this new term changes/renormalizes the observable quantity E as

$$E_{ren} = \lim_{L\to\infty} (E - \frac{L^2}{2}) = -\frac{a^2}{2}$$

- This is called renormalization in quantum theories. A classical symmetry ($\delta S_{cl} = 0$) in order to survive in the quantum regime must satisfy

$$\delta S_{ren} = 0$$

Anomalies

- Sometimes classical symmetries fail to survive the quantization procedure. We call this phenomenon a quantum anomaly, e.g.

$$\partial_\mu J^\mu_{Acl} = 0, \quad \partial_\mu J^\mu_{Bcl} = 0 \Rightarrow \partial_\mu J^\mu_{Aren} = 0, \quad \partial_\mu J^\mu_{Bren} \neq 0$$

- Since the discovery of the axial anomaly by Adler, Bell and Jackiw in 1969, the anomalies played a central role in theoretical physics.
- Many kinds of classical symmetries were found to be anomalous in certain circumstances, such as translation symmetry, scale symmetry, supersymmetry etc.

- In some cases the existence of (gauge) anomalies lead to inconsistencies, so their cancellation helps us build viable physical models.
- In other cases (global) anomalies are linked to observable effects and explain experimental data, since they allow classically forbidden processes to occur. One such example is the pion decay into two photons.

Outlook

- Classical symmetries are any set of transformations that leave the action invariant.
- A classical symmetry in order to survive in the quantum theory, besides the action has to respect the renormalization procedure. If it fails then we have a quantum anomaly.
- Since their discovery, anomalies have been extremely useful in explaining experimental data and helping us build viable and consistent physical models.

New supersymmetry anomaly

- We consider the free massless Wess-Zumino model

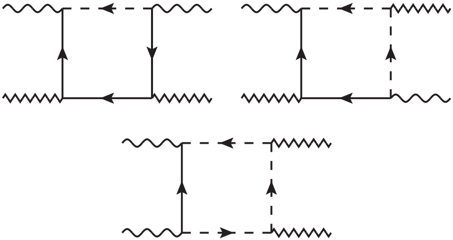
$$S = \int d^4x \left[-\partial_\mu \phi^* \partial^\mu \phi - \frac{1}{2}(\bar{\psi}\gamma^\mu \partial_\mu \psi) \right]$$

  which is invariant under supersymmetry transformations of the form

$$\delta\psi \sim \phi \,, \qquad \delta\phi \sim \psi$$

  and U(1) field transformations.
- The corresponding conserved currents are the supercurrent S^μ and the R-current R^μ.
- We compute the 4-point correlation function of two supercurrents and two R-currents $\langle T(S^\mu(x_1)\bar{S}^\nu(x_2)R^\kappa\,(x_3)\,R^\lambda\,(x_4))\rangle$.

- The Feynman diagrams for the connected part of this correlation function have the following form:



All the diagrams are linearly divergent. After regulating with either momentum cut-off or Pauli-Villars we find that the above 4-point correlation function is anomalous, i.e.

$$\partial_\mu \left\langle T(S^\mu(x_1)\bar{S}^\nu(x_2)R^\kappa\left(x_3\right)R^\lambda\left(x_4\right)\right)\rangle = A_Q^{\nu\kappa\lambda}$$

where $A_Q^{\nu\kappa\lambda}$ is the supersymmetry anomaly. This new anomaly may have implications in phenomenology, early universe cosmology and localization computations.

References

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[2] I. Papadimitriou, *Supersymmetry anomalies in N=1 conformal supergravity*, JHEP (2019), arXiv:1902.06717

[3] G. Katsianis, I. Papadimitriou, K. Skenderis, M.Taylor, *Computation of supersymmetric anomalies*, to appear