Twisted non-Abelian vortices

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Progress in Fluid/Gravity Correspondence
Institute of Theoretical Physics, Aristotle University of Thessaloniki,
16-20 February 2015

Outline

Motivation

Localised solutions

- Classical solutions
- Important in Quantum Theory as well
- Non-perturbative

Vortices and strings

- A vortex is a 2D solution
- Can be the cross section of a string
- Strings may play an important role in confinement
- non-Abelian vortices have nice mathematical properties too

Theory considered

Bosonic sector of $N = 2$ supersymmetric $SU(2) \times U(1)$ gauge theory, $SU(2)$ flavor symmetry.
$S = \int \mathrm{d}^4 x \mathcal{L}$,

$$\mathcal{L} = -\frac{1}{4g_1^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4g_2^2} G^a_{\mu\nu} G^{\mu\nu a} + \mathrm{Tr}(D_\mu \Phi)^\dagger D^\mu \Phi - (V_1 + V_2) \,,$$

where $D_\mu \Phi = (\partial_\mu - i A_\mu \sigma^0/2 - i C^a_\mu \sigma^a/2)\Phi$
Potential

$$V_1 = \frac{\lambda_1}{8} (\mathrm{Tr}\, \Phi^\dagger \Phi - 2\xi)^2 \,, \quad V_2 = \frac{\lambda_2}{8} (\mathrm{Tr}\, \Phi^\dagger \sigma^a \Phi)^2$$

Properties of this theory:

- scalar sector of a supersymmetric theory
- possesses many localized solutions (strings, etc.)

Spontaneous symmetry breaking

Let

$$\Phi = \begin{pmatrix} \phi_1 & \psi_1 \\ \phi_2 & \psi_2 \end{pmatrix}$$

with this notation:

$$V_1 = \frac{\lambda_1}{8}(\phi^\dagger\phi + \psi^\dagger\psi - 2\xi)^2\,, \quad V_2 = \frac{\lambda_2}{8}\left[(\phi^\dagger\phi - \psi^\dagger\phi)^2 + 4|\psi^\dagger\phi|^2\right]\,,$$

i.e., vacuum: both ϕ, ψ normalized to ξ and orthogonal
Symmetry breaking pattern:

$$U(1) \times SU(2) \times SU(2)_{\text{global}} \to SU(2)_{\mathrm{CF}}$$

where $SU(2)_{CF}$ preserves the VEV,
E.g., choosing $\langle\Phi\rangle = \xi\mathbb{1}$: $SU(2)_{CF}$ acts as $\Phi \to V\Phi V^\dagger$

Color-flavor locking: gauge and color symmetry both broken spontaneously, $SU(2)_{\mathrm{CF}}$ remains unbroken

Topology permits vortex solutions

Some vortex solutions

An (n_1, n_2) vortex:

$$\Phi = \begin{pmatrix} \phi_1(r)\mathrm{e}^{in_1\vartheta} & \\ & \psi_2(r)\mathrm{e}^{in_2\vartheta} \end{pmatrix}, \qquad \begin{aligned} A_\vartheta &= a(r) \\ C^3_\vartheta &= c_3(r) \end{aligned}$$

with real radial functions
Radial equation for ϕ_1, ψ_2, a, c_3 solved numerically

Further solutions generated: orientational normal modes

$$\Phi \to V \Phi V^\dagger, \quad V \in SU(2)$$

explicitly:

$$\Phi = \chi_+ \mathbb{1} + \chi_- n^a \sigma^a, \quad c_a = n^a \tilde{c}_3$$

where $\chi_\pm = (\phi_{1D} + \phi_{2D})/2$.
(Shifman etal., Auzzi etal.)

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BPS

Energy as sum-of-squares if $\lambda_i = g_i^2$ (fixed by SUSY):

$$\mathcal{E}_{\rm BPS} = \frac{1}{4g_1^2}\left[F_{ik} \pm \frac{g_1^2}{2}\epsilon_{ik}(\mathrm{Tr}\,\Phi^\dagger\sigma^0\Phi - 2\xi)\right]^2 + \frac{1}{4g_2^2}\left[G_{ik}^a \pm \frac{g_2^2}{2}\epsilon_{ik}\,\mathrm{Tr}\,\Phi^\dagger\sigma^a\Phi\right]^2$$
$$+\frac{1}{2}\mathrm{Tr}(D_i\Phi \pm i\epsilon_{ik}D_k\Phi)^\dagger(D_i\Phi \pm i\epsilon_{im}D_m\Phi) \pm \frac{\xi}{4}F_{ik}\epsilon_{ik} \mp \epsilon_{ik}\partial_i\,\mathrm{Tr}(\Phi^\dagger D_k\Phi)\,.$$

minimal energy: all squares vanish:

$$F_{ik} = \mp\frac{g_1^2}{2}\epsilon_{ik}(\mathrm{Tr}\,\Phi^\dagger\sigma^0\Phi - 2\xi)\,,$$
$$G_{ik}^a = \mp\frac{g_2^2}{2}\epsilon_{ik}\,\mathrm{Tr}\,\Phi^\dagger\sigma^a\Phi\,,$$
$$D_i\Phi = \mp i\epsilon_{ik}D_k\Phi\,,$$

Energy:

$$E_{\rm BPS} = \int \mathrm{d}^2x\,\mathcal{E}_{\rm BPs} = 2\pi\xi\varphi$$

where φ is the number of flux quanta

BPS multi-vortices

Multi vortex solutions also possible

- No interaction between vortices
- Moduli: positions, orientations

Moduli matrix approach (Eto etal.):

- $\Phi = S(x+iy, x-iy)^{-1}\Phi_0(x+iy)$
  Φ_0 holomorphic given, zeros of its determinant: position of vortices
- $S = S_1 S_2$, $\Omega_i = S_i S_i^\dagger$, $\Omega_1 = exp(\psi)$
- one equation for $\Omega = SS^\dagger$, reduced to a holomorphic splitting problem

$$\partial_{\bar z}(\Omega_2 \partial_z \Omega_2^{-1}) = -\frac{g_2^2}{4}\left(\Phi_0\Phi_0^\dagger\Omega_2^{-1} - \frac{\mathbb{1}}{N}\,\mathrm{Tr}\,\Phi_0\Phi_0^\dagger\Omega_2^{-1}\right)\mathrm{e}^{-\psi}\,,$$

$$\partial_{\bar z}\partial_z\psi = -\frac{g_1^2}{4N}\left(\mathrm{Tr}(\Phi_0\Phi_0^\dagger\Omega_2^{-1})\mathrm{e}^{-\psi} - N\xi\right)\,.$$

Also for more general gauge groups

Adding twist

Straight string: translation invariance along axis z:

$$\Phi(x^\mu) = \Phi(x^i)\exp\left(\frac{i}{2}M\omega_\alpha x^\alpha\right)\,,$$

$$A_\mu(x^\nu) = (A_i(x^j), A_\alpha(x^j))\,,$$

$$C^a_\mu(x^\nu) = (C^a_i(x^j), C^a_\alpha(x^j))\,,$$

Decoupling

$$\omega^2 = -\omega_\alpha\omega^\alpha = 0$$

ensures that the equations for $\Phi(x^i)$, C^a_i, A_i are unchanged ($i = 1,2$)

$$A_\alpha = \omega_\alpha A\,, \quad C^a_\alpha = \omega_\alpha C^a$$

The out-of-plane components satisfy a Gauss-constraint

Solutions equivalent to solving mass deformed theory

- adjoint scalars: out-of-plane gauge field components
- mass matrix – twisting matrix

(Collie, Eto etal., Gorsky etal.)

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Gauss constraint

Gauss constraint for out-of-plane fields:

$$D_i^2\left[\frac{1}{g_1^2}A\sigma^0+\frac{1}{g_2^2}C^a\sigma^a\right]=-\frac{1}{2}\left[\Phi(\Phi M-C\Phi)^\dagger+(\Phi M-C\Phi)\Phi^\dagger\right]$$
$$=-\Phi M\Phi^\dagger+\frac{1}{2}\{C,\Phi\Phi^\dagger\}\,,$$

Physical quantities, like momentum in string axis direction and energy:

$$E=E_{\rm BPS}+\frac{\omega_0^2+\omega_3^3}{4}\int \mathrm{d}^2xQ\,,$$

where $E_{\rm BPS}=2\pi\xi\varphi$ (no. of flux quanta) and

$$P=\frac{1}{2}\omega_0\omega_3\int \mathrm{d}^2xQ$$

$$Q=\mathrm{Tr}\left[\Phi^\dagger(\Phi M-C\Phi)M\right]\,,$$

and

$$J=\int \mathrm{d}^2x\omega_0\frac{1}{2}\,\mathrm{Tr}\left[\Phi^\dagger\Phi(NM+MN)-2\Phi^\dagger C\Phi N\right]$$

where $\partial_\vartheta\Phi=iN\Phi$.

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An Ansatz for rotationally symmetric solutions in the plane

$$\Phi(x^i) = \begin{pmatrix} \phi_1(r) & \psi_1(r)\mathrm{e}^{iN\vartheta} \\ \phi_2(r) & \psi_2(r)\mathrm{e}^{iN\vartheta} \end{pmatrix} ,$$
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Minimal Ansatz: ϕ_i, ψ_i real.
$c_2 = 0$: consistency condition
Diagonalizable: elementary or (n_1, n_2) vortex, $V\Phi_D V^\dagger$
Coincident composite vortices (Shifman, Auzzi): $N = -1$, flux 2
(Auzzi, Shifman, Yung)
Parameter α: angle of $(\phi_1(\infty), \phi_2(\infty))$ and $(1, 0)$

Small α

- ϕ_1, ψ_2, a, c_3 of unit magnitude
- c_1, ϕ_2, ψ_1 small

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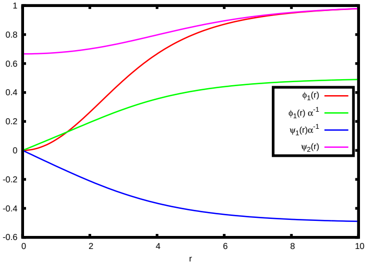
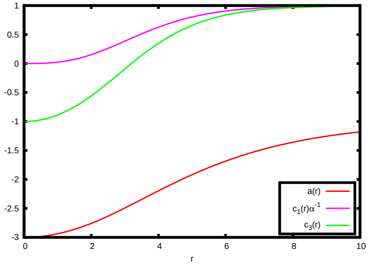
Perturbative framework





Expansion in α:

$$\phi_1 = \phi_1^{(0)} + \alpha^2 \phi_1^{(2)} + \ldots, \qquad \psi_1 = \alpha \psi_1^{(1)} + \ldots,$$

$$\phi_2 = \alpha \phi_2^{(1)} + \ldots, \qquad \psi_2 = \psi_2^{(0)} + \alpha^2 \psi_2^{(2)} + \ldots,$$

and

$$a = a^{(0)} + \alpha^2 a^{(2)} + \ldots, \qquad c_1 = \alpha c_1^{(1)} + \ldots.$$

$$c_3 = c_3^{(0)} + \alpha^2 c_3^{(2)} + \ldots,$$

Twisting the elementary vortex

Reminder: elementary vortex: $\Phi = V\Phi_D V^\dagger$:

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Ansatz for out-of-plane fields: let $M = m^0\sigma^0 + m^a\sigma^a$,

$$A = m^0 \,, \quad C^a = (mn)n^a + \tilde{m}^a C(r) \,, \quad \tilde{m}^a = m^a - (mn)n^a \,,$$

yielding one equation

$$\frac{1}{r}(rC')' - \frac{c_3^2}{r^2} C = g_2^2 \left[\chi_+^2 (C-1) + \chi_-^2 (C+1)\right] \,.$$

Note: m^0 and parallel part to n^a: pure gauge

$$Q = \left(m^2 - (nm)^2\right) \left(C(r)(\chi_-^2 - \chi_+^2) + (\chi_-^2 + \chi_+^2)\right) \,,$$

(see also Collie, Eto etal.)

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Twisting the coincident composite vortex

Gauss constraints:

$$\nabla^2 A = -g_1^2 J_3^0 \,, \qquad D_i^2 C^a = -g_2^2 J_3^a \,,$$

Two cases considered: $m^0 = -m^3 = s/2$ and $m^1 = m/2$

Simplest case: $m^0 = -m^3 = s/2$

$A = A(r)$, $C^a = C^a(r)$, $C^2 = 0$

$$Q = -s\left((A-s)(\psi_1^2+\psi_2^2) + 2C^1\psi_1\psi_2 + C^3(\psi_1^2-\psi_2^2)\right) \,.$$

- For small α: energy difference very small

$$E = E_{\mathrm{BPS}} + \frac{\omega_0^2+\omega_3^2}{4}\int \mathrm{d}^2 x Q \,, \quad E_{\mathrm{BPS}} = 4\pi\xi$$

- $\alpha = 0$: bgr. agrees with elementary, m^0, m^3 twist pure gauge

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Twisting the coincident composite vortex

The other case: $m^1 = m/2$

$A = 0$, $C^{1,3} = \hat{C}^{1,3}\cos\vartheta$, $C^2 = \hat{C}^2\sin\vartheta$

$$Q = \frac{m}{2}\Big[\frac{m}{2}(\phi_1^2+\phi_2^2+\psi_1^2+\psi_2^2) - (\phi_1\psi_2+\phi_2\psi_1)\hat{C}^1 + (\phi_1\psi_2-\phi_2\psi_1)\hat{C}^2 - (\phi_1\psi_1-\phi_2\psi_2)\hat{C}^3\Big]\,,$$

- energy difference $O(1)$ (for small α too)

$$E = E_{\mathrm{BPS}} + \frac{\omega_0^2+\omega_3^4}{4}\int \mathrm{d}^2 x Q\,, \quad E_{\mathrm{BPS}} = 4\pi\xi$$

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Symmetries

Rotational symmetry of the static solution

$$\Phi(r,\vartheta) = \exp\left(i\frac{N}{2}\varphi\right)\Phi(r,\vartheta-\varphi)\exp\left(-i\frac{N}{2}\varphi\sigma^3\right) ,$$

i.e., rotation compensated by internal symmetries
Twisted solution:

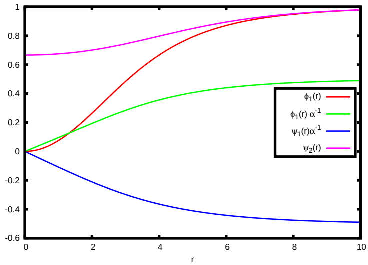
$$\Phi(x^\mu) = \Phi(x^i)\exp\left(\frac{i}{2}\omega_\alpha x^\alpha M\right)$$

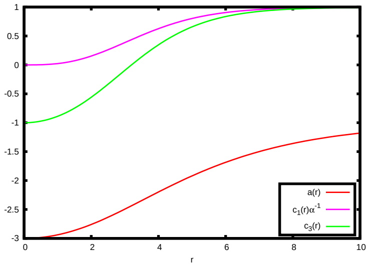
Also rotationally symmetric if $[M,\sigma^3] = 0$
m^0, m^3-twisted vortex more symmetric, lower energy

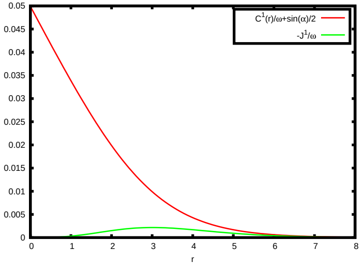
Stability

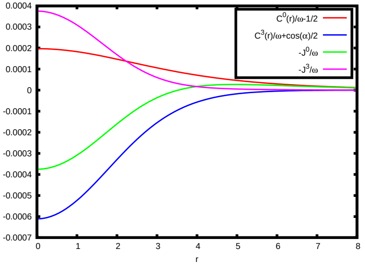
- static BPS solution: minimal energy
- twisted vortex carries conserved charge
  it is also localised to the core

Numerical solutions: $m^0 = -m^3 = s/2$









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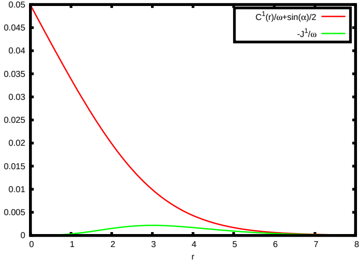
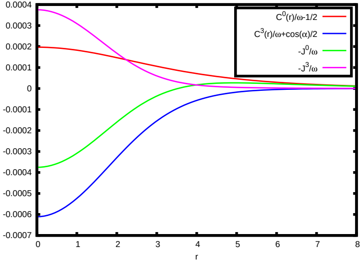
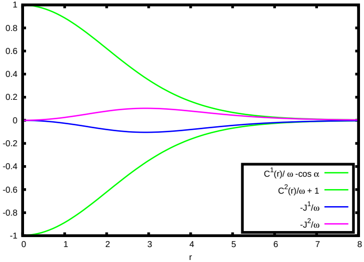
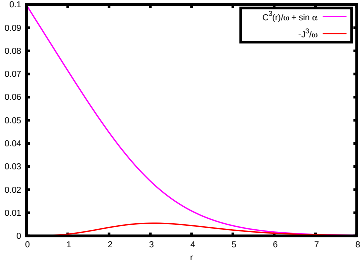
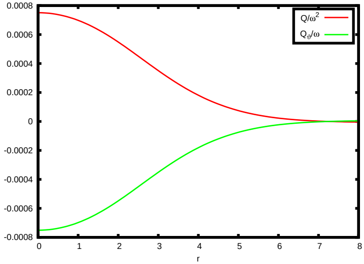


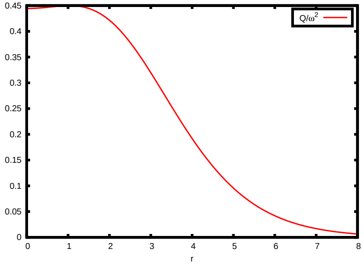






Energy and angular momentum density





- $s^{-2} \int \mathrm{d}^2 x Q = 0.02488$
- also gives $-J/\omega_0$

- $m^{-2} \int \mathrm{d}^2 x Q = 24.89$
- No net angular momentum

Note: Energy of static (BPS) vortex $2\pi\xi\varphi$, here $\varphi = 2$
A remark: m^1 case: constraint for $A = 0$, consistent, reduces Gauss constraints to quadrature

Conclusions

- Static vortices well known
  - in the Abelian Higgs model
  - in non-Abelian gauge theories
  - BPS case (SUSY)
- With multiple fields, twisted strings also possible
- Known for the elementary vortex
- Interesting properties for the composite coincident vortex
  - Energy difference sometimes very small
  - Momentum in the direction of the string axis
  - Rotating and no net angular momentum case

THANK YOU

FOR

YOUR

ATTENTION!

References

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