Steady states in conformal field theories

Based on work with A. Karch, H-C. Chang and I. Amado.

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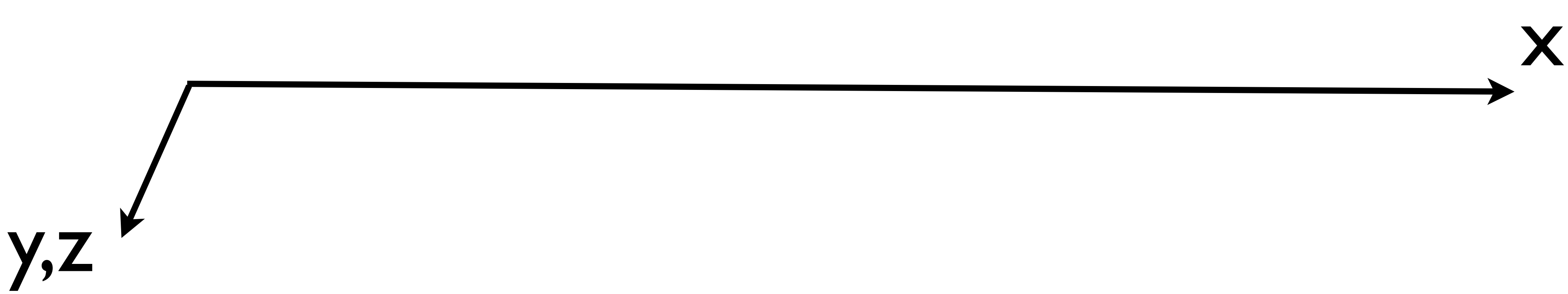

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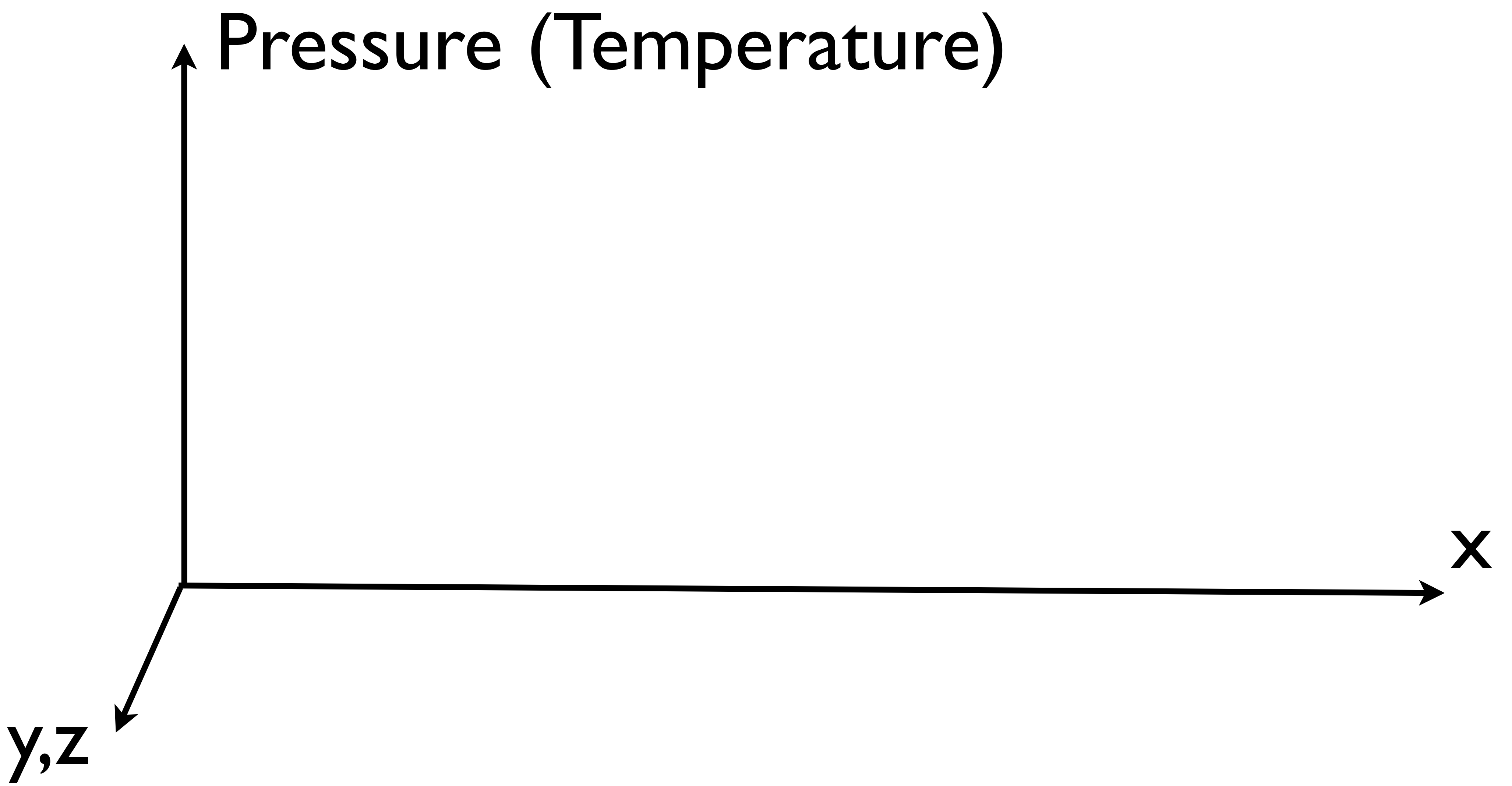

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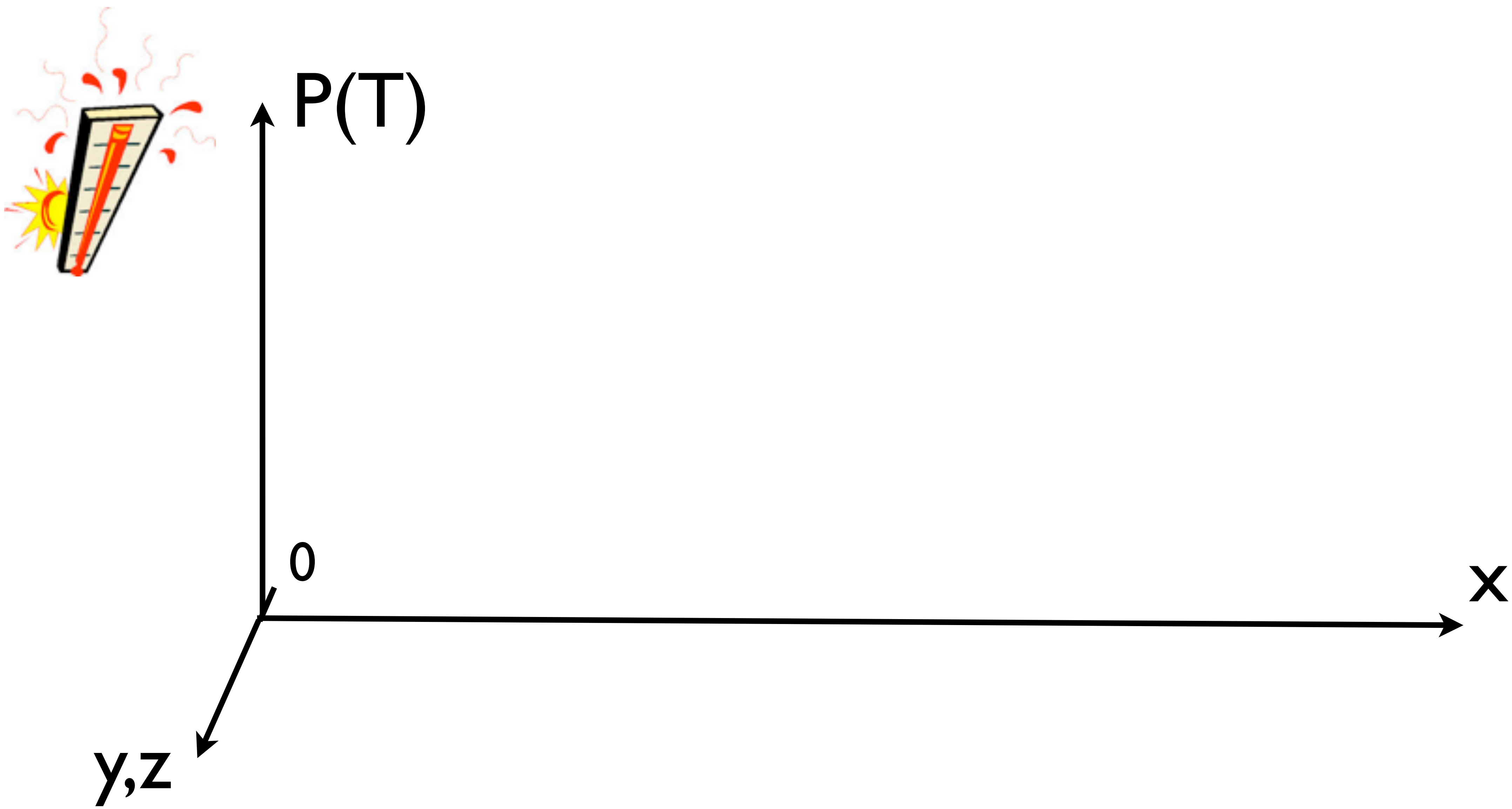

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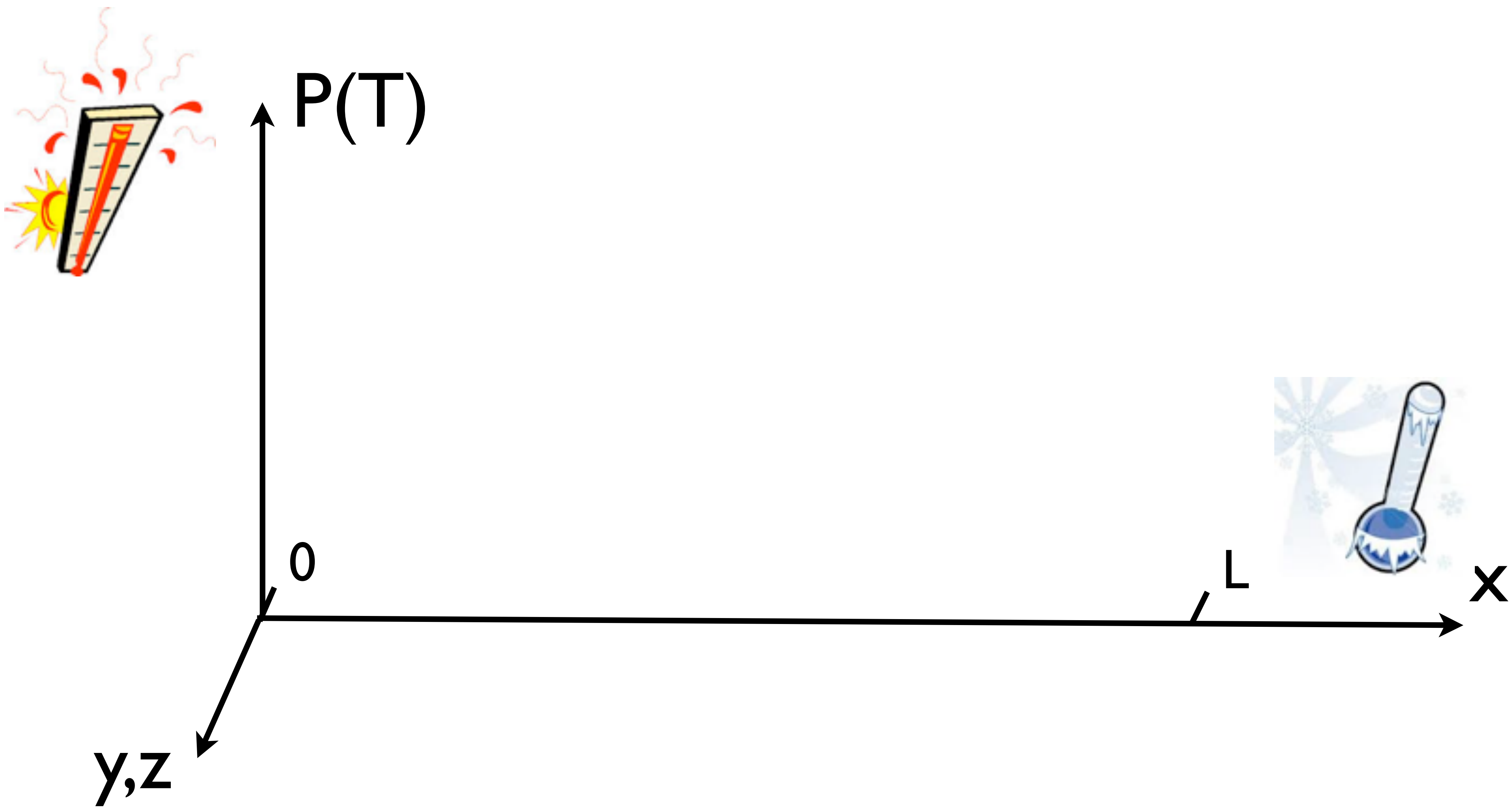

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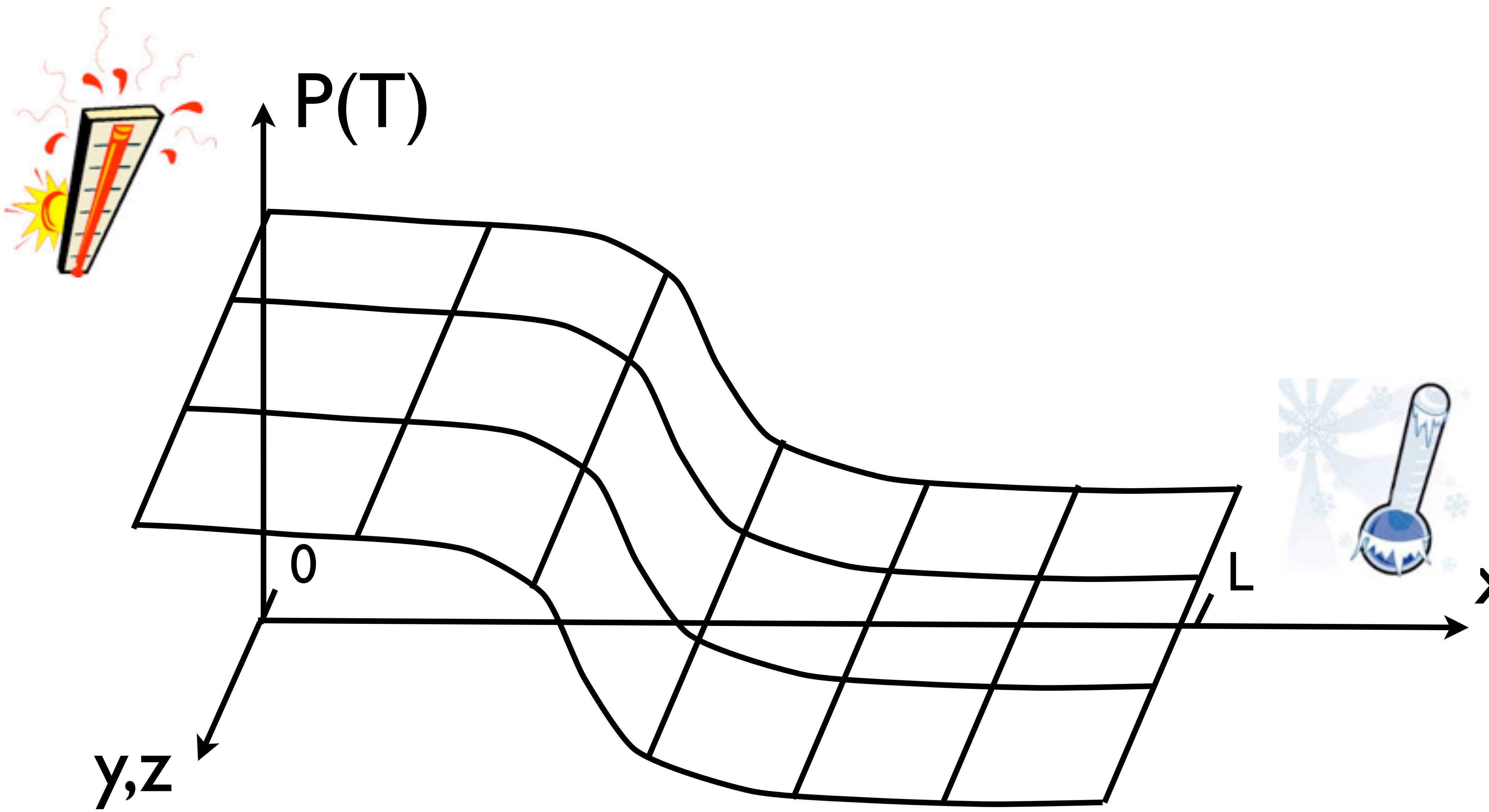

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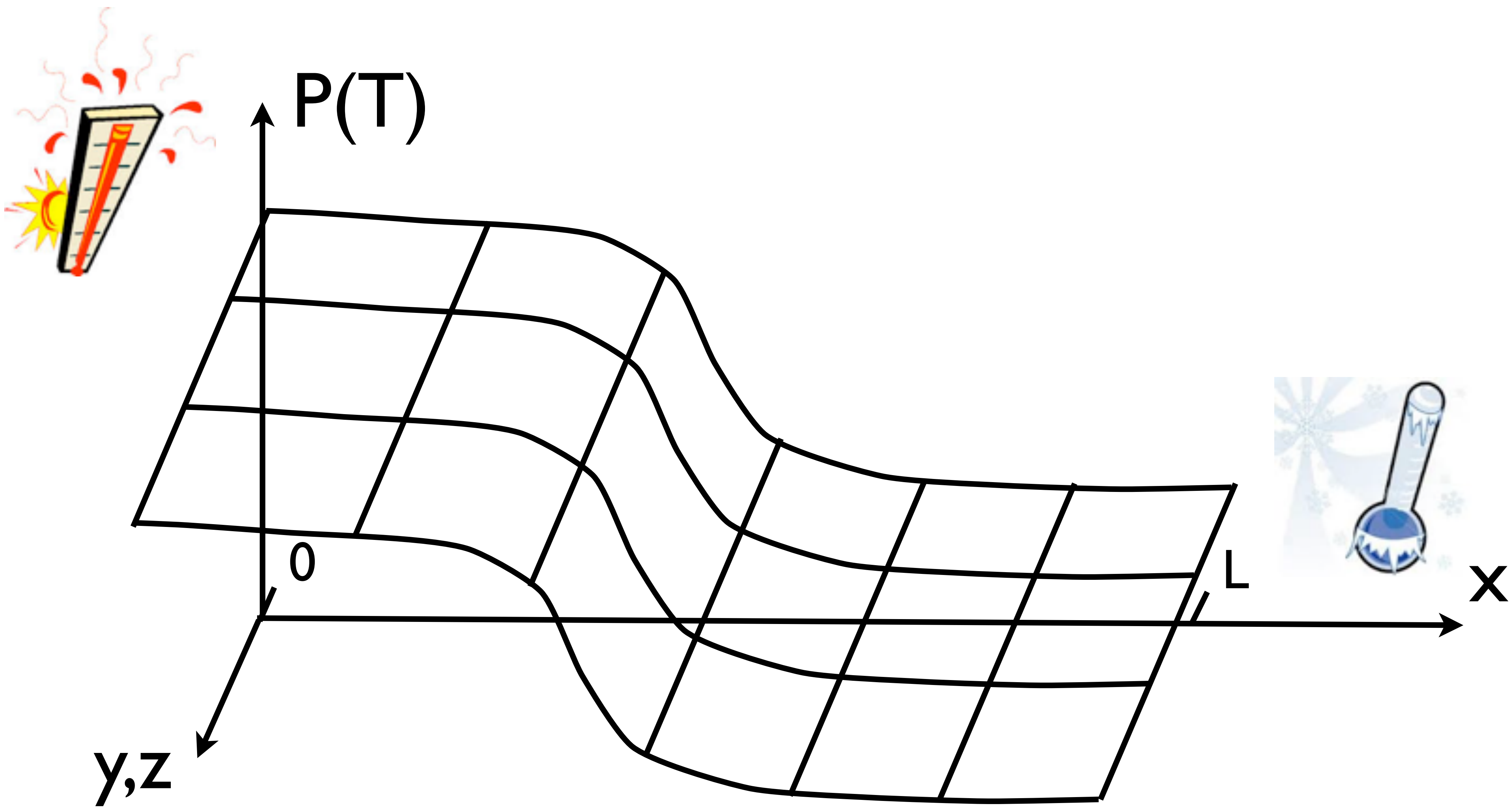

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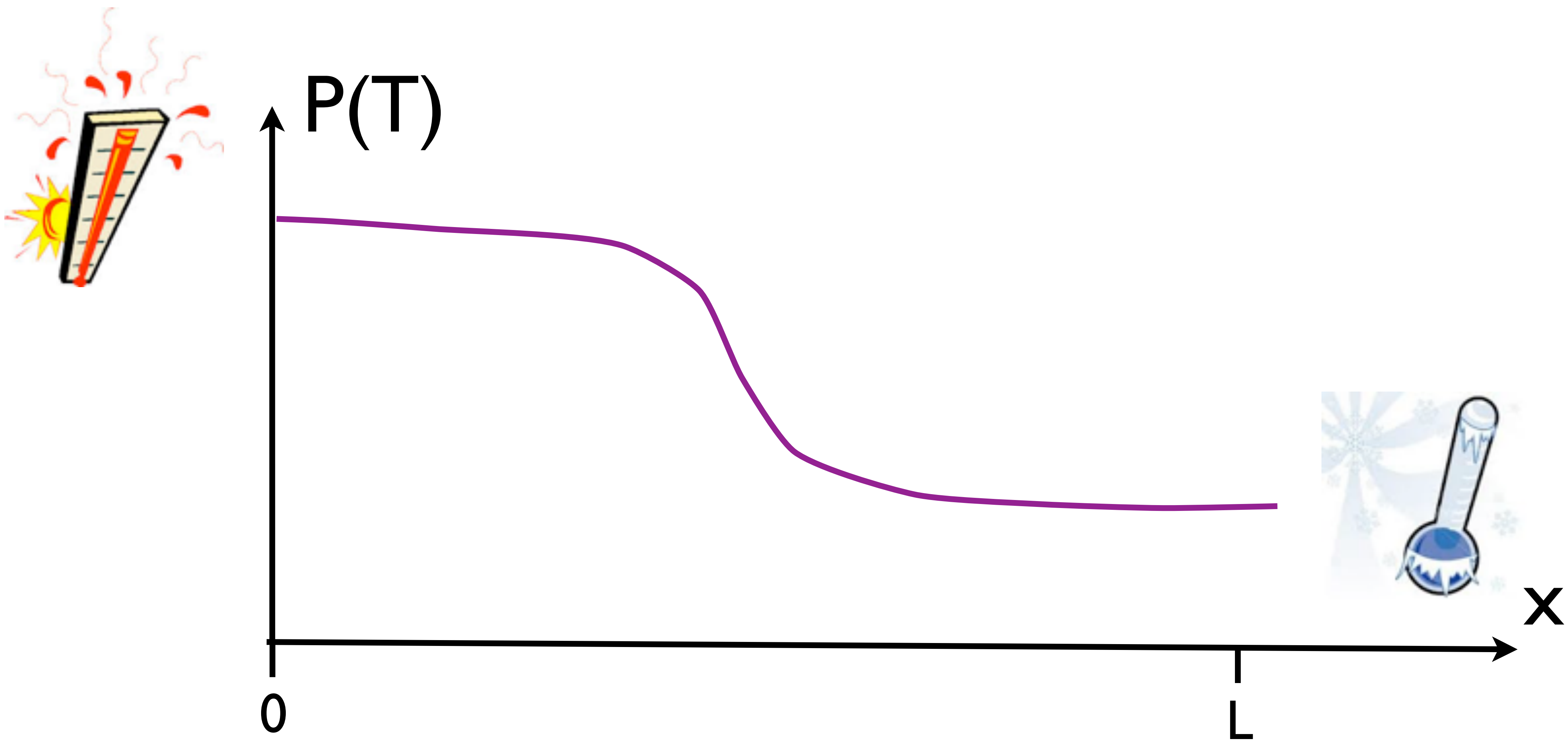

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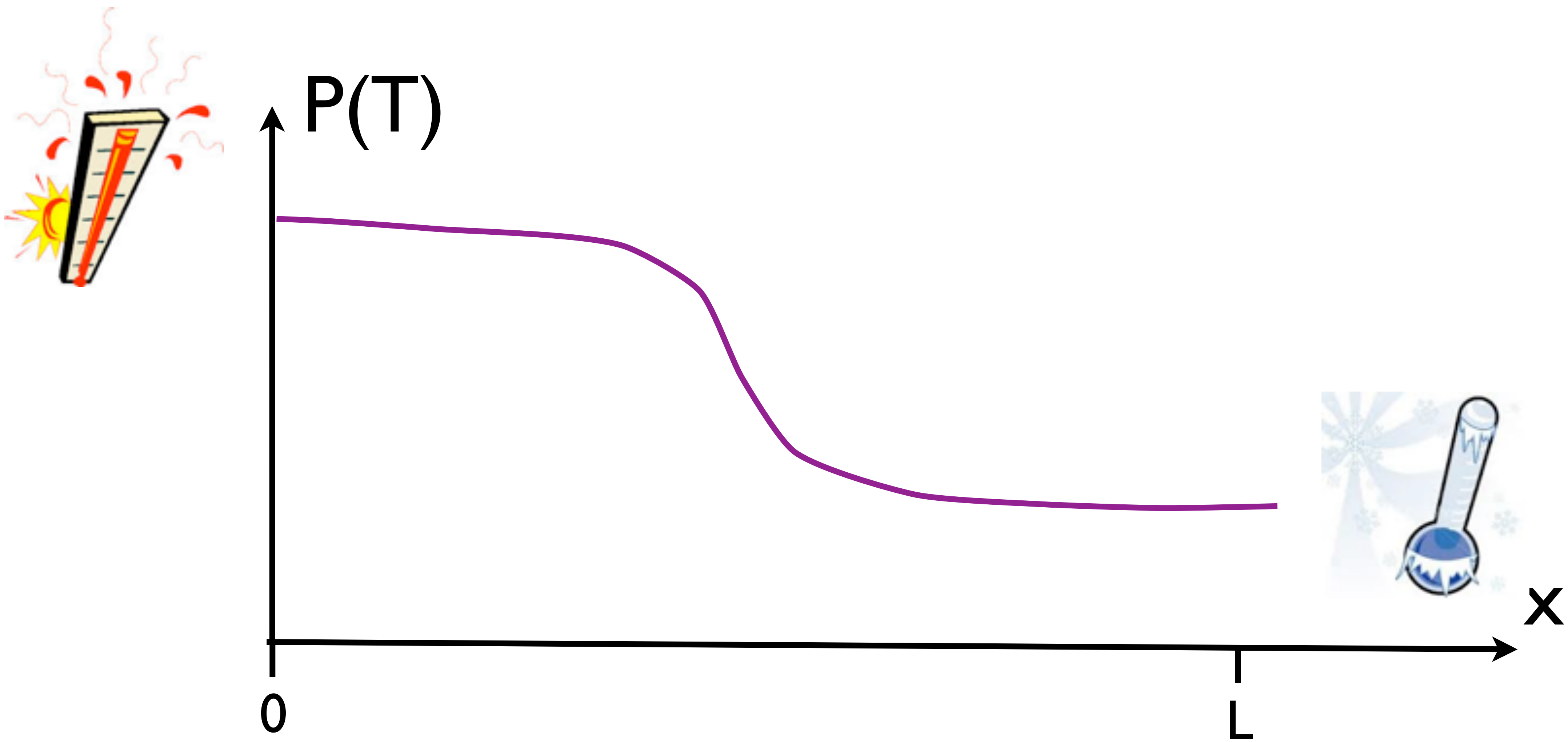

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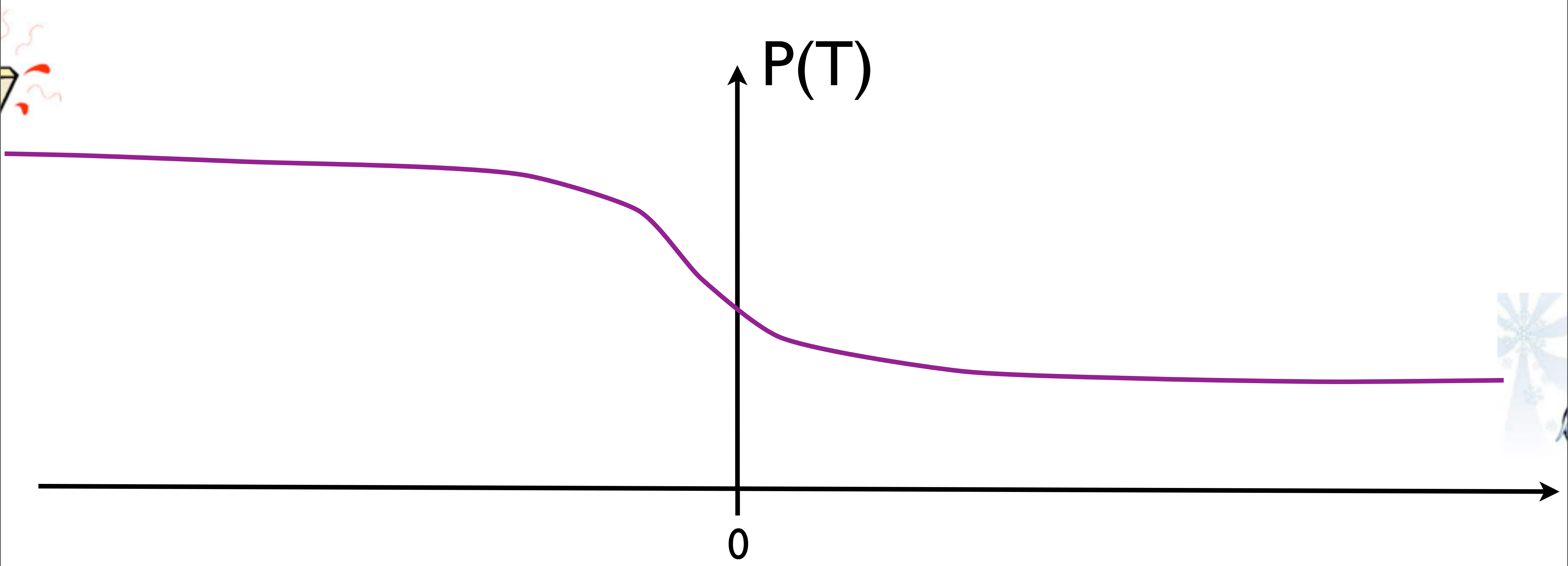

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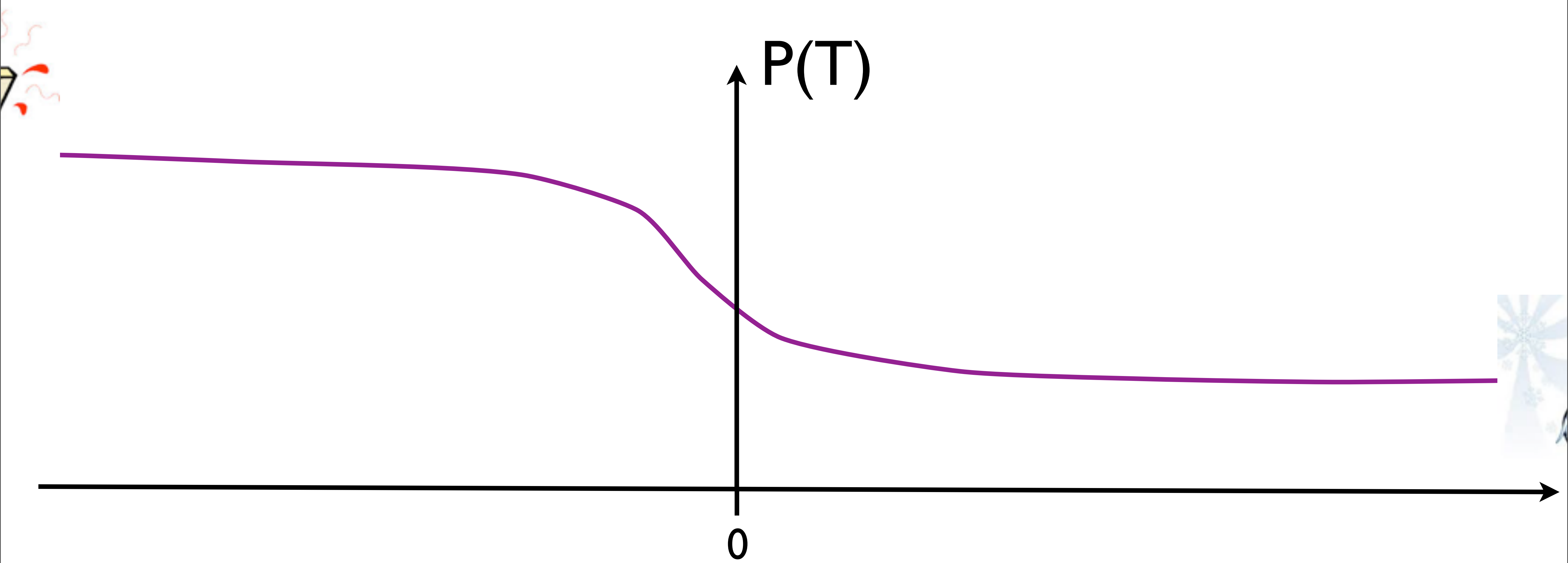

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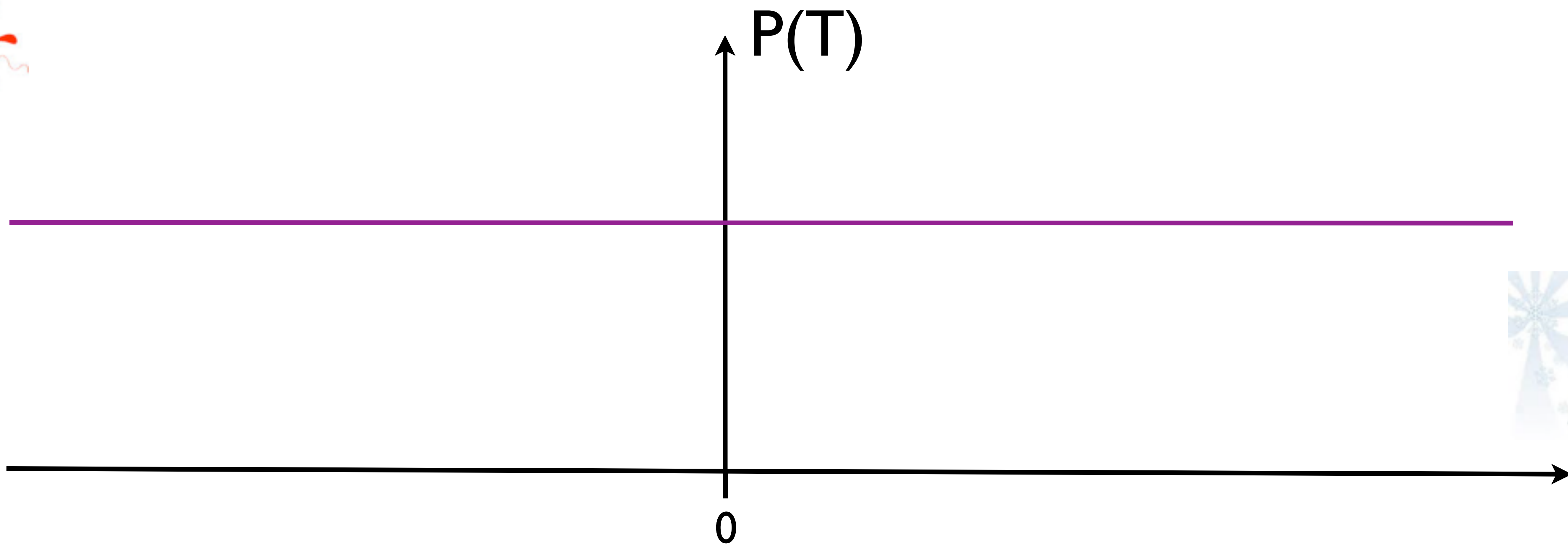

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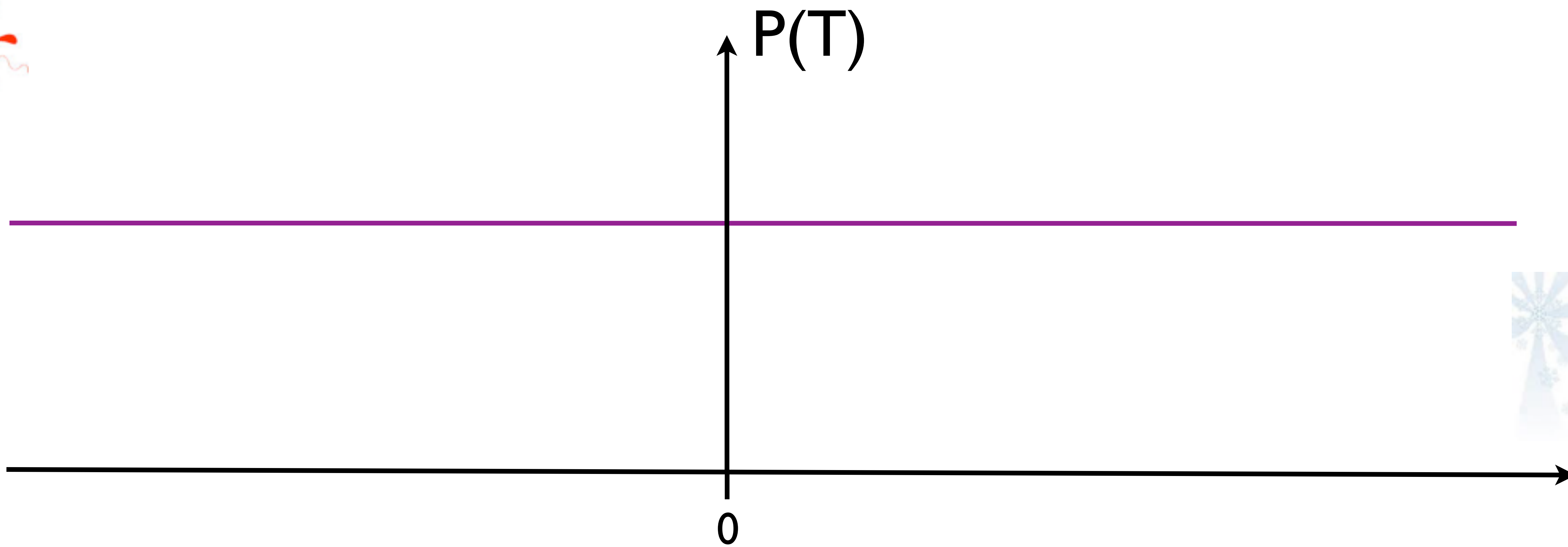
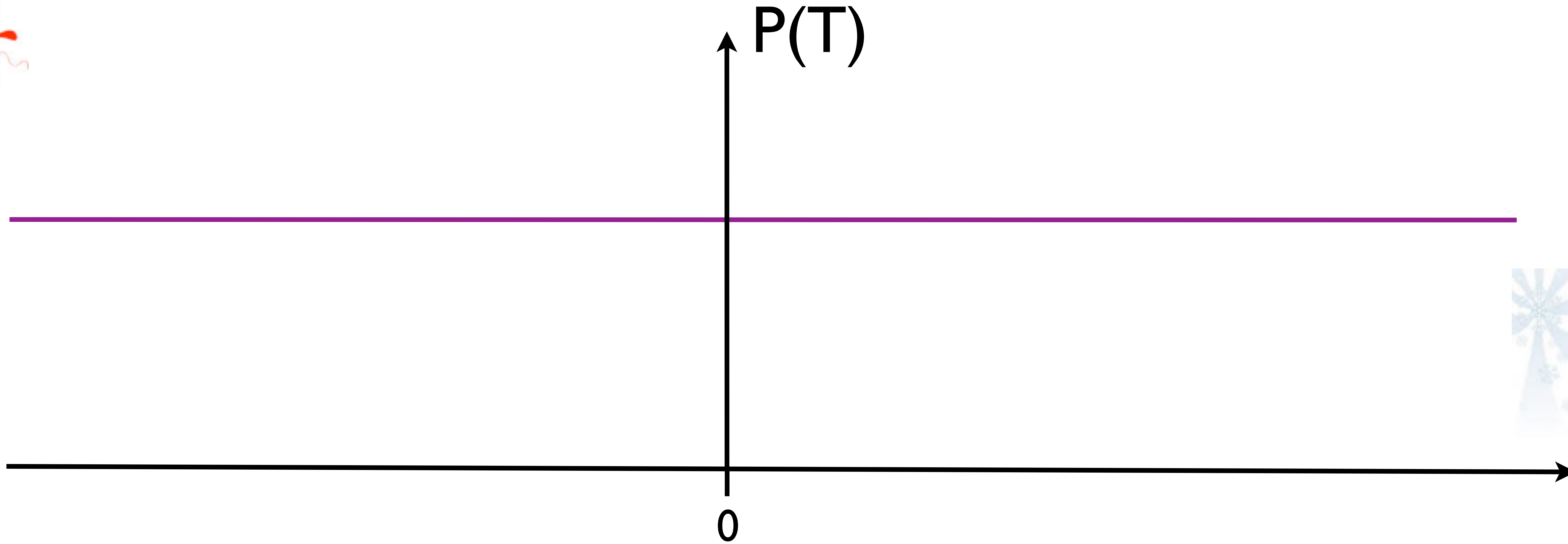

P(T)
0

Conjecture: If the field theory thermalizes quickly then the late time steady state is universal.



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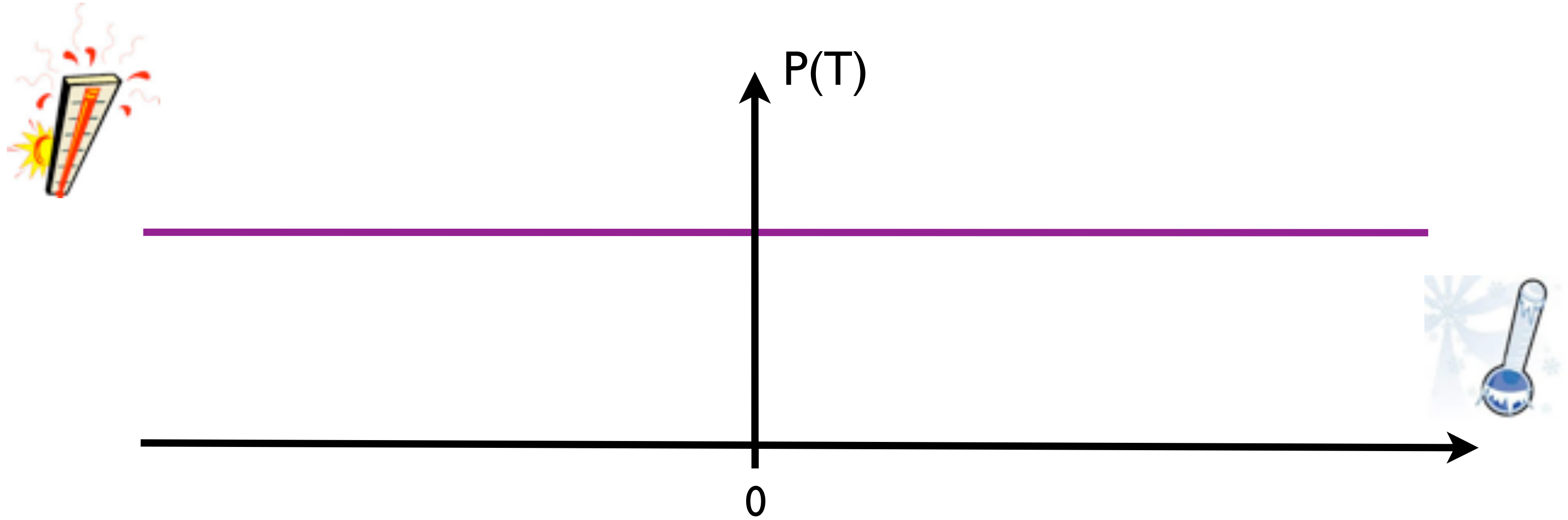

The pressure at late times will take one of 2 values:

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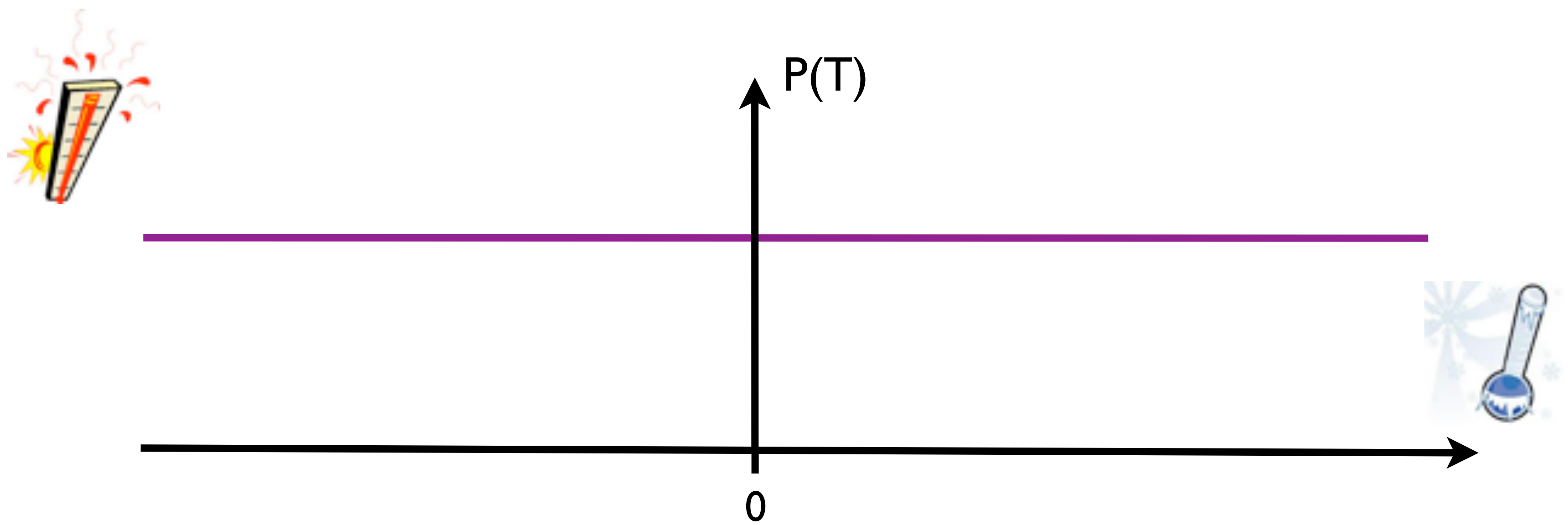

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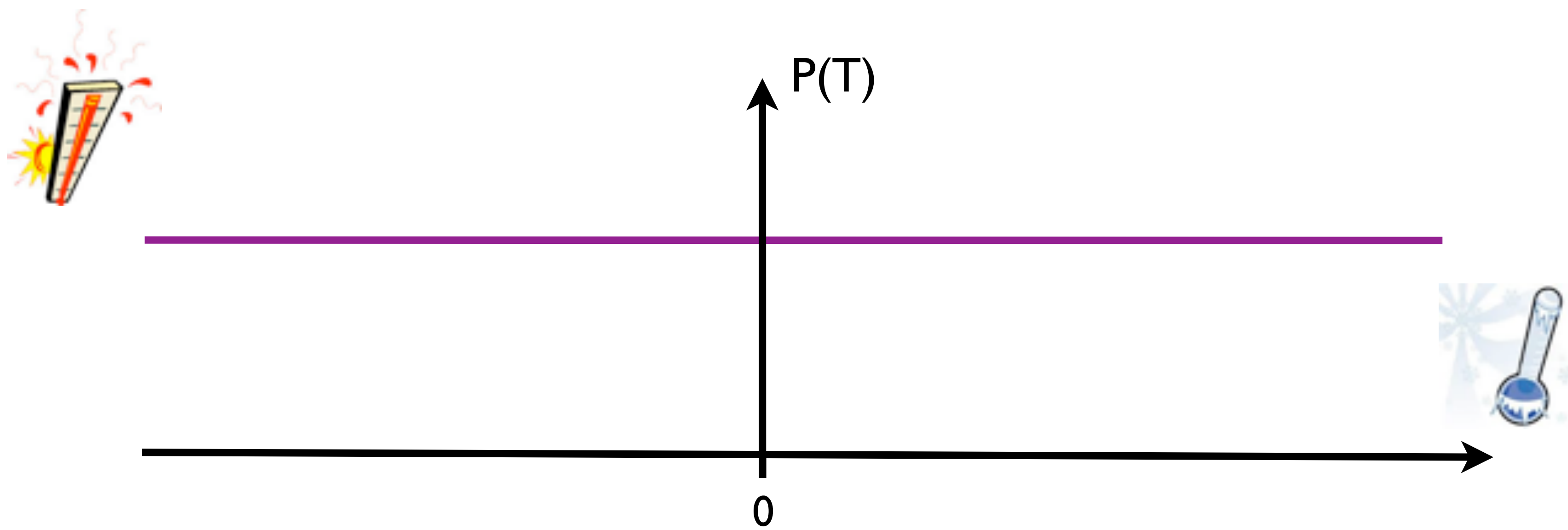

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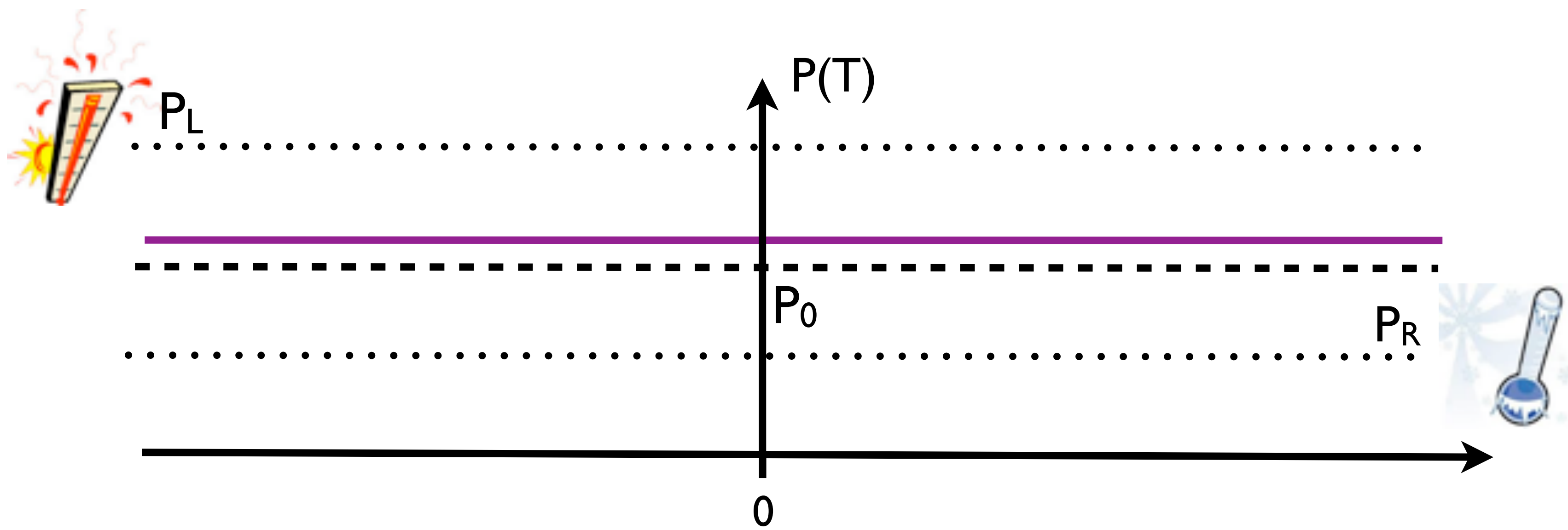

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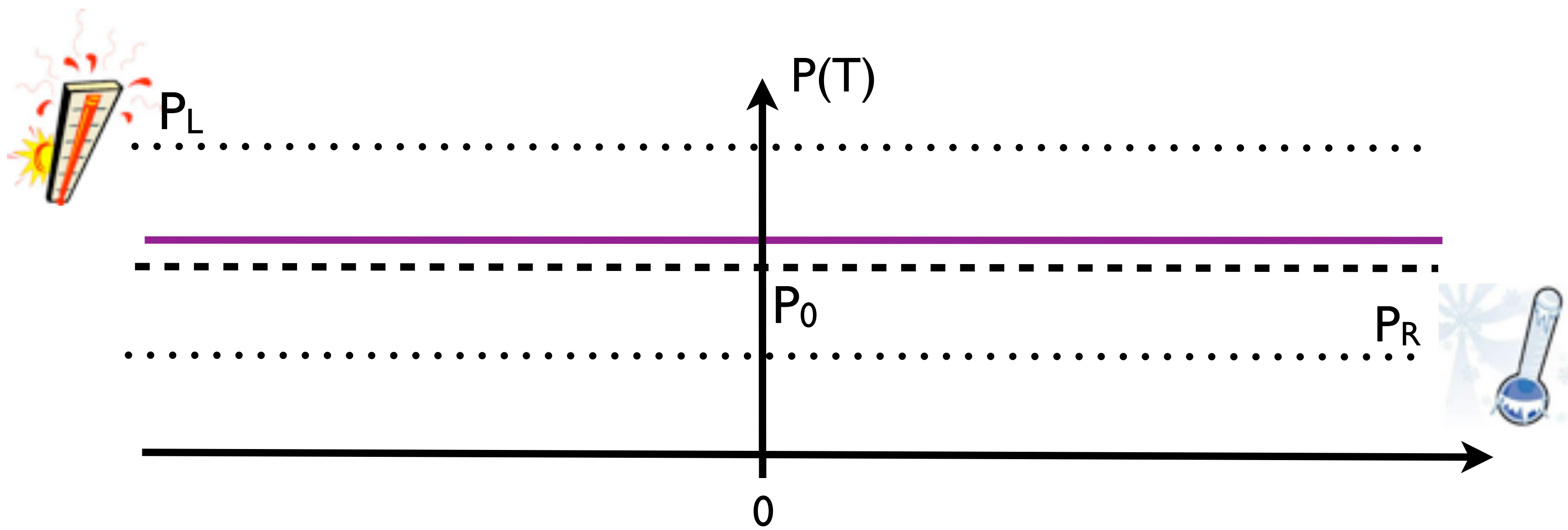

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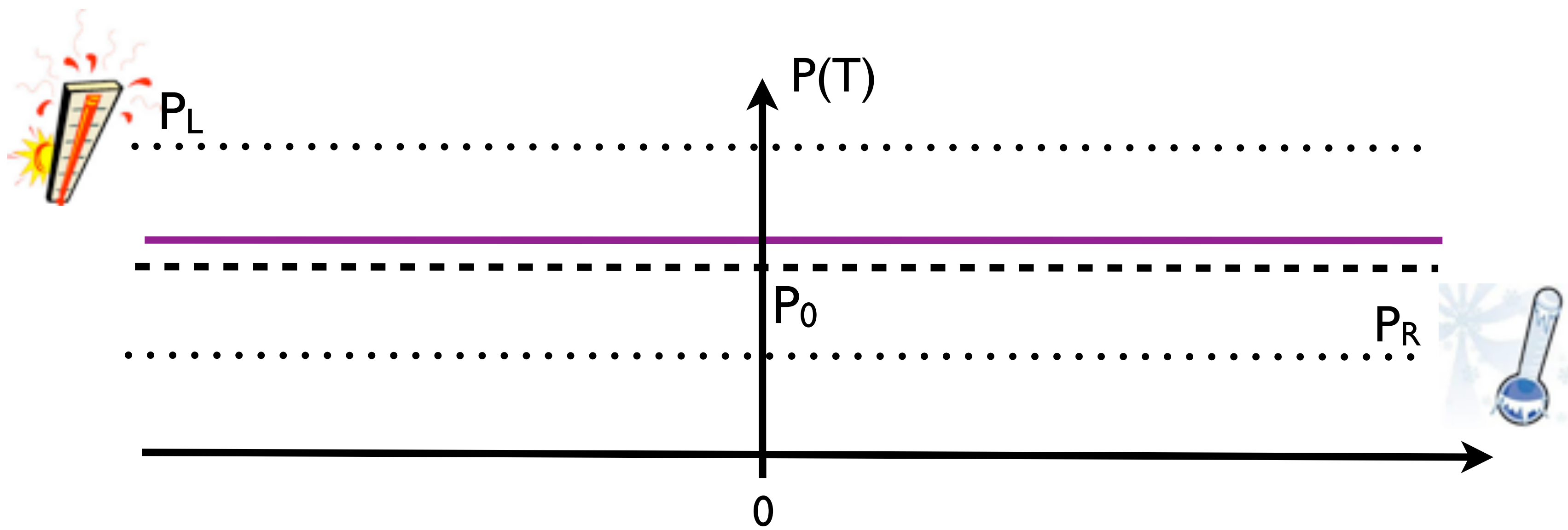

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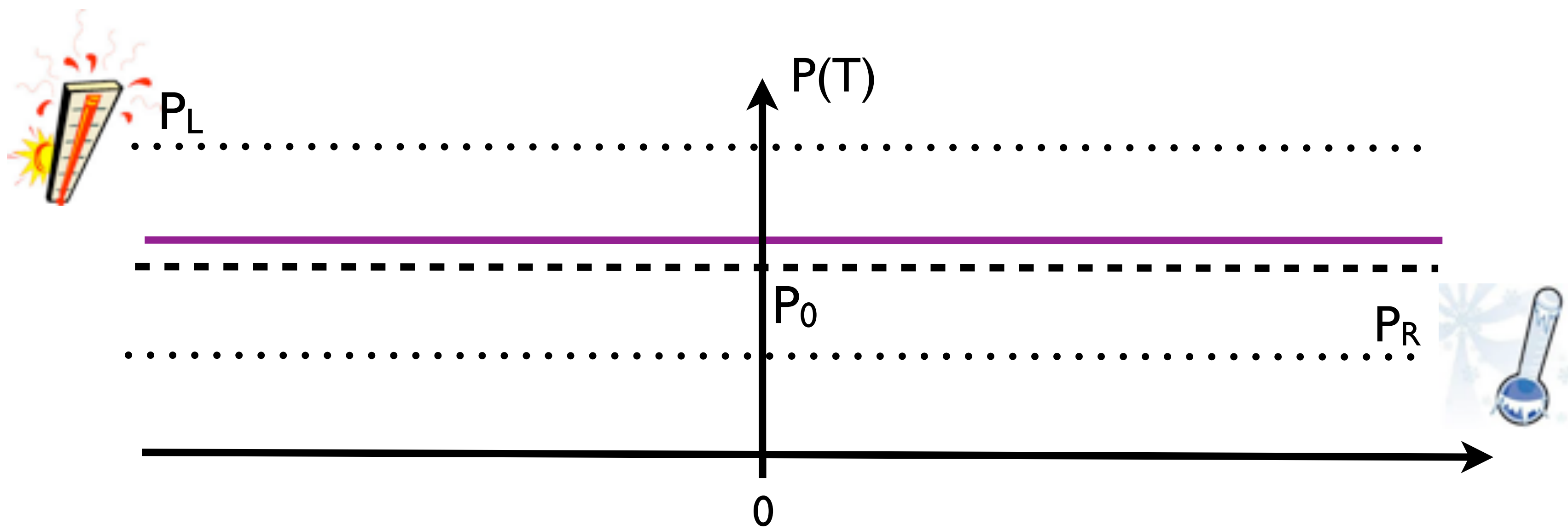

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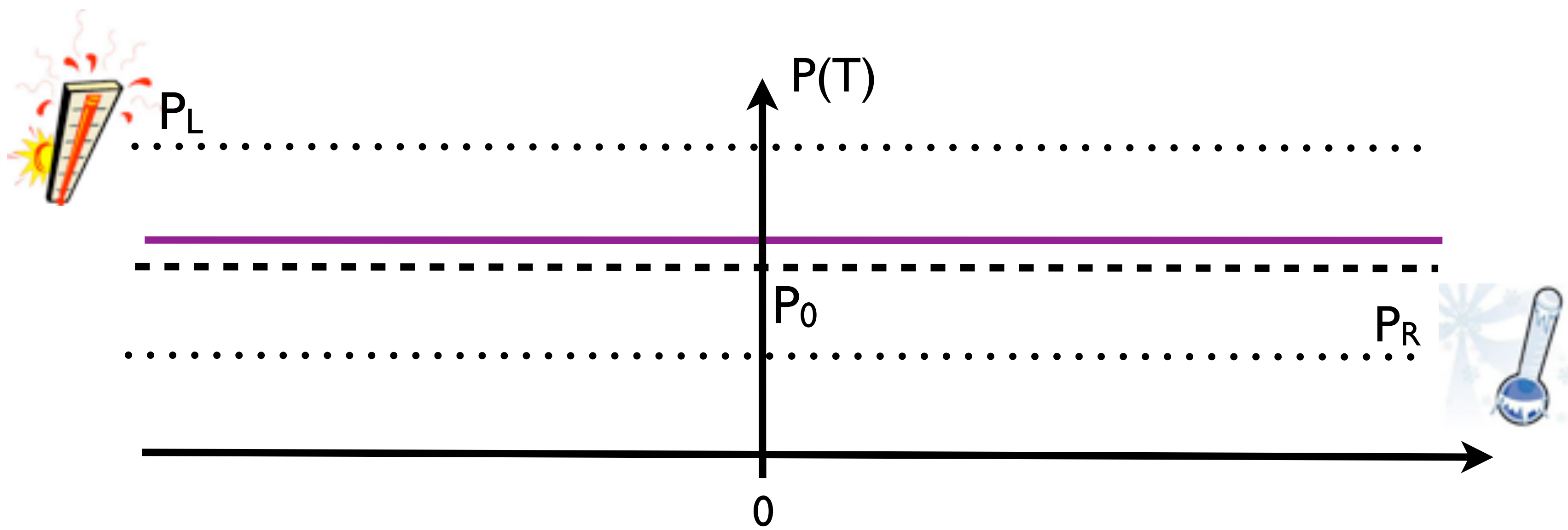

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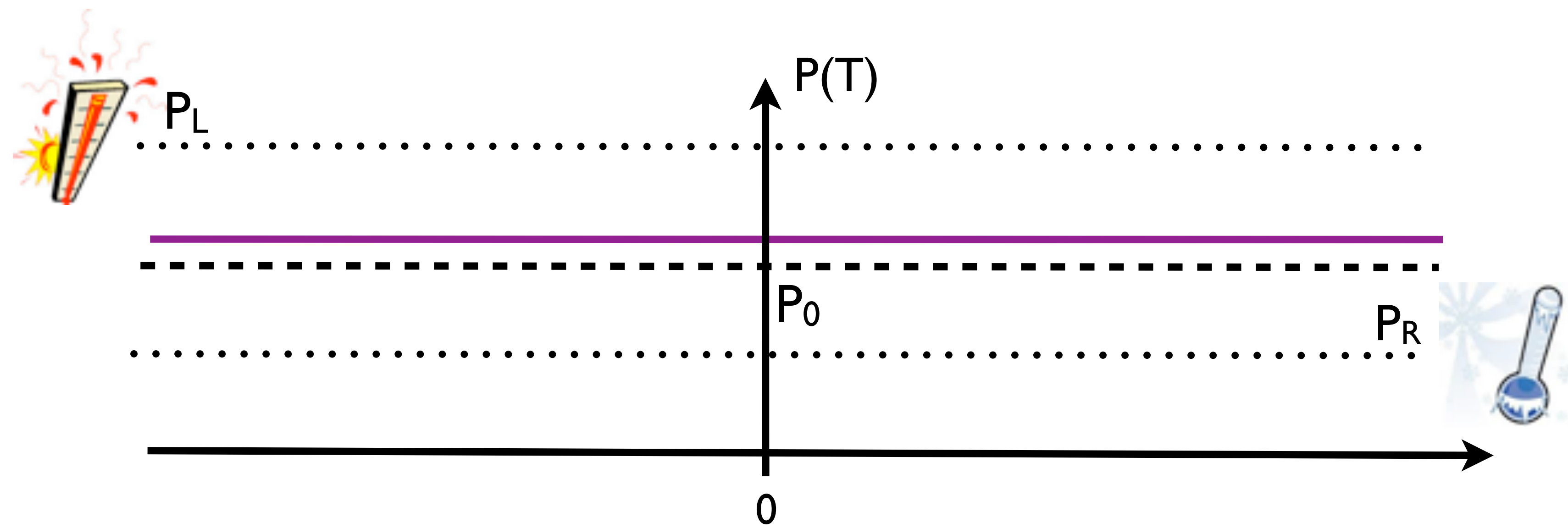

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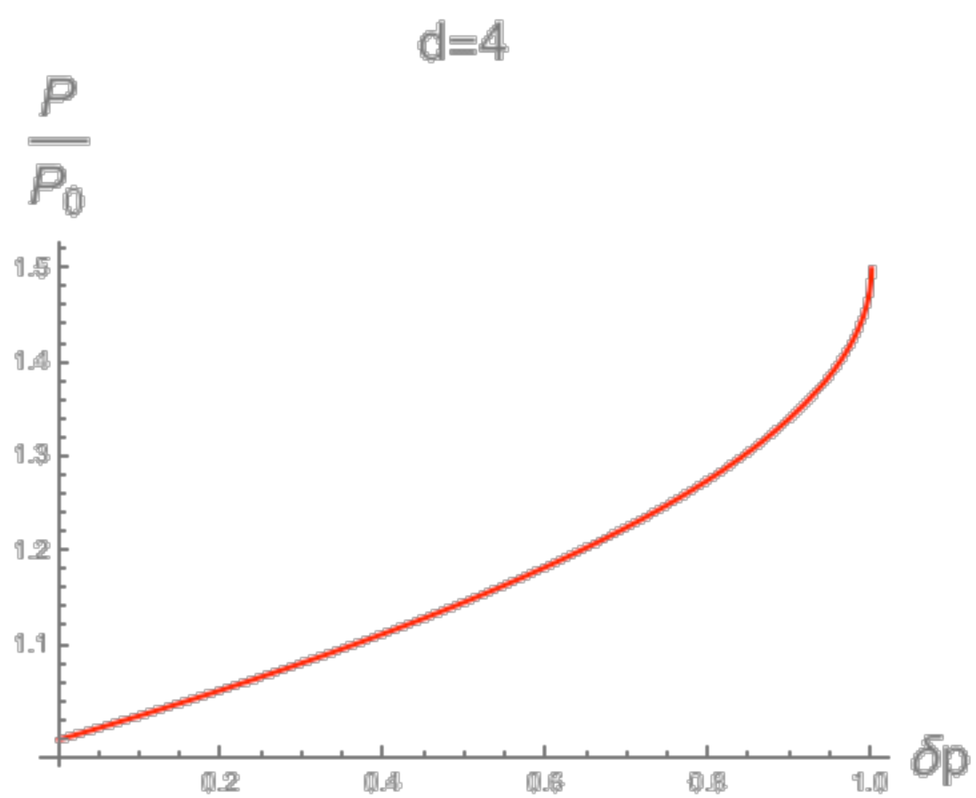

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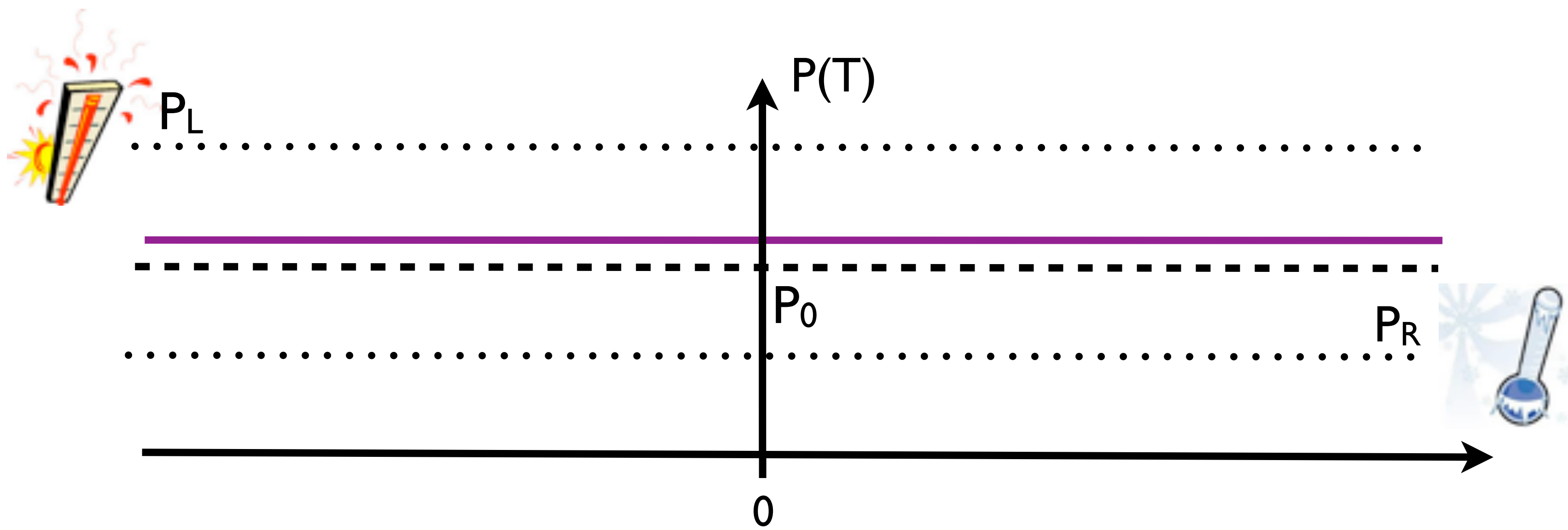

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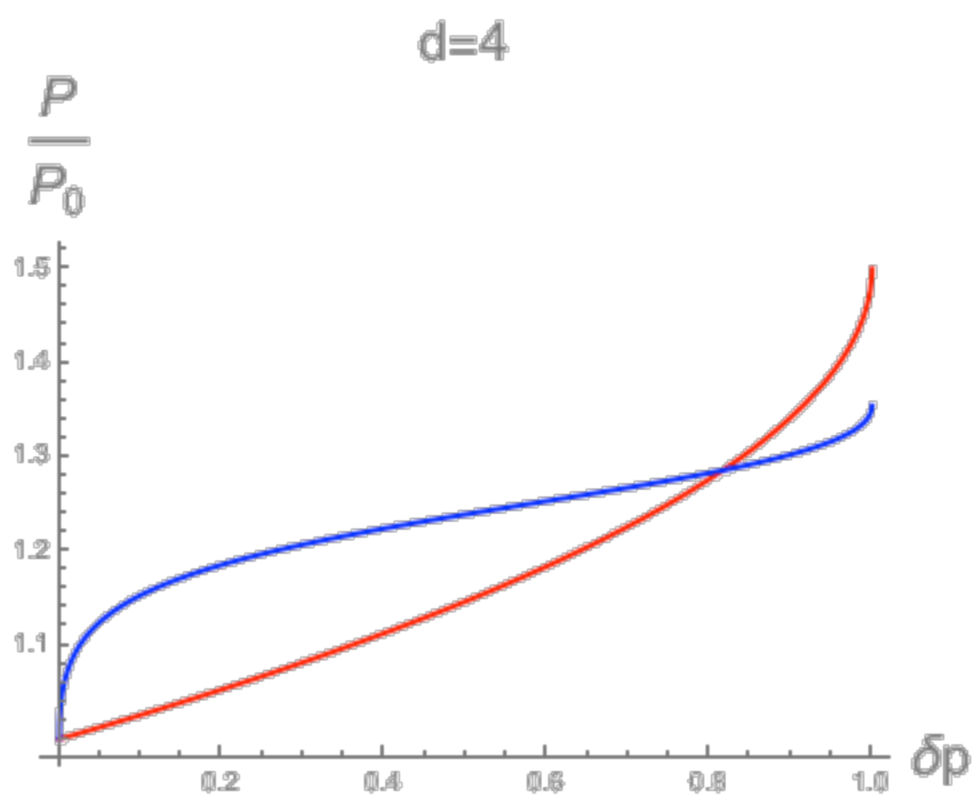

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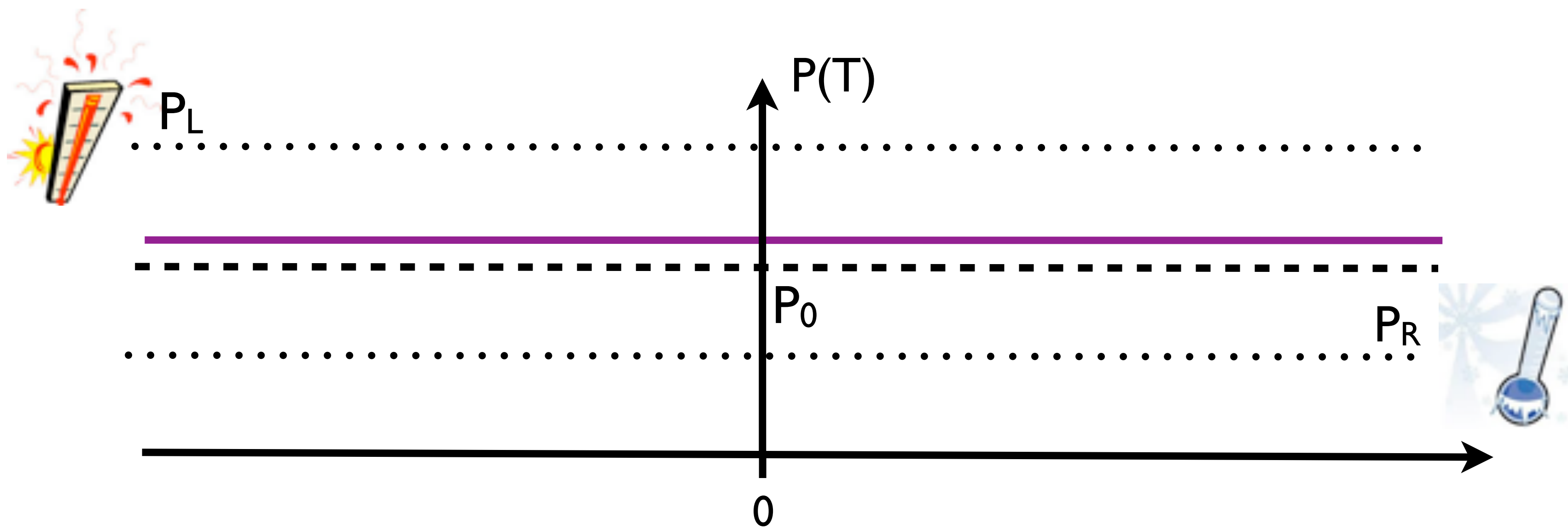

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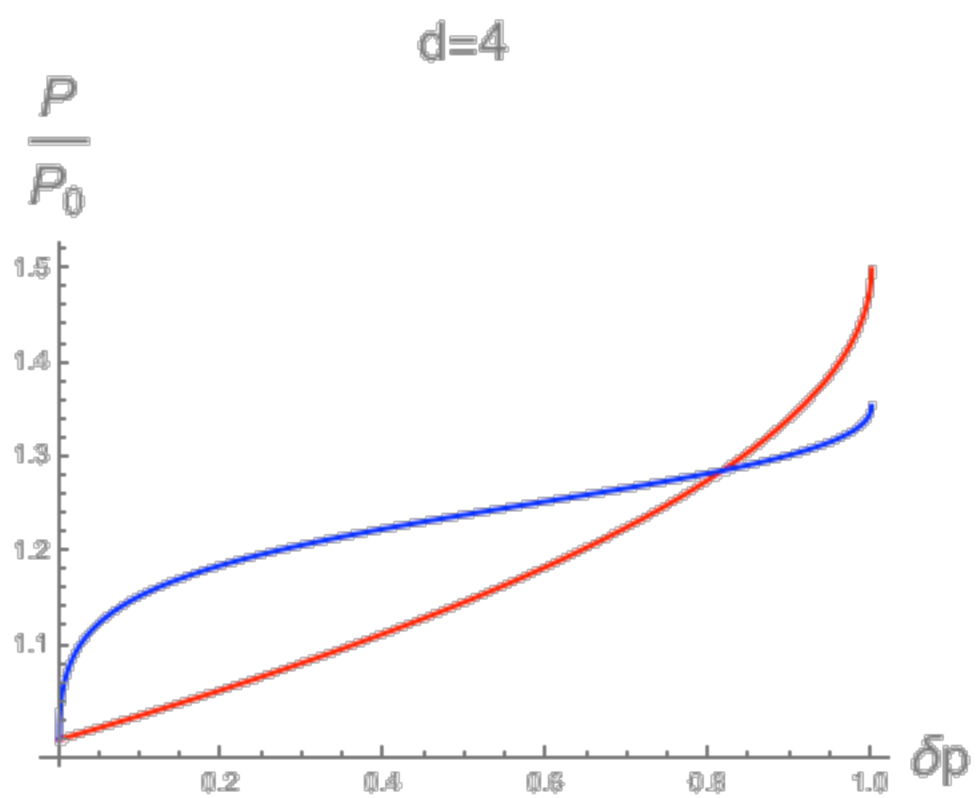

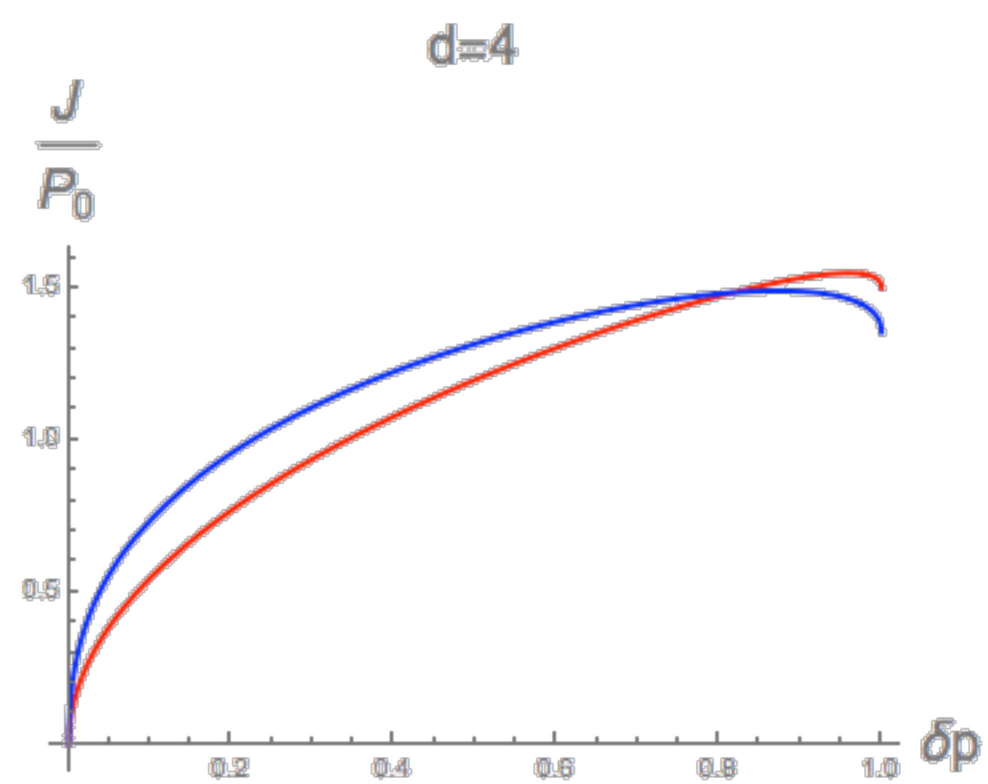

Plan:

- Prove the conjecture for 2d CFT's
- Prove the conjecture in idealized case
- Motivate the conjecture
- Provide evidence for the conjecture in non trivial configurations

Steady states in 2d CFT's

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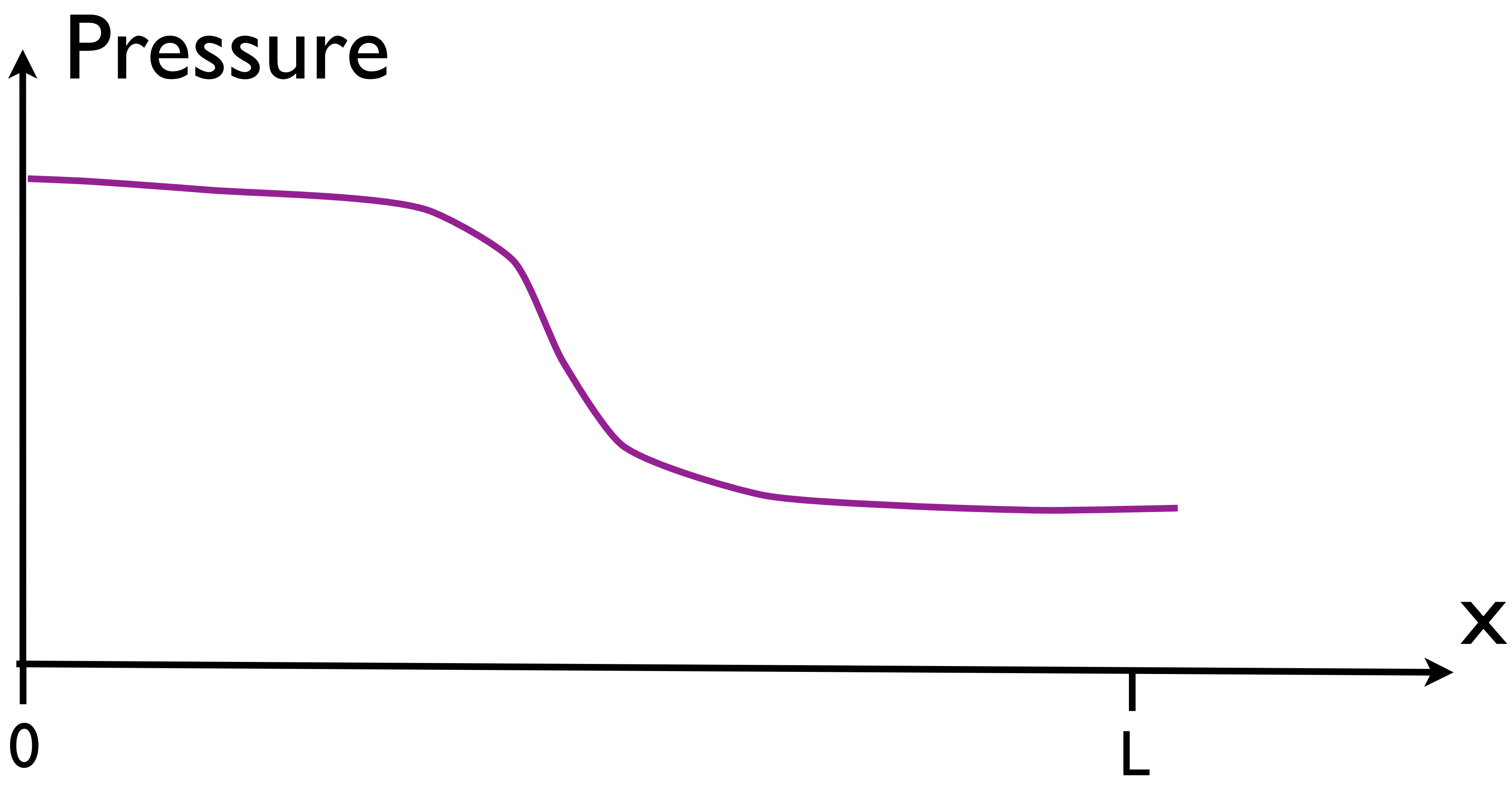

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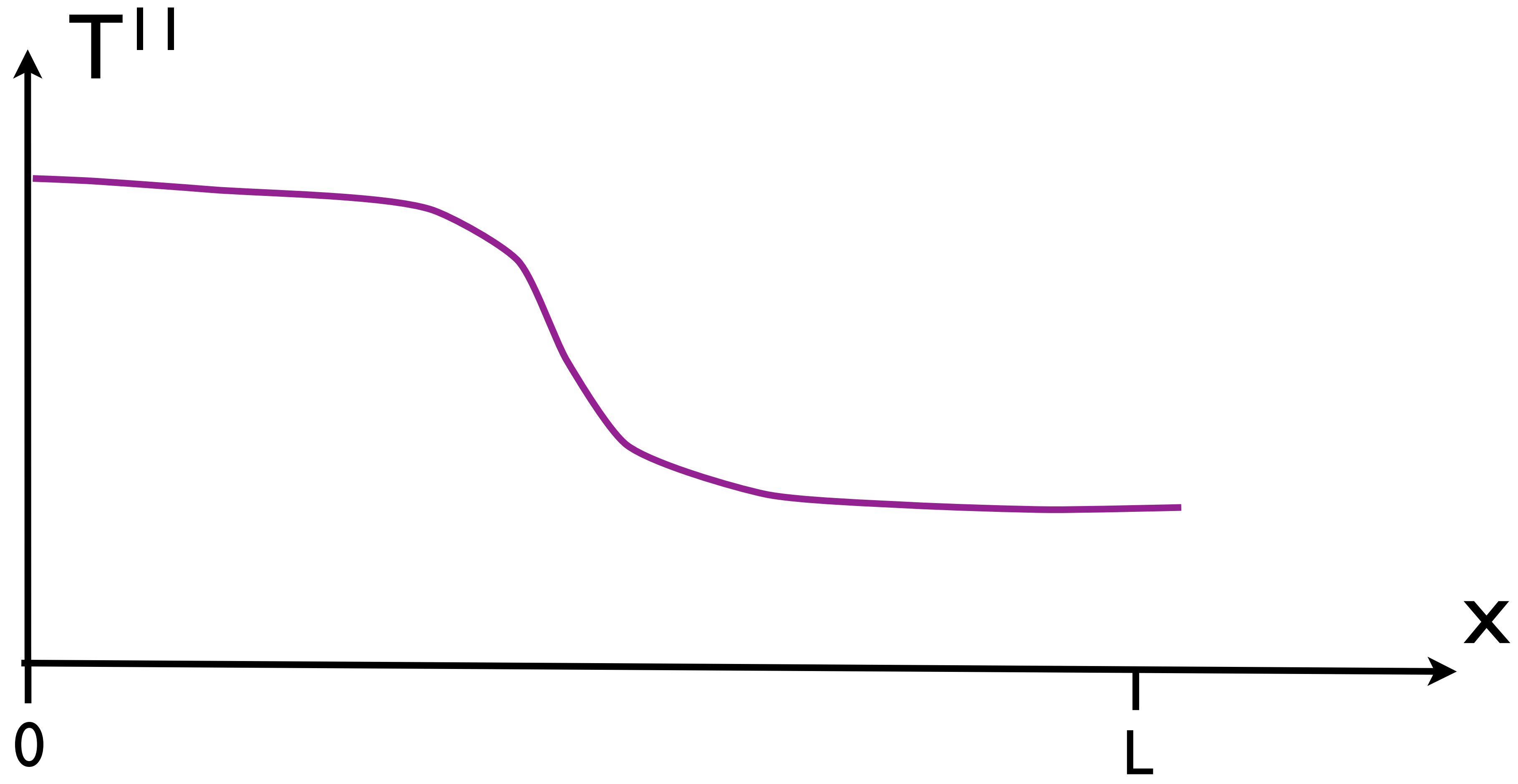

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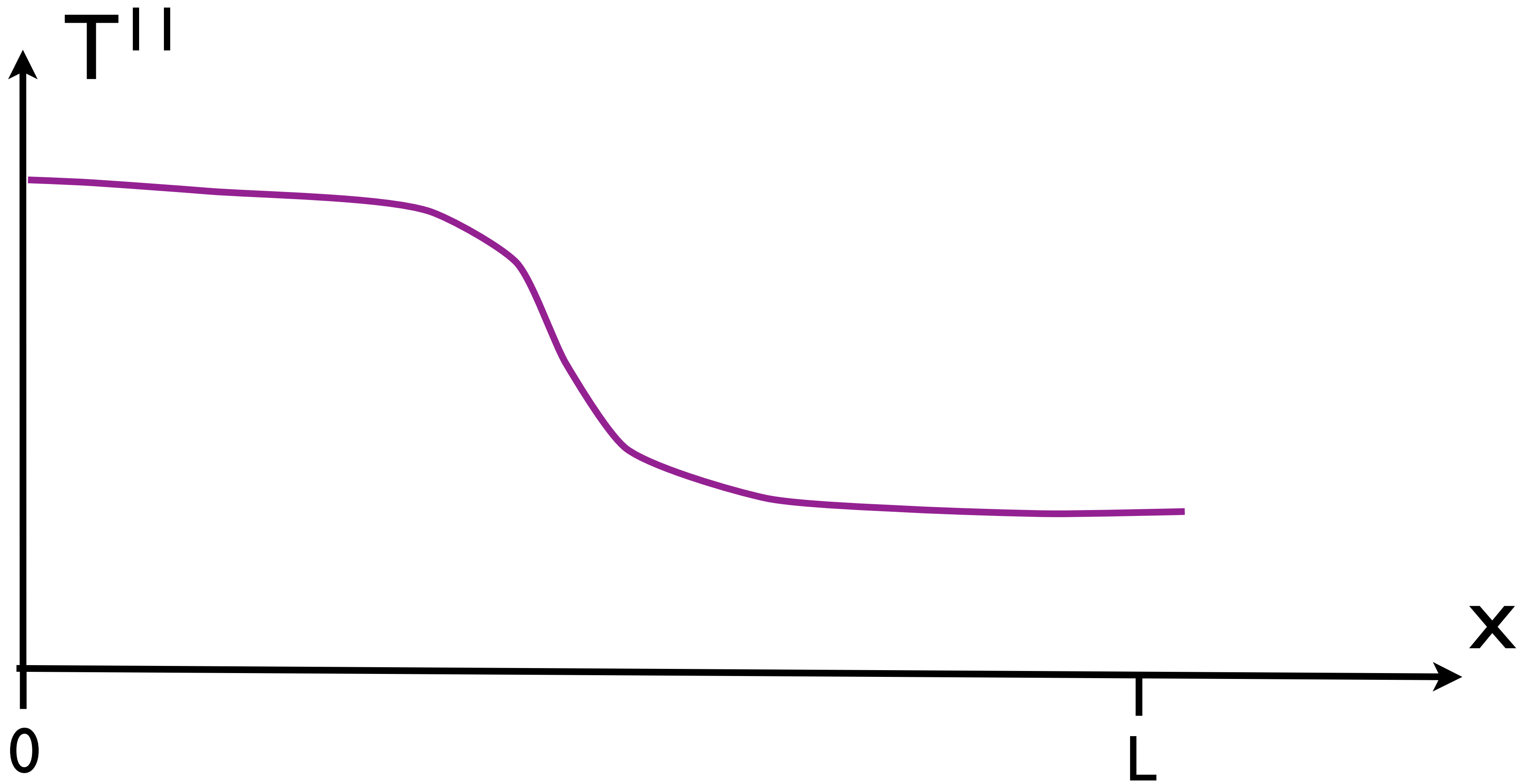

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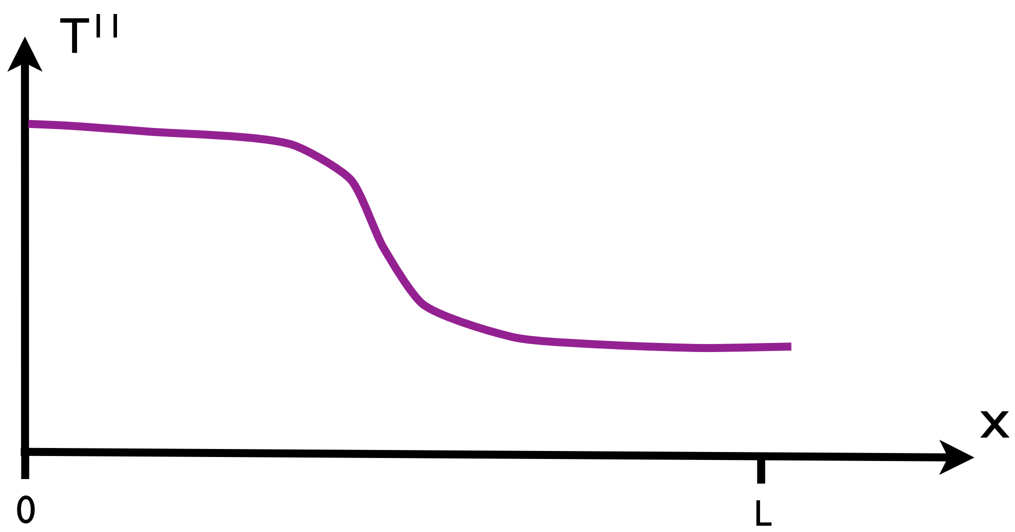

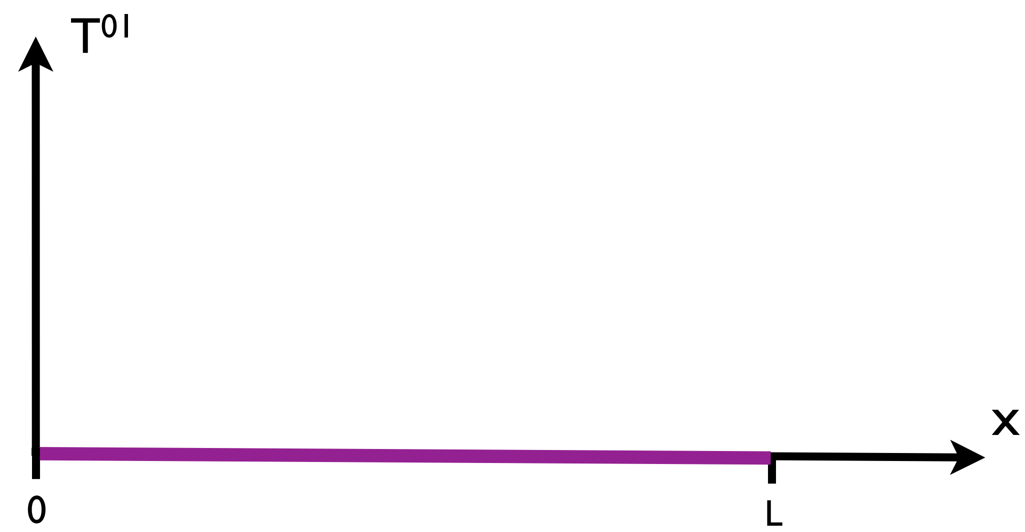

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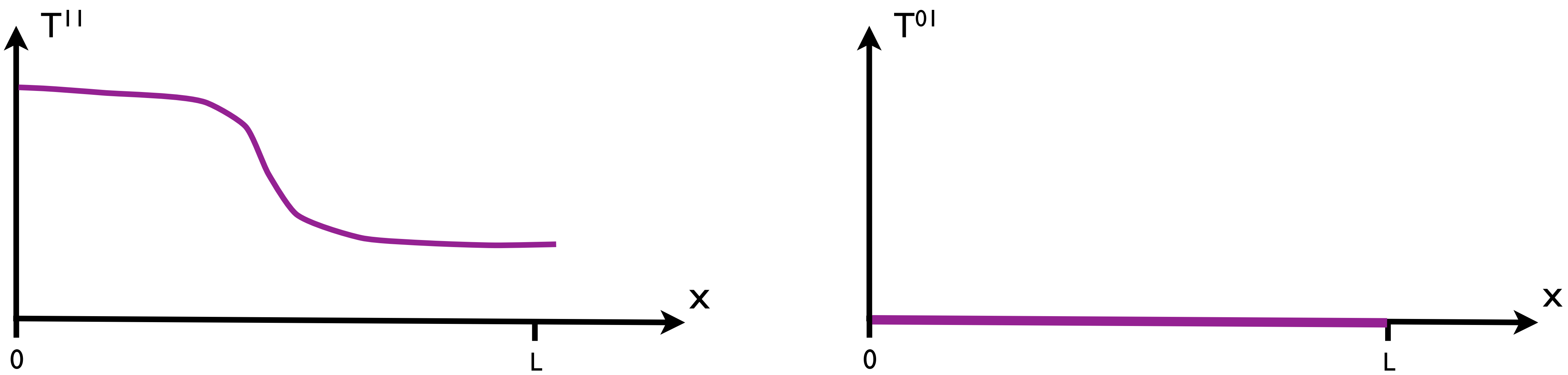

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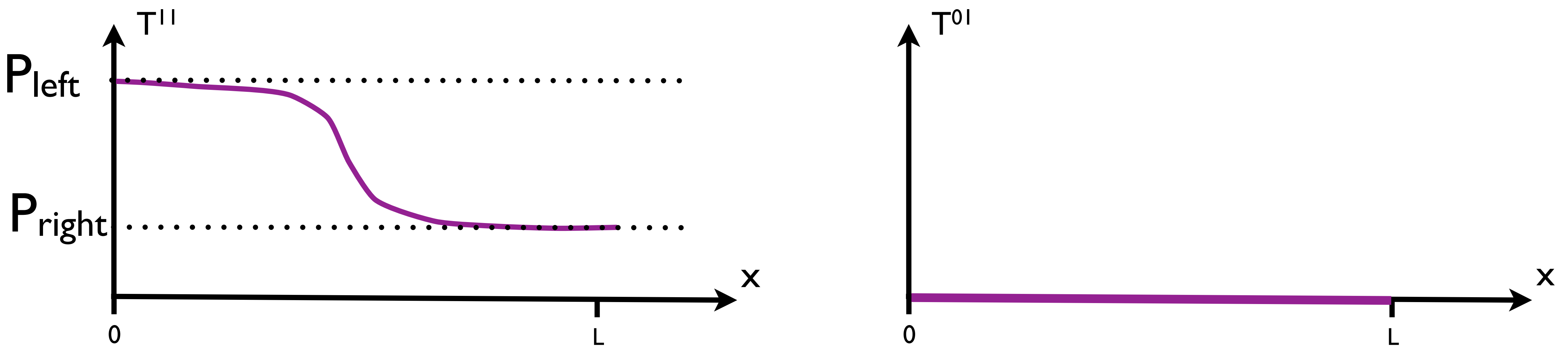

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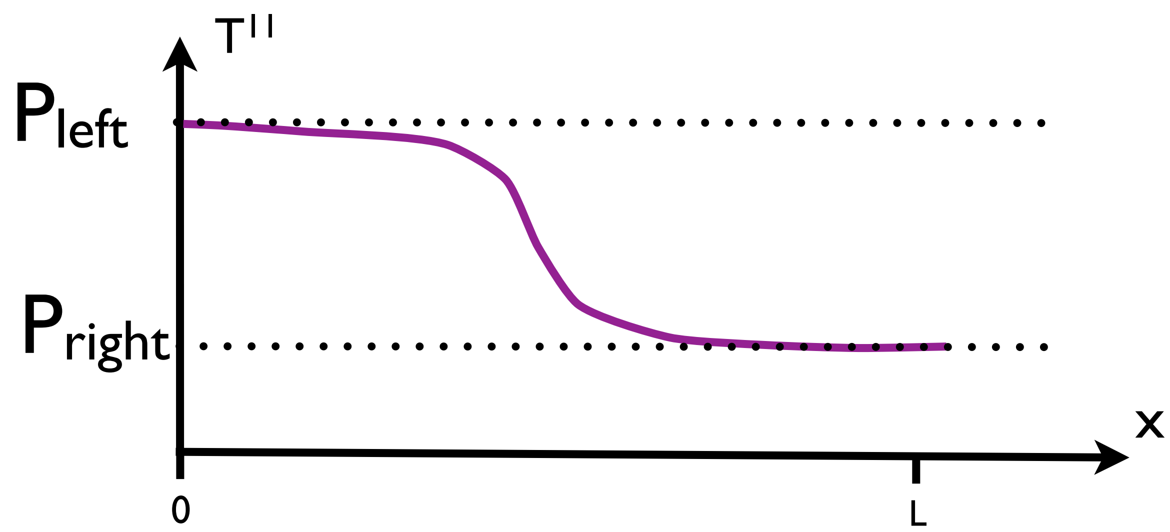

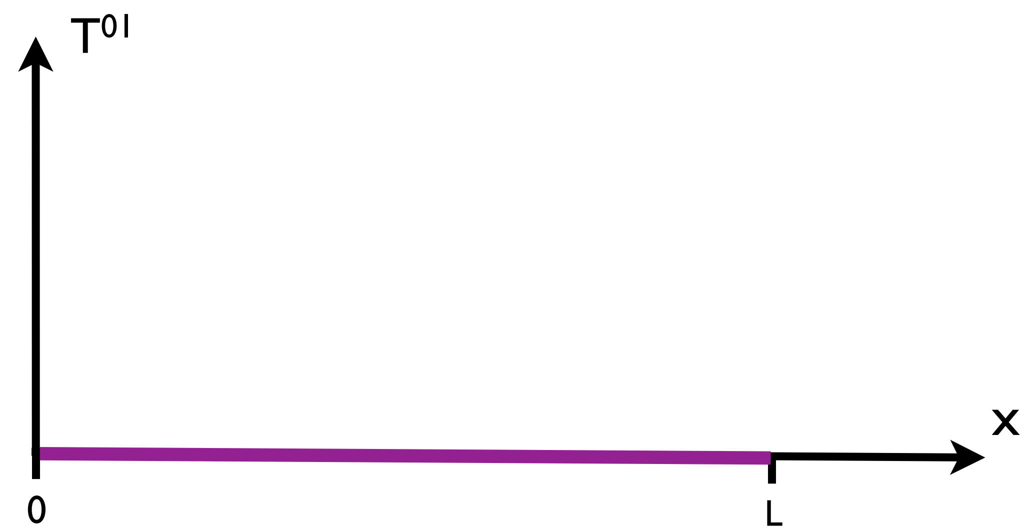

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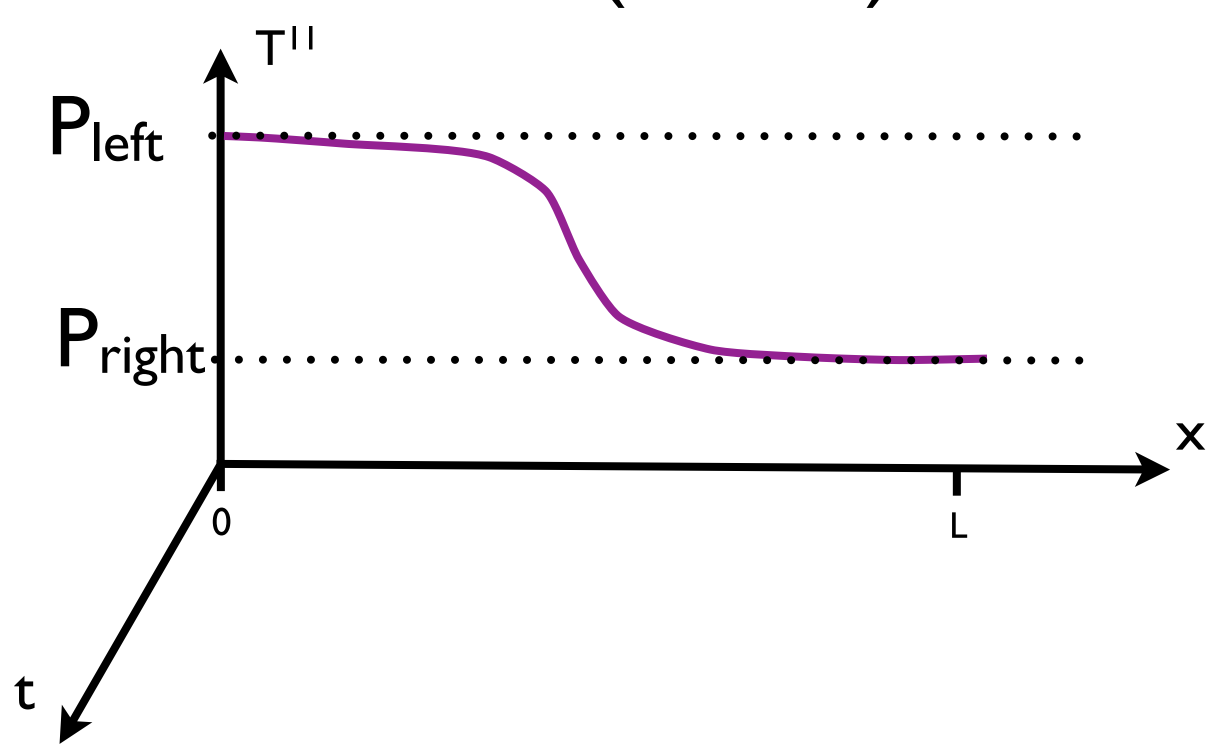

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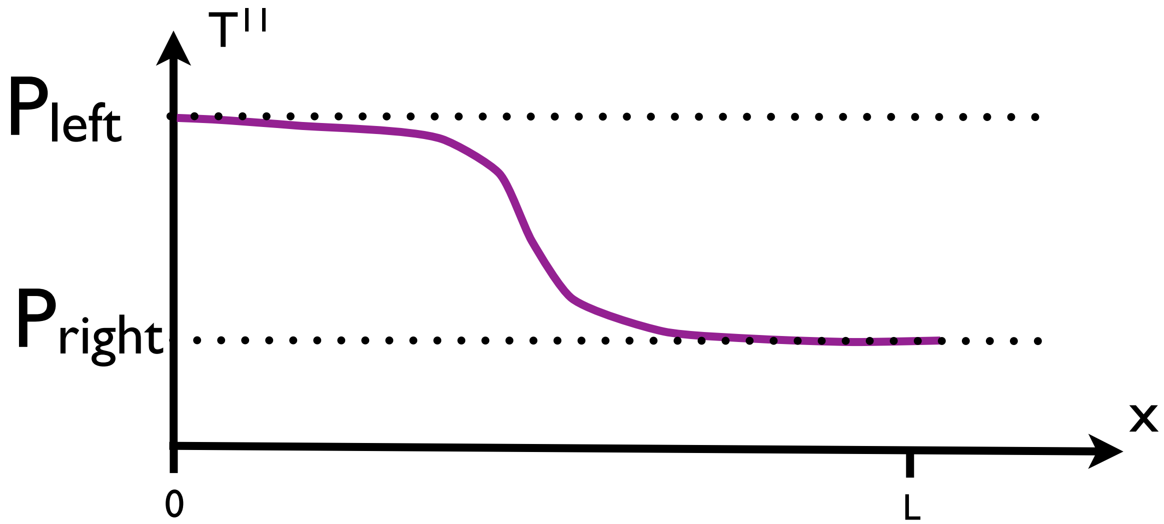

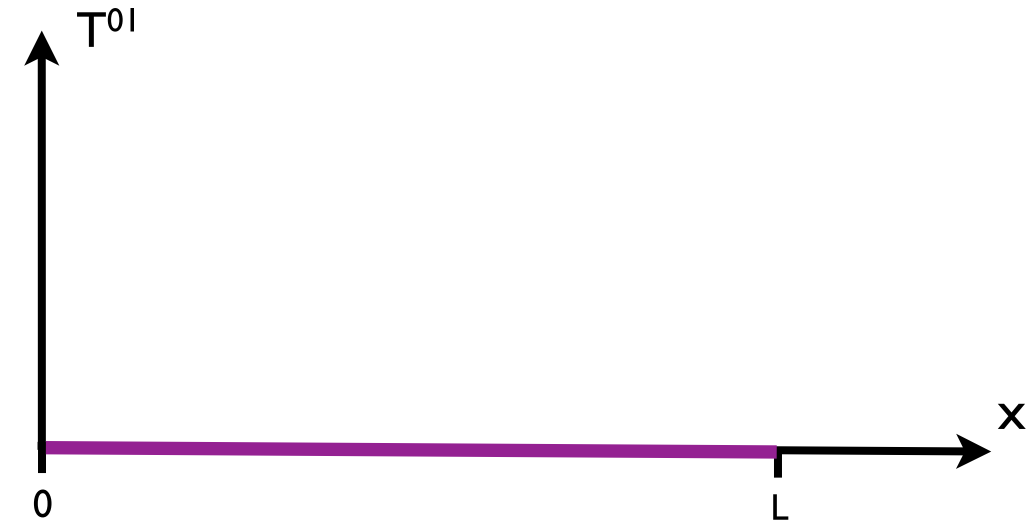

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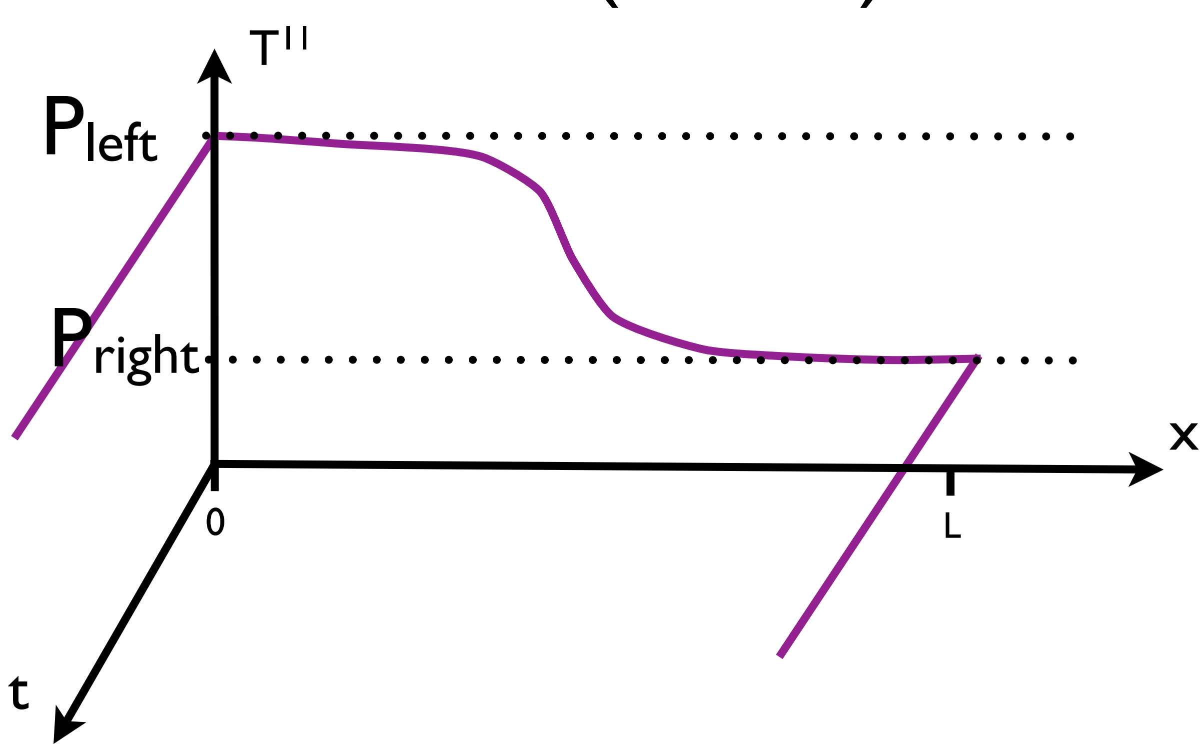

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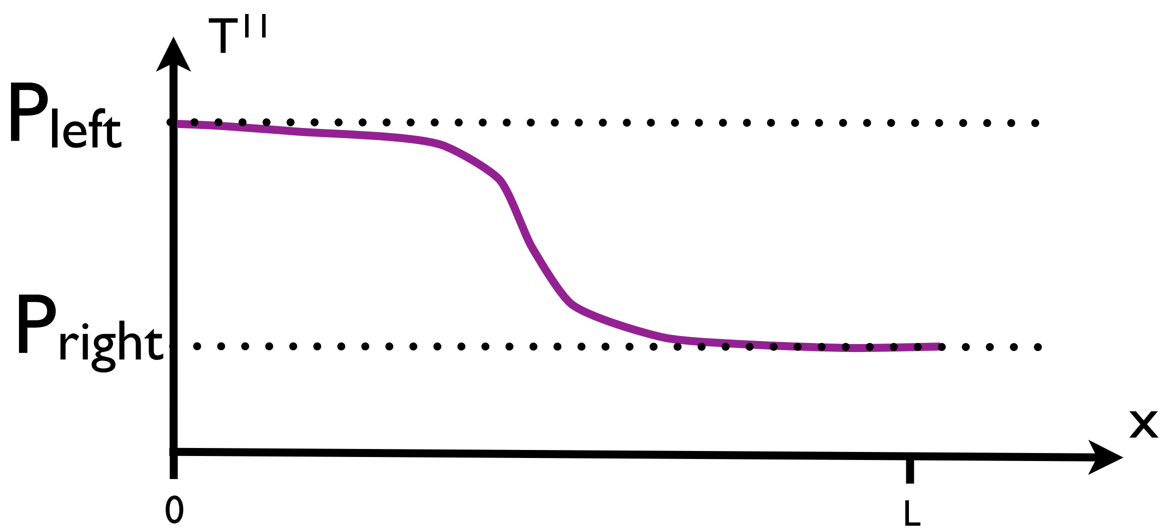

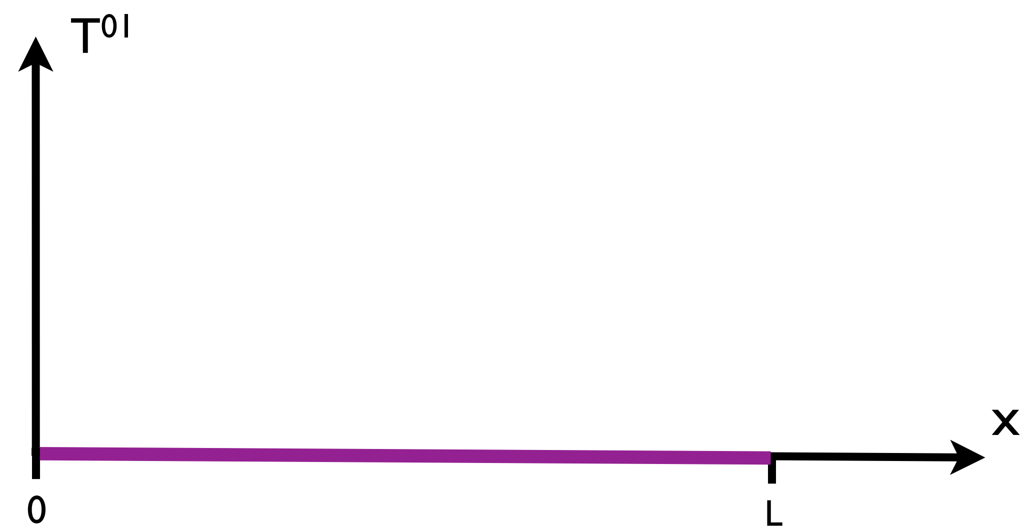

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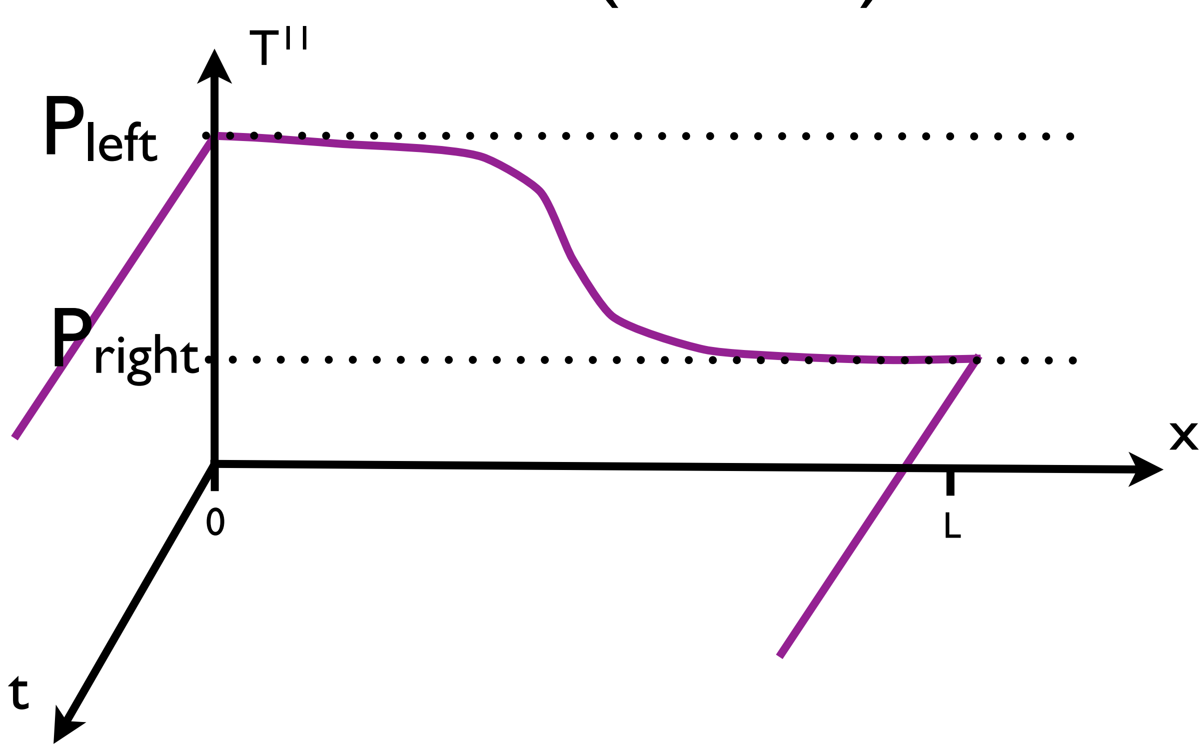

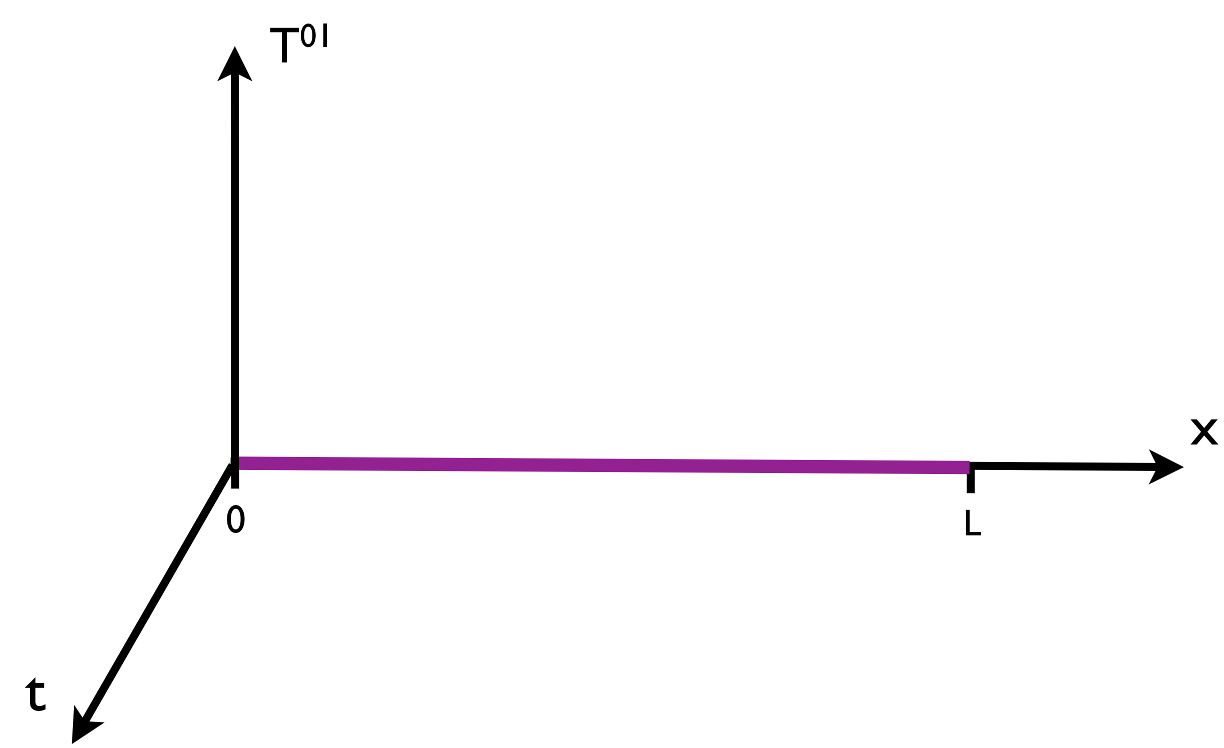

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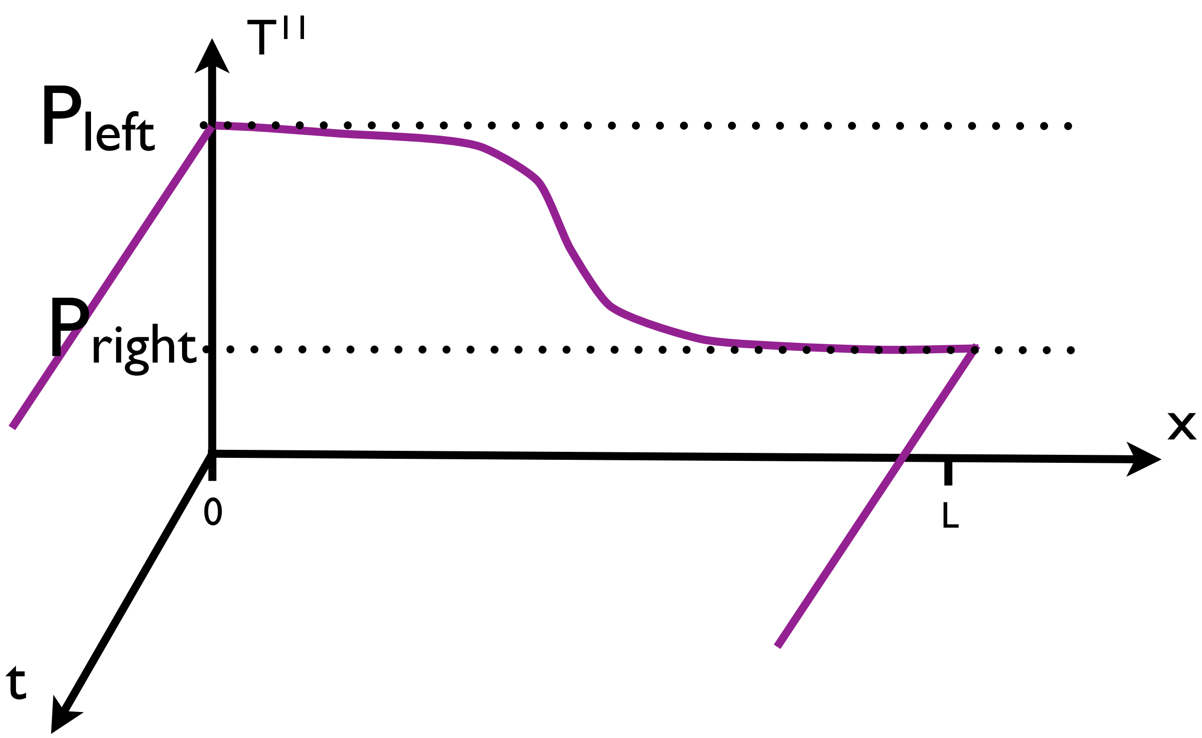

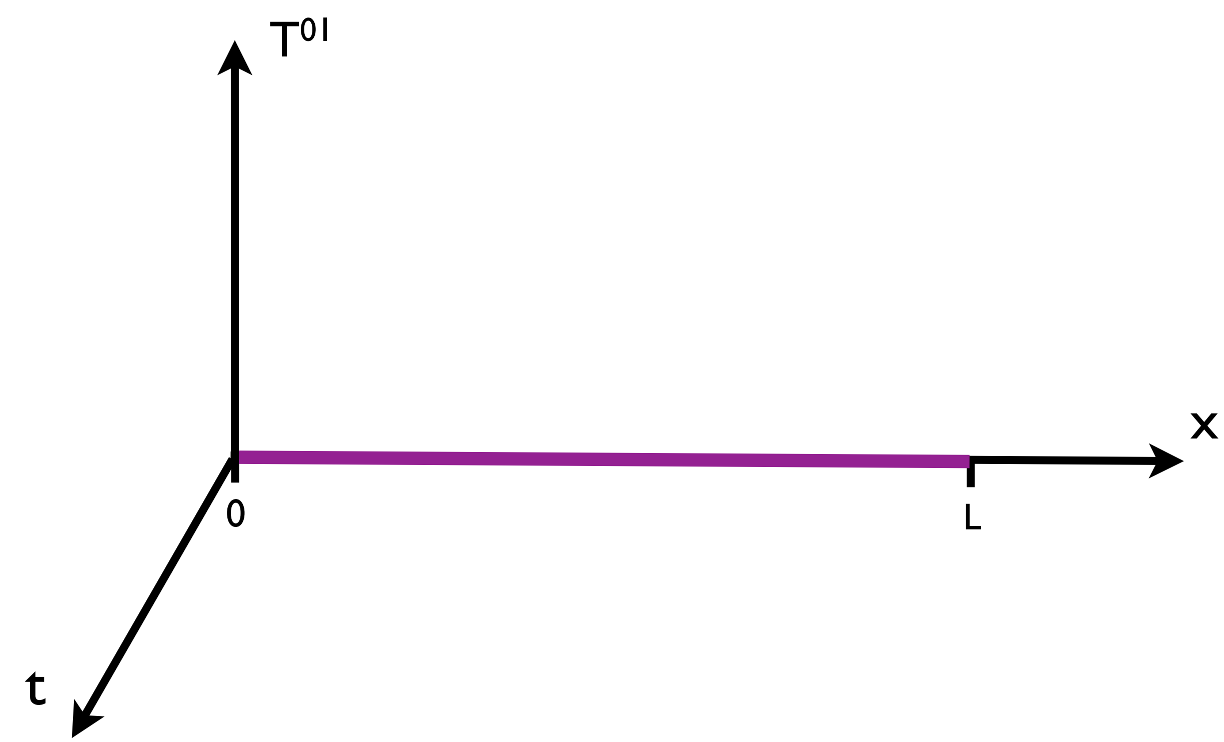

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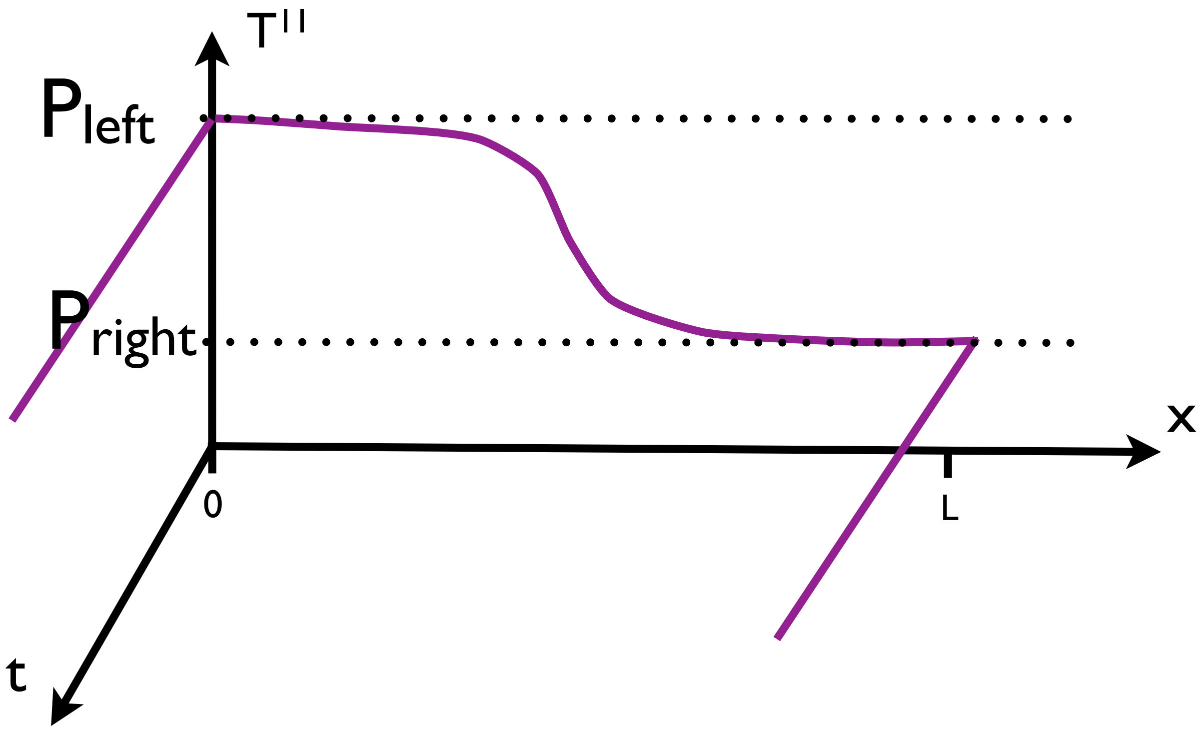

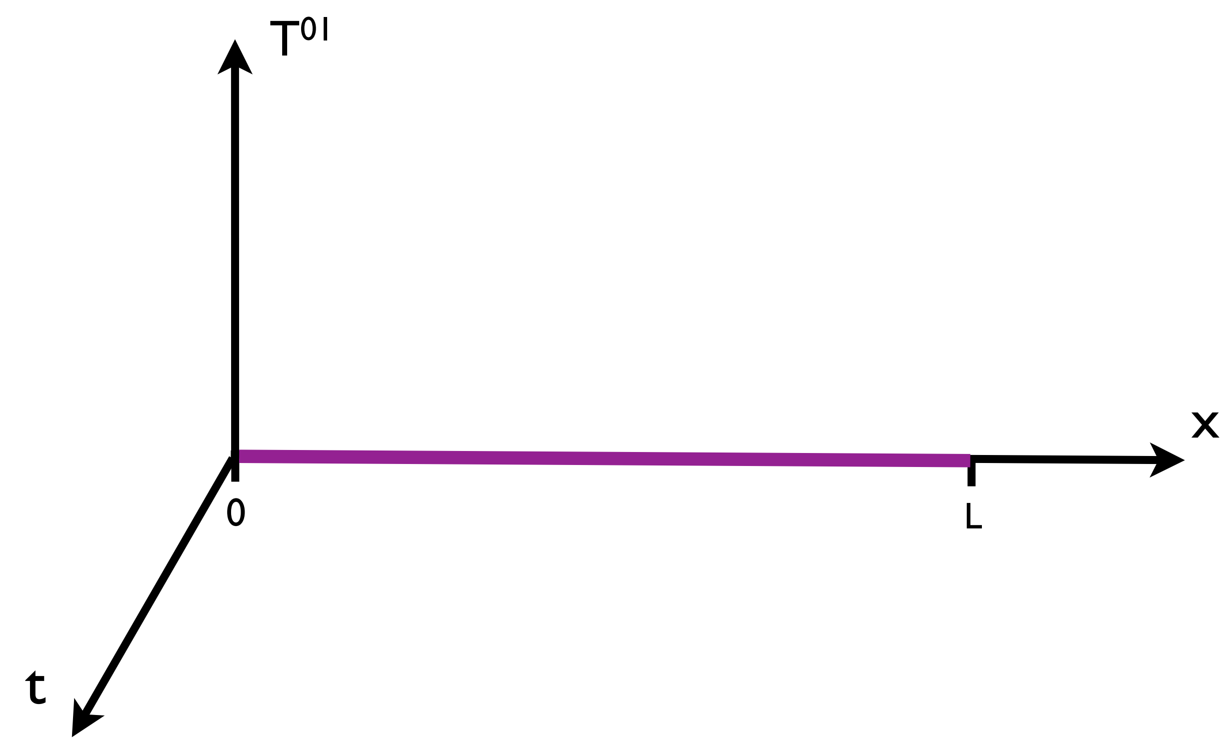

What are T^{11} and T^{01} for all t and x?

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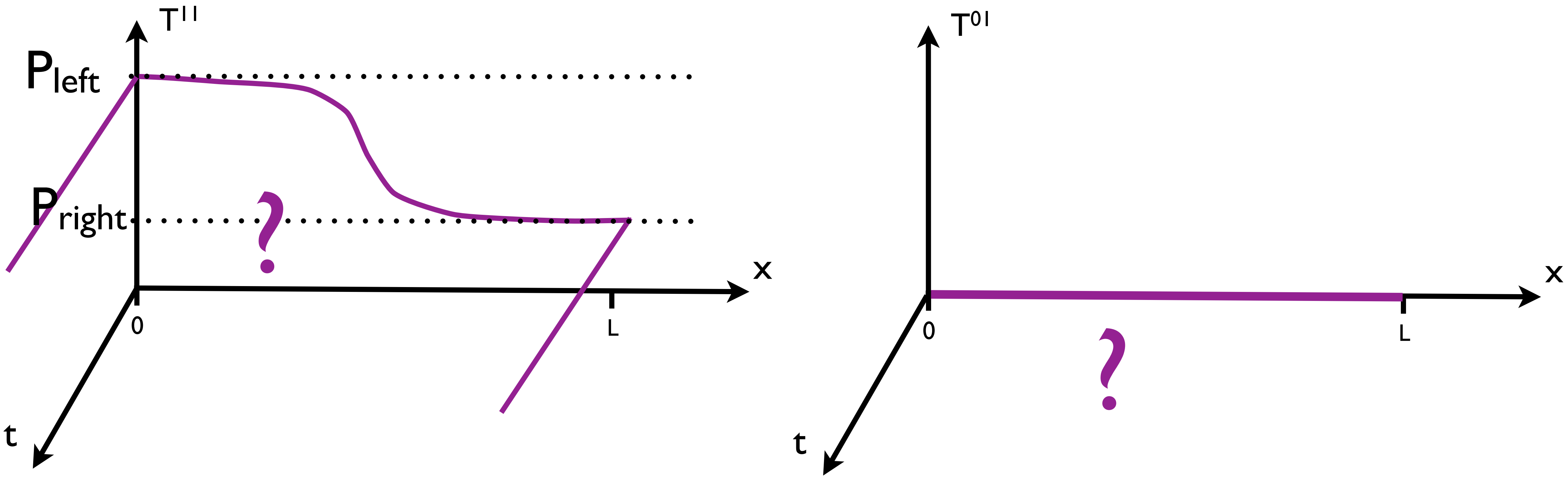

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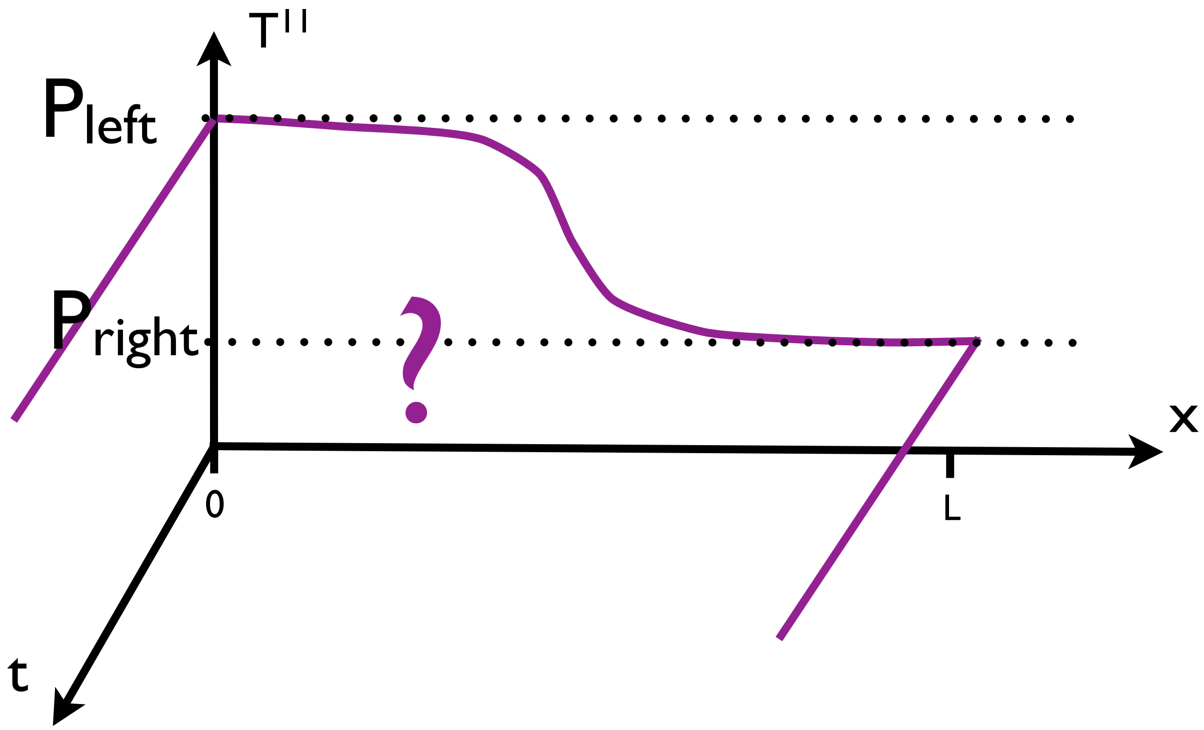
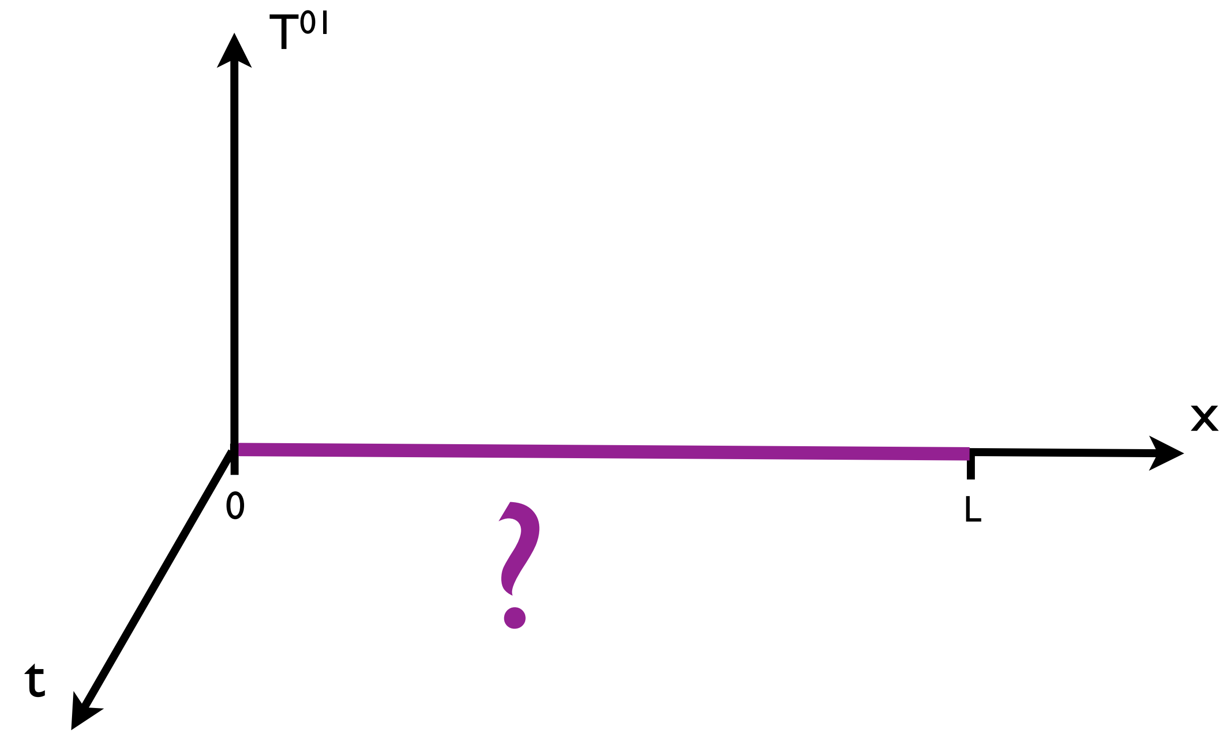
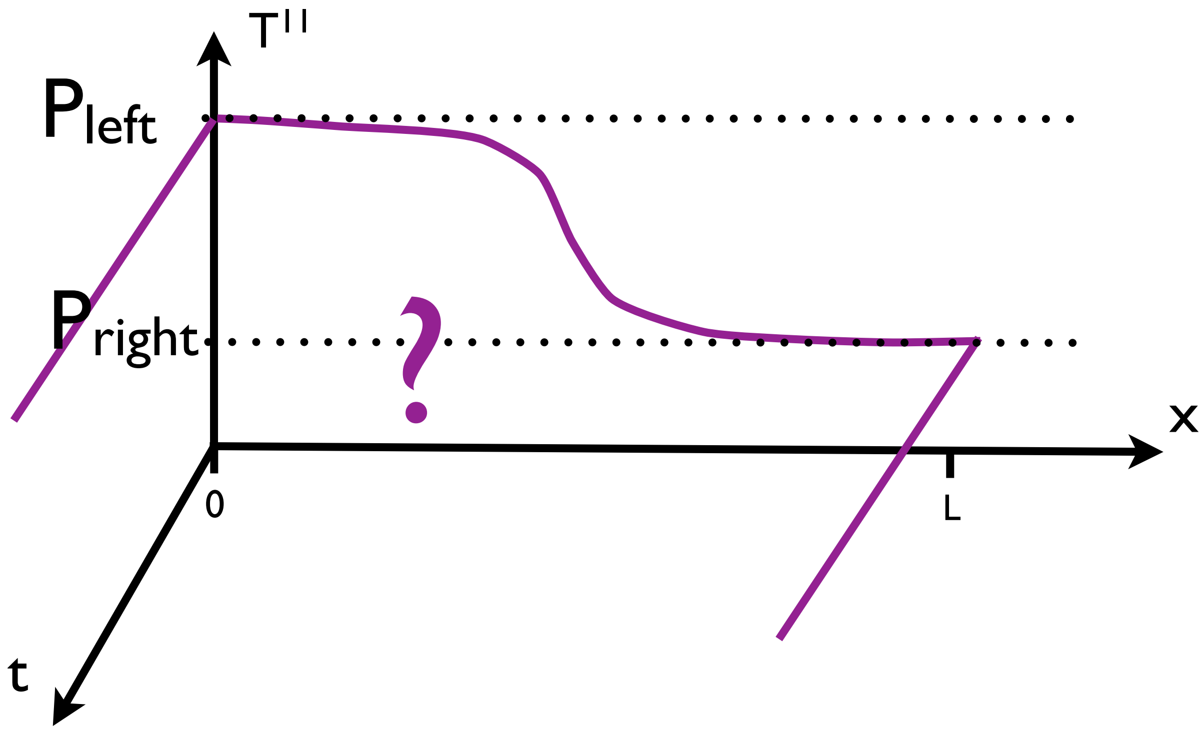


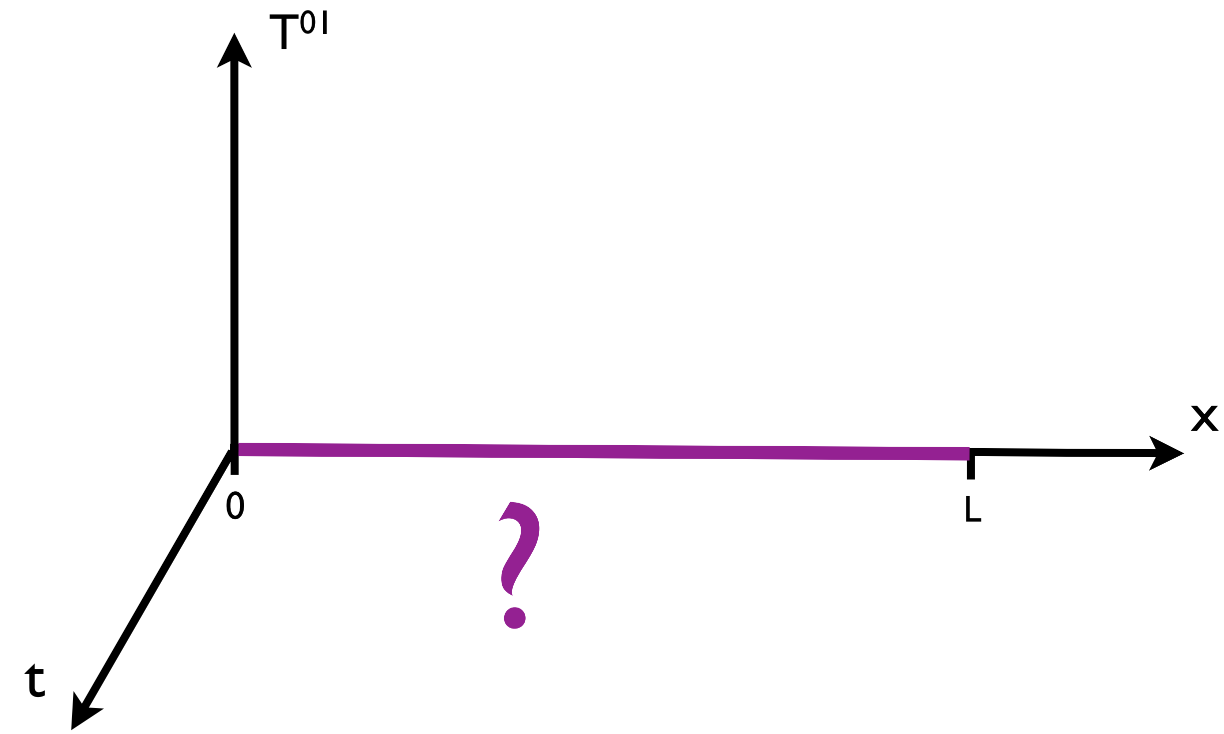


For a conformal theory (using $ds^2 = dzd\bar{z}$)

$$T^{zz} = T(z) \qquad T^{\bar{z}\bar{z}} = \bar{T}(\bar{z}) \qquad T^{\bar{z}z} = 0$$

Steady states in 2d CFT's





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In the $ds^2 = -dt^2 + dx^2$ coordinate system

$$T^{\mu\nu} = \begin{pmatrix} T_+(t+x) + T_-(-t+x) & T_-(-t+x) - T_+(t+x) \\ T_-(-t+x) - T_+(t+x) & T_+(t+x) + T_-(-t+x) \end{pmatrix}$$

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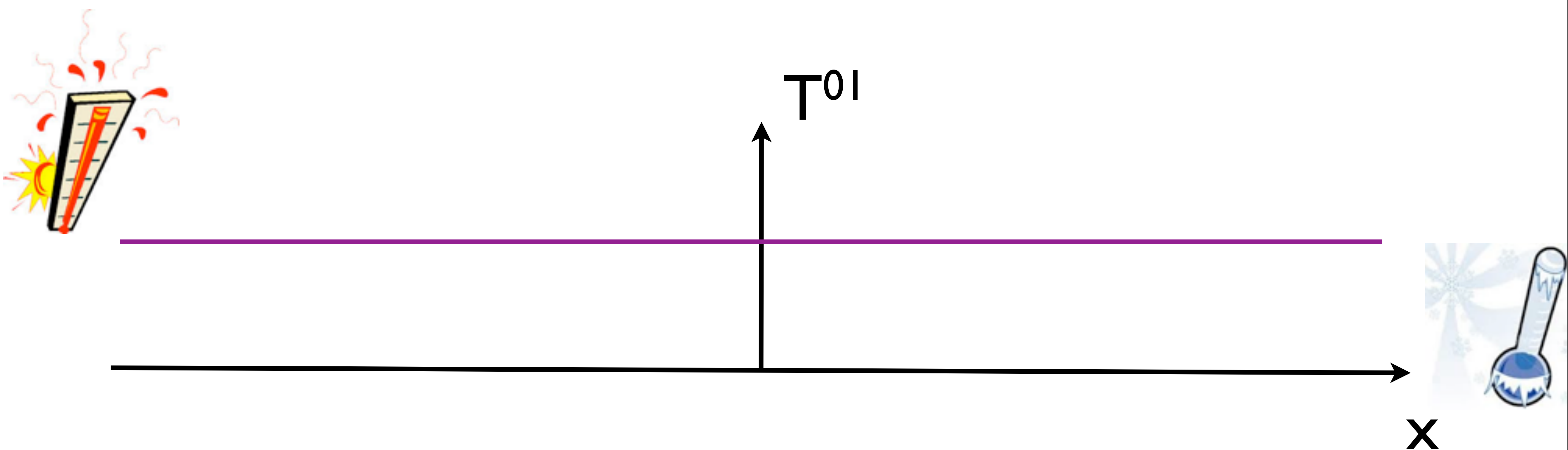
Steady states in 2d CFT's

The exact same analysis can be used to consider more complicated configurations:



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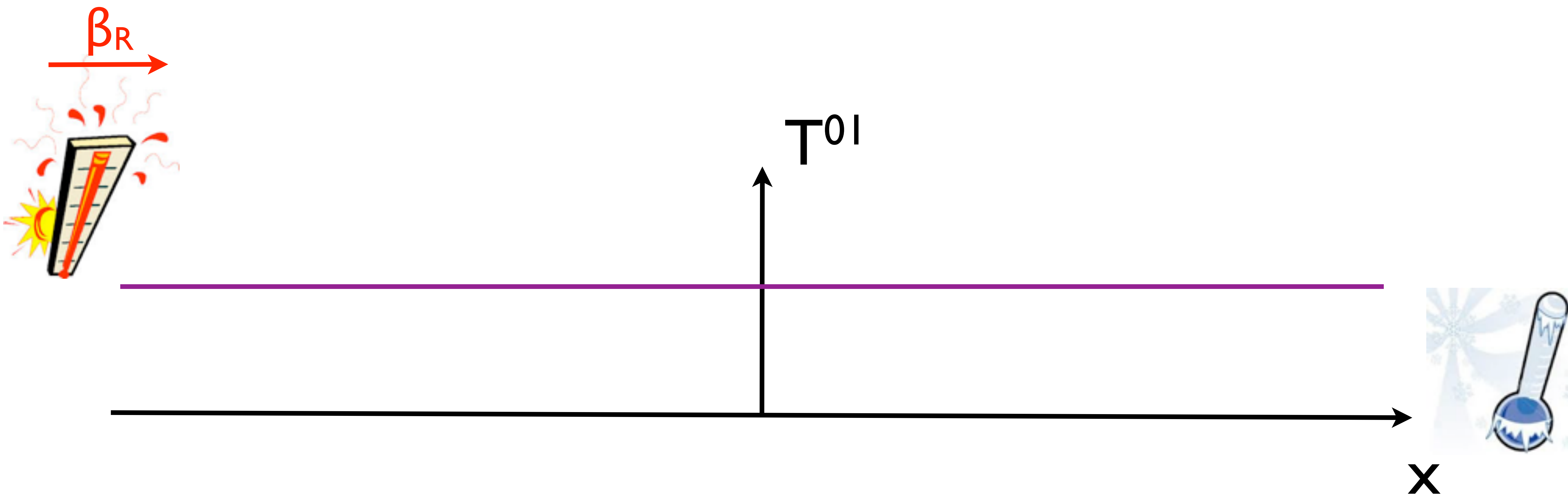

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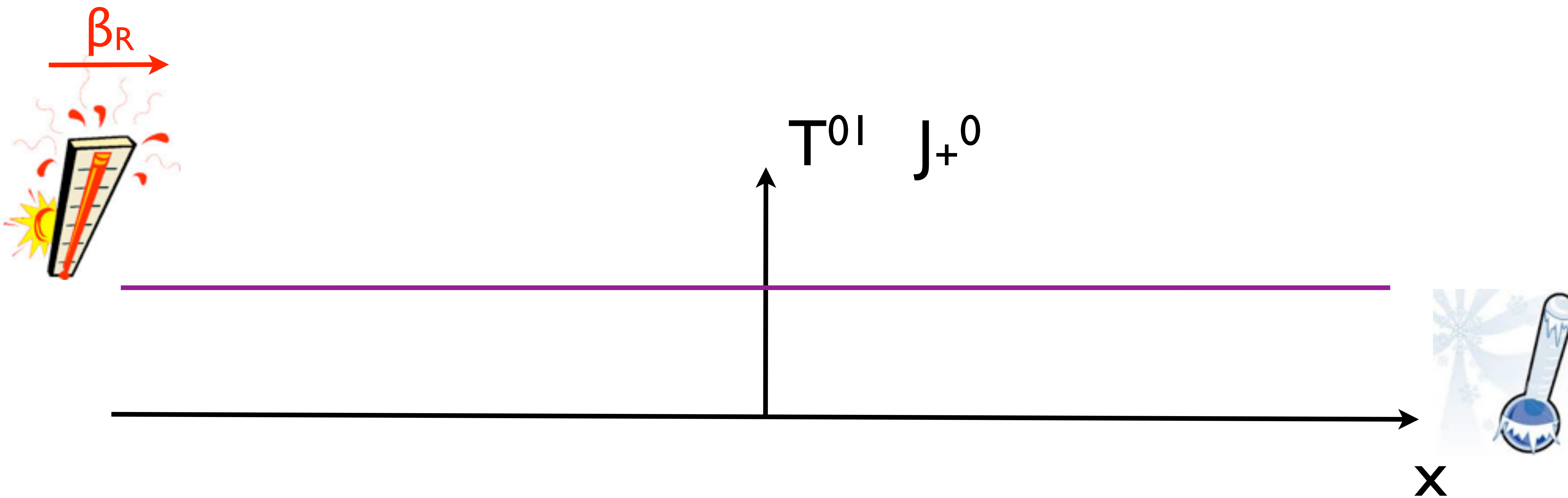

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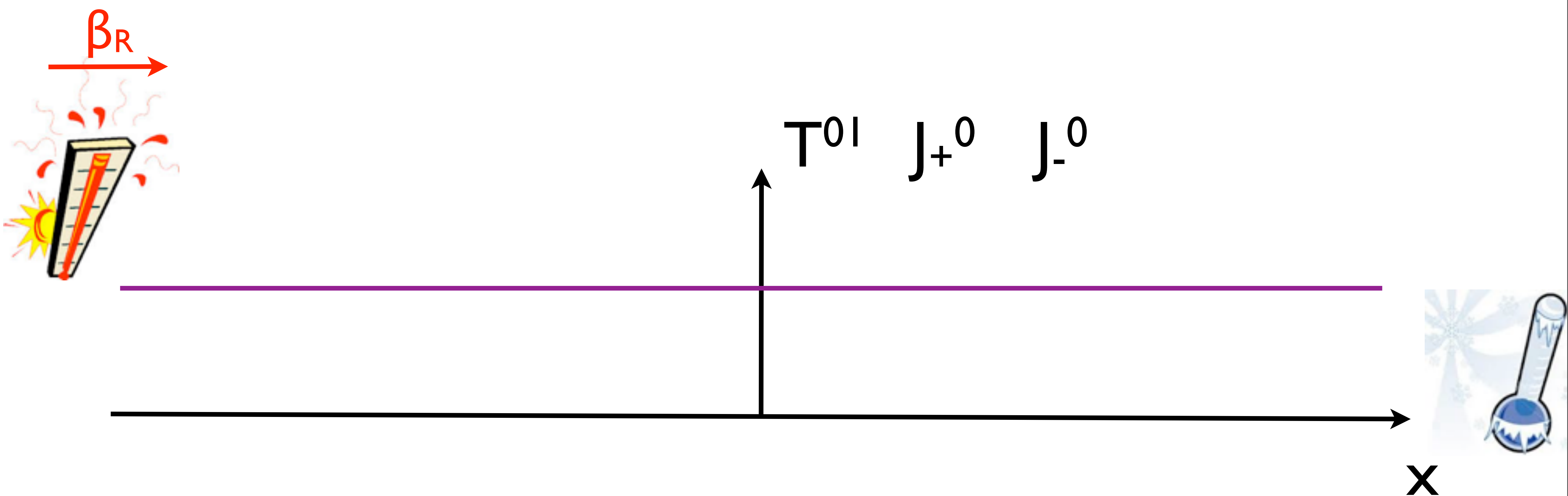

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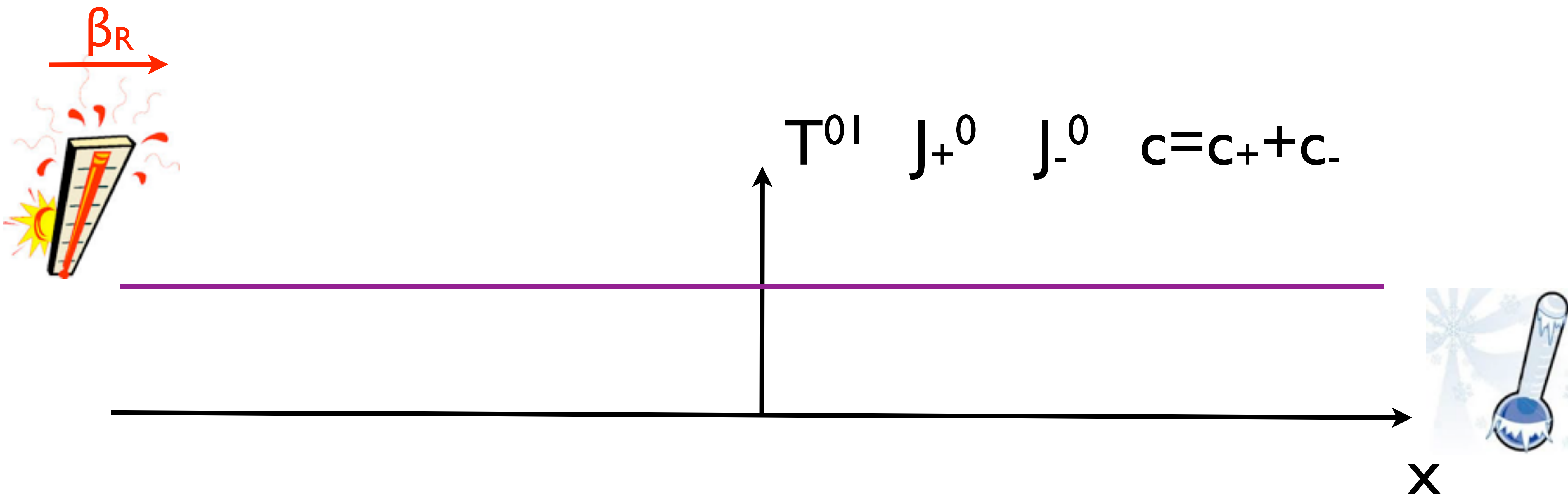

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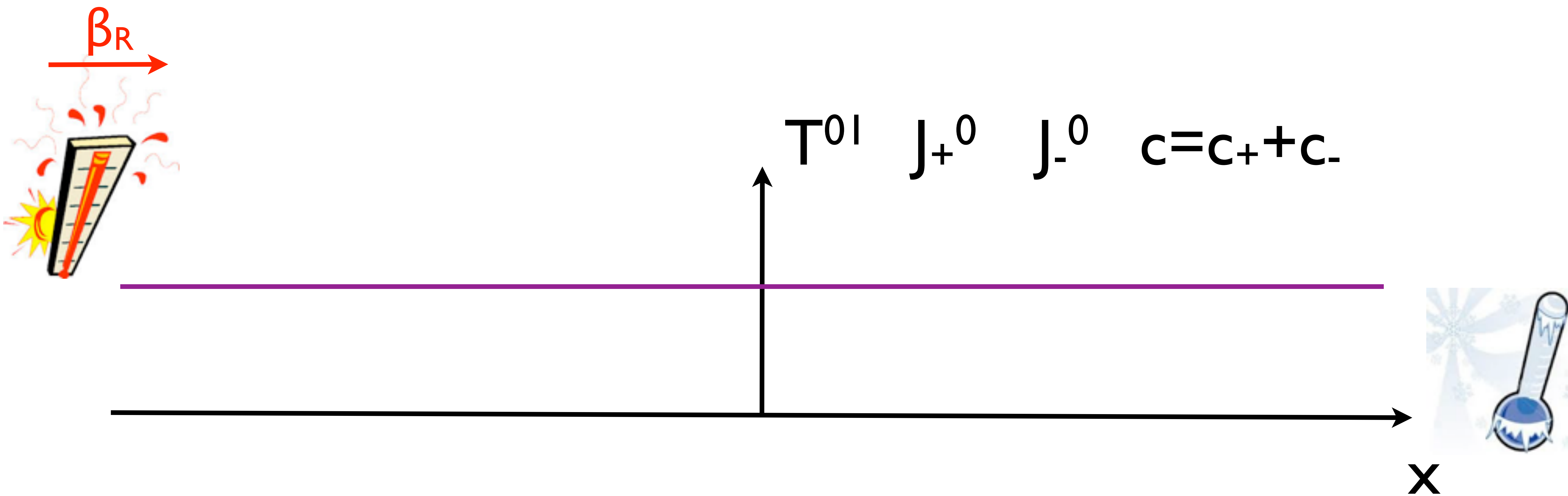

$$T^{01}(t \to \infty) = \frac{\pi}{12}\left(c_- T_L^2 - c_+ T_R^2 \frac{1-\beta_R}{1+\beta_R}\right) + \frac{1}{2\pi}\left(k_- \mu_L^- - k_+ \mu_R^+ \frac{1-\beta_R}{1+\beta_R}\right)$$

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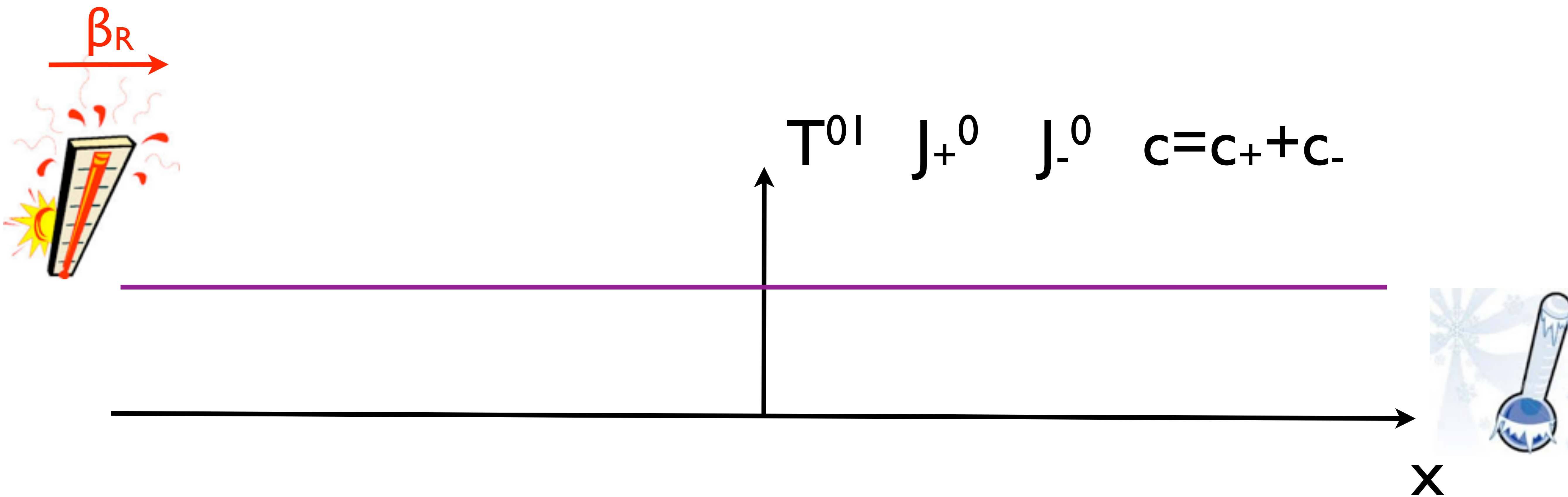

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(Note that: $T^{01}(t\to\infty)=\frac{\pi}{12}\left(c_-T_L^2-c_+T_R^2\right)$)

Steady states in 2d CFT's

Main ingredient:

$$T^{\mu\nu} = \begin{pmatrix} T_+(t+x) + T_-(-t+x) & T_-(-t+x) - T_+(t+x) \\ T_-(-t+x) - T_+(t+x) & T_+(t+x) + T_-(-t+x) \end{pmatrix}$$

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It follows from:

$$\partial_\mu T^{\mu\nu} = 0\,, \quad T^\mu{}_\mu = 0$$

Generalizing to higher dimensions

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We need more input.

Higher dimensions: an idealized case

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energy density — 4-velocity — Pressure

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speed of sound

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So we can use the same strategy as before to obtain the late time behavior of the pressure and velocity:

At $x \to \mp\infty$ we impose that the system is connected to a heat bath. This determines the $t \to \infty$ behavior

$$T^{00}(t \to \infty) = (d-1)P_0 \,, \quad T^{01}(t \to \infty) = \frac{\Delta P}{c_s} \,, \quad T^{11}(t \to \infty) = P_0$$

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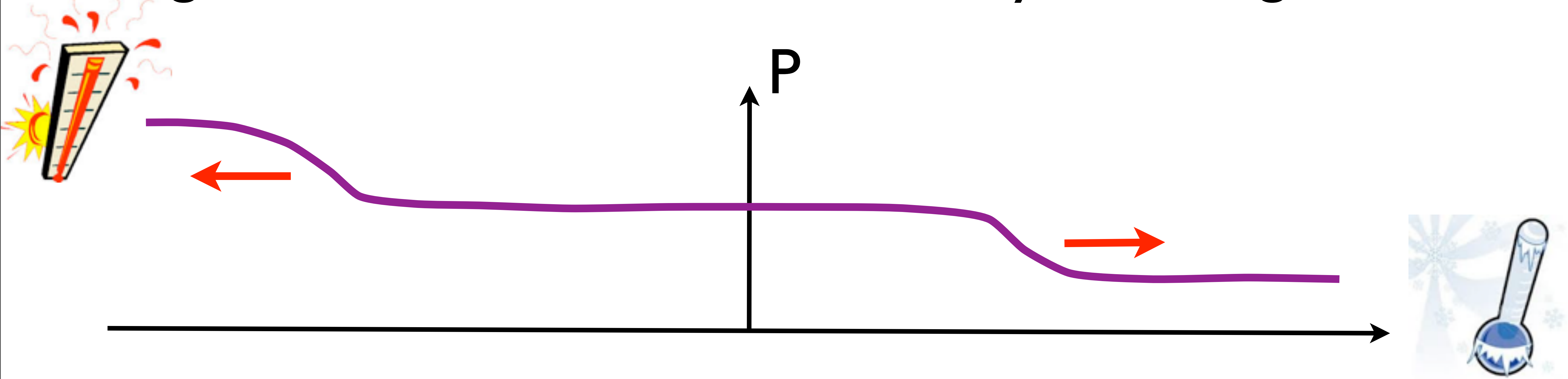
At late times sound modes propogating towards the heat bath generated an intermediate steady state region.



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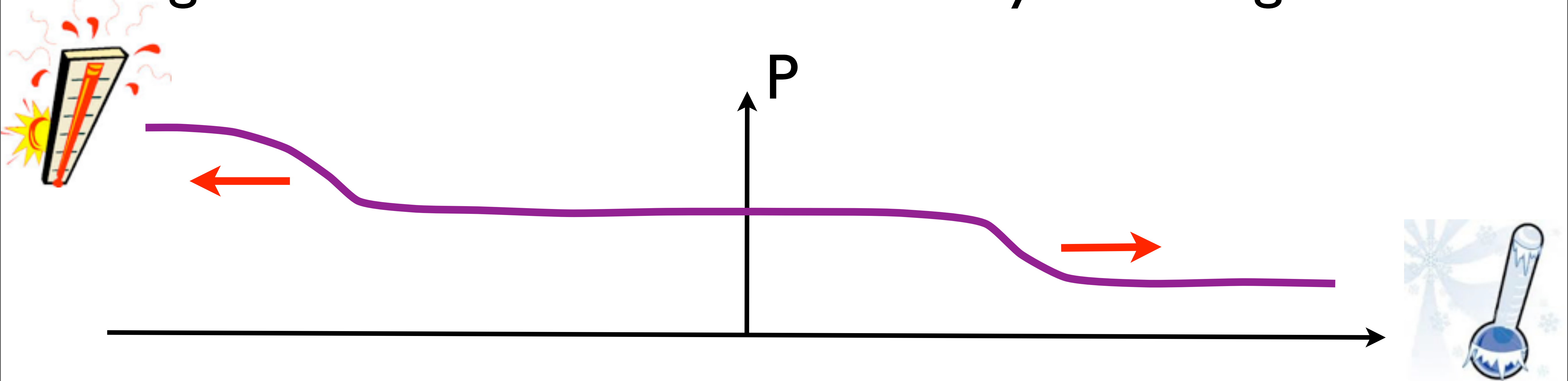

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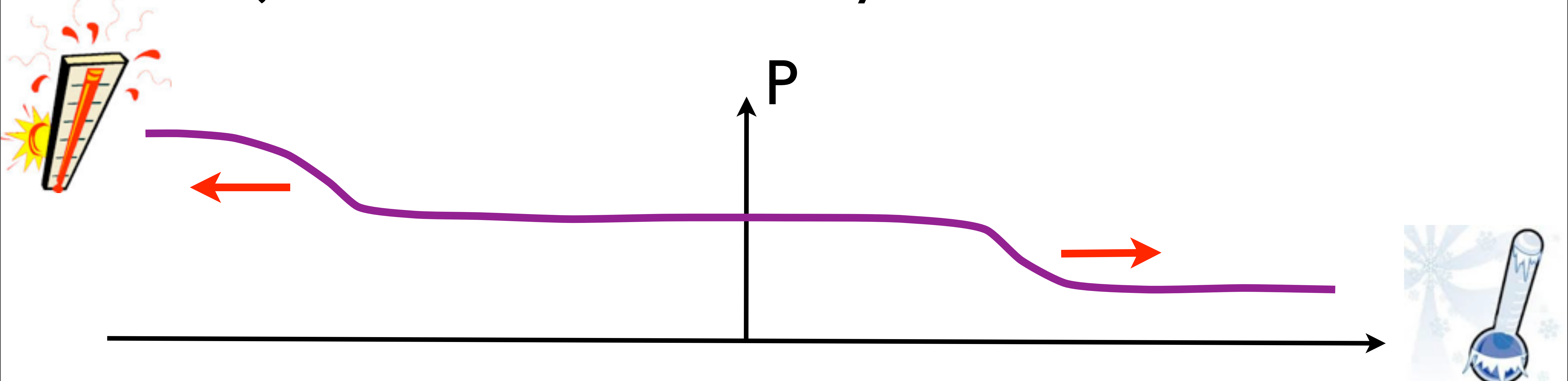

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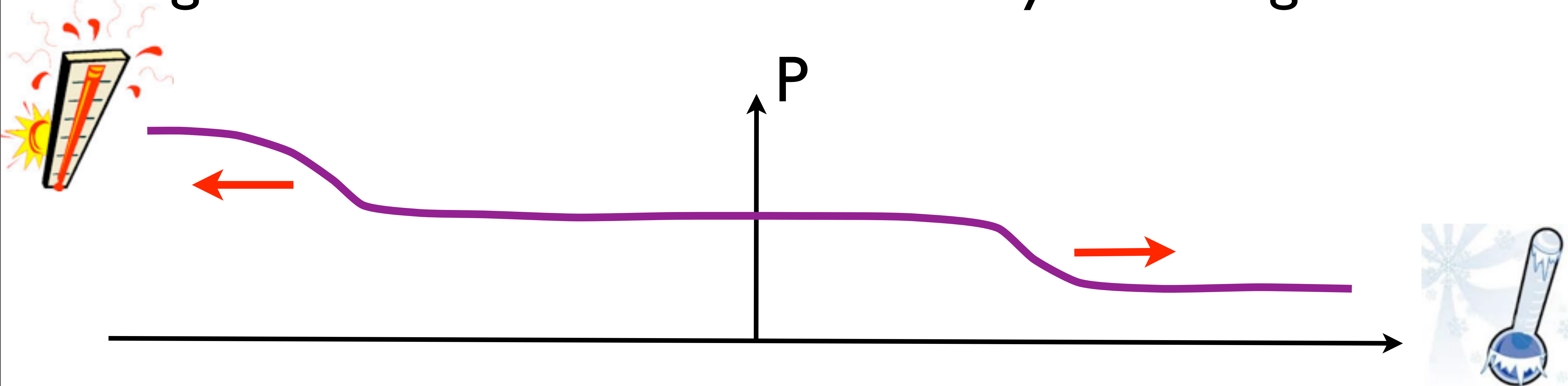

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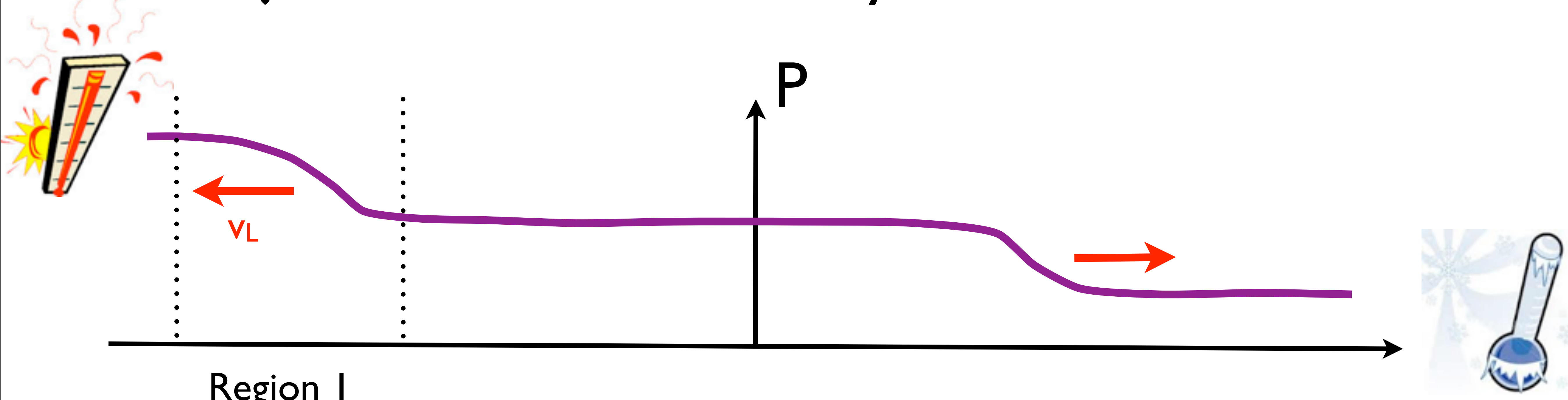

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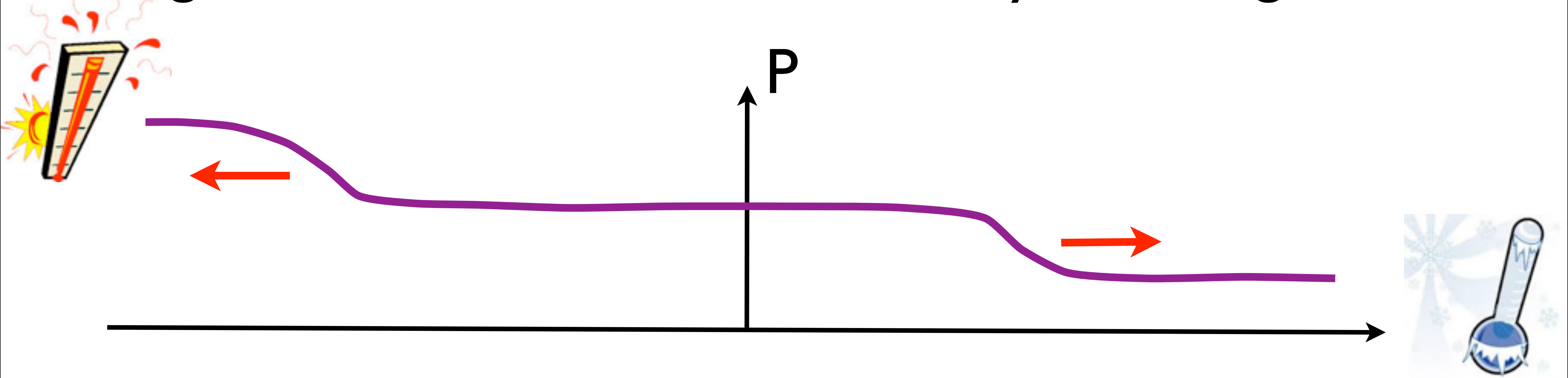

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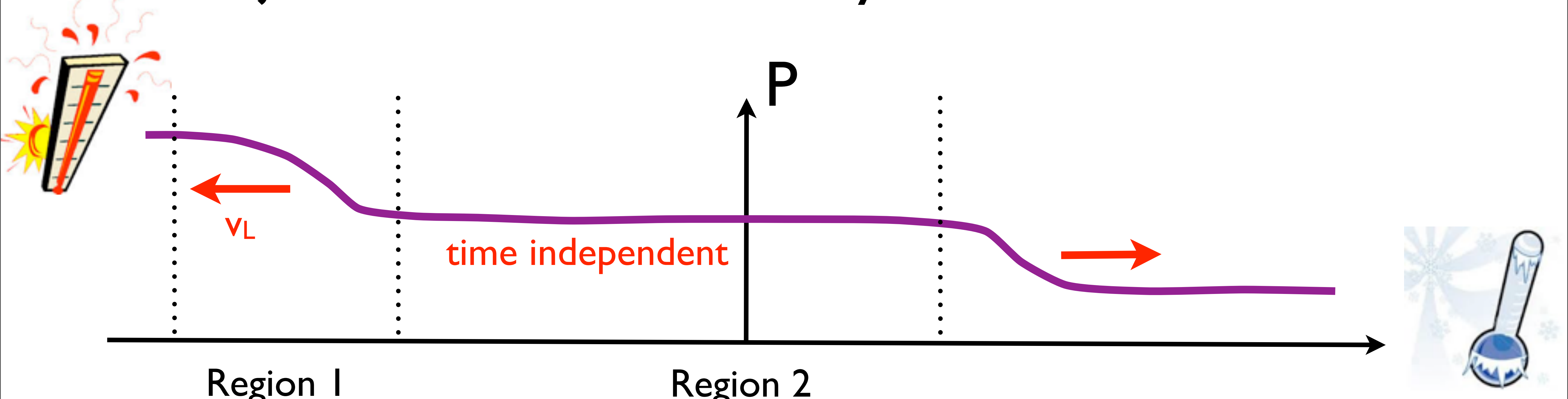

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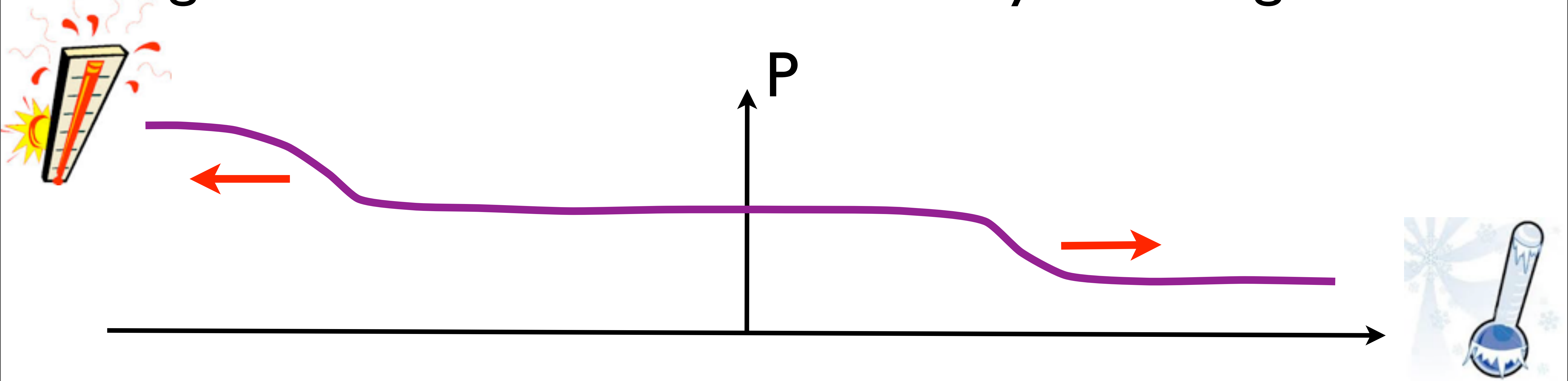

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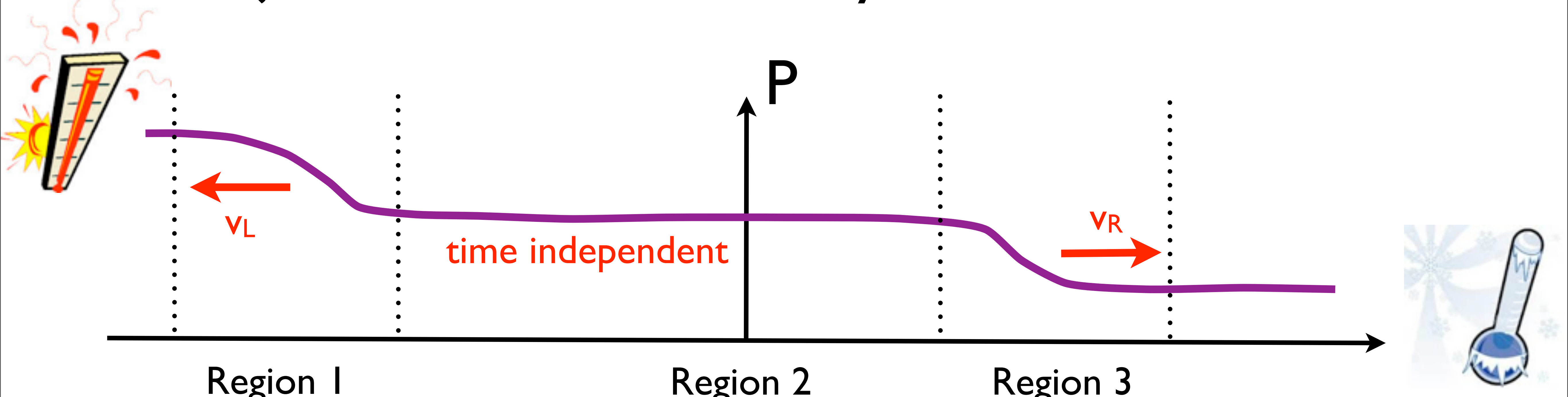

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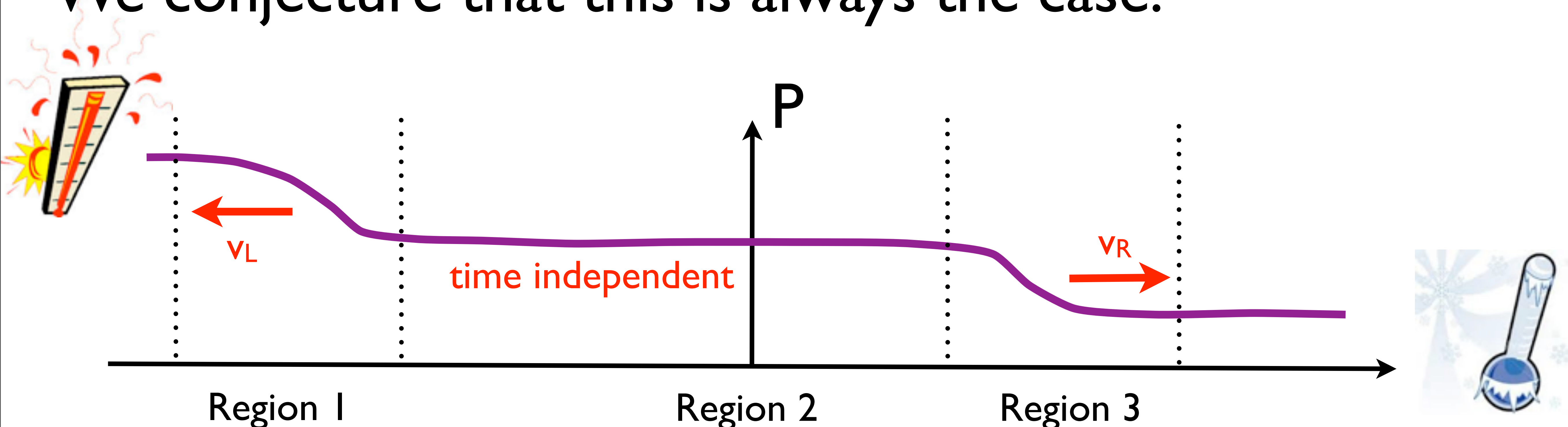

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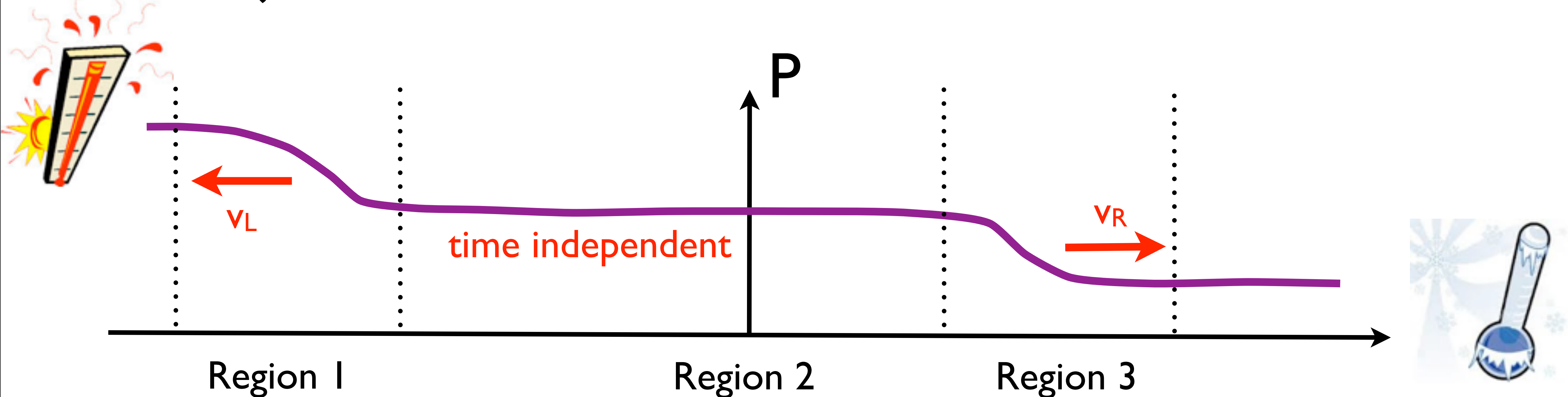

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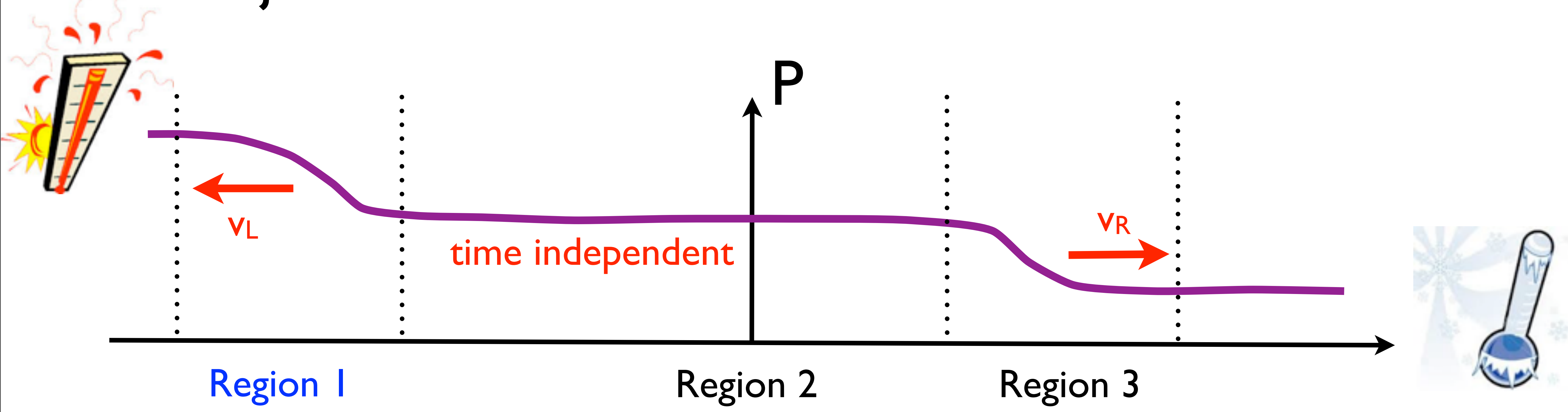

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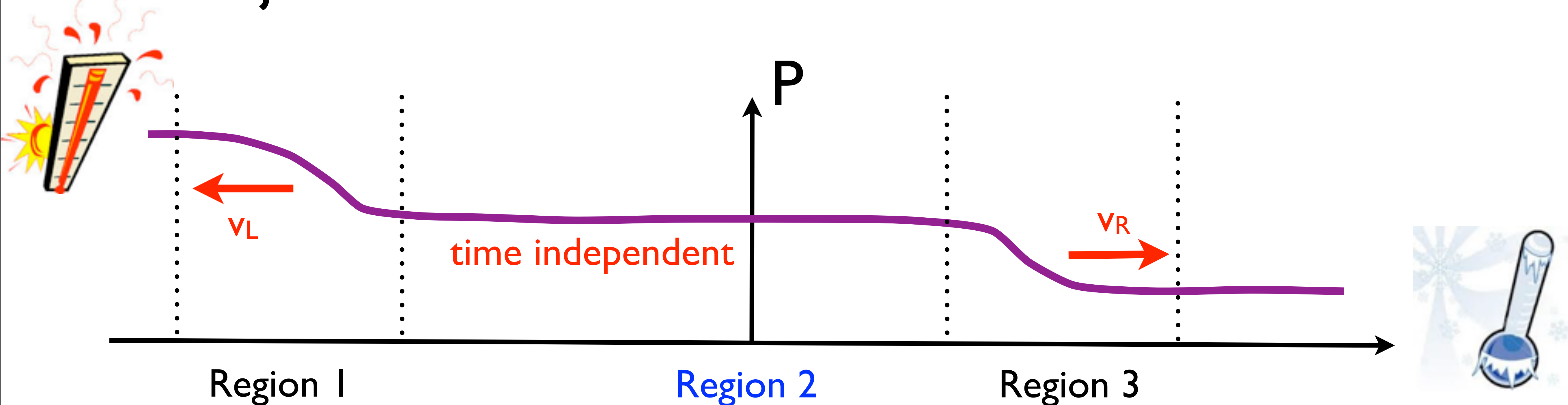

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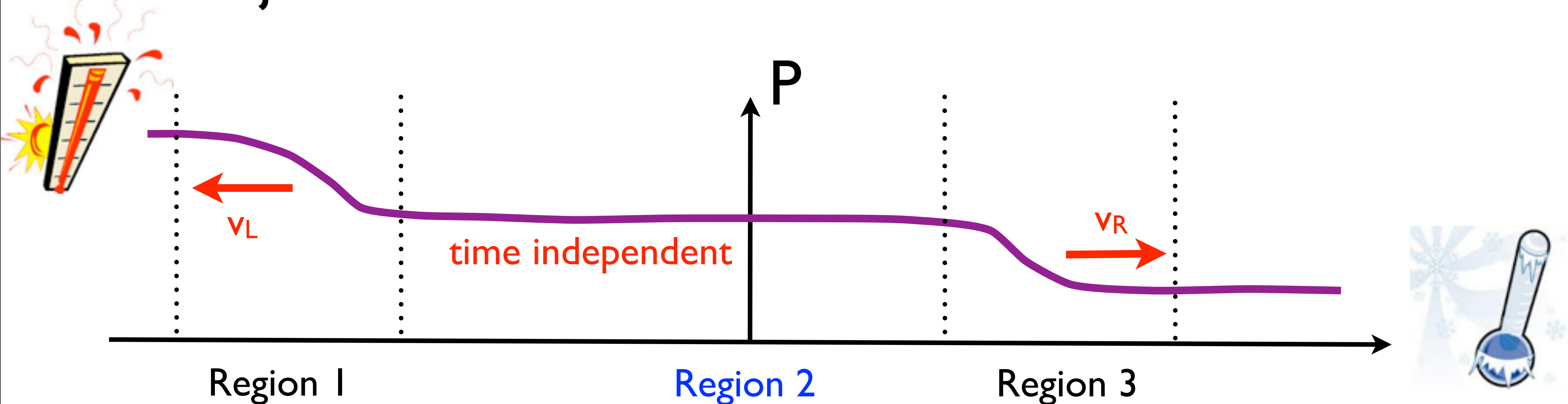

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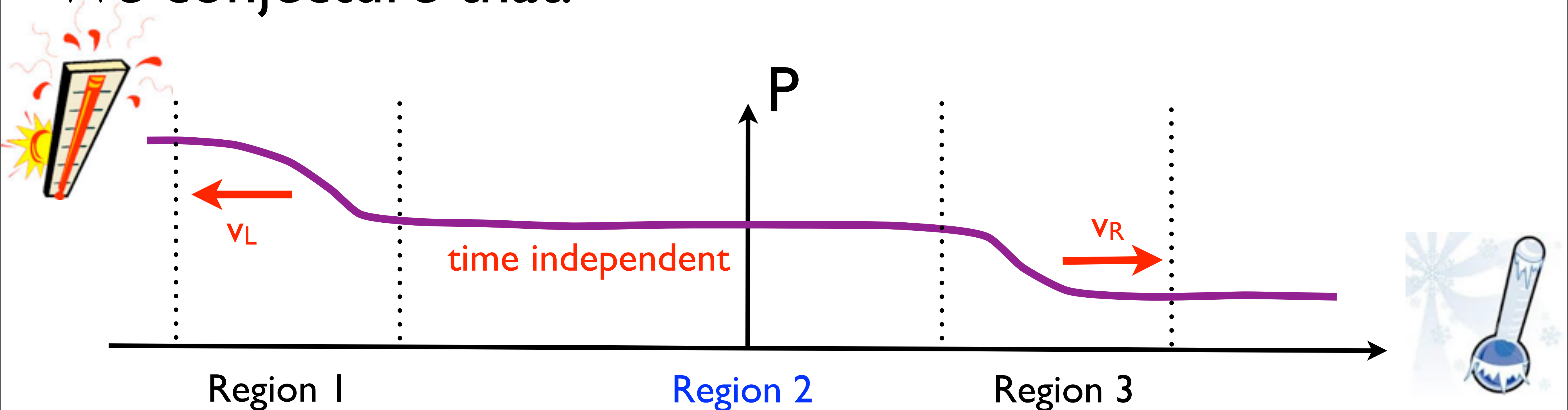

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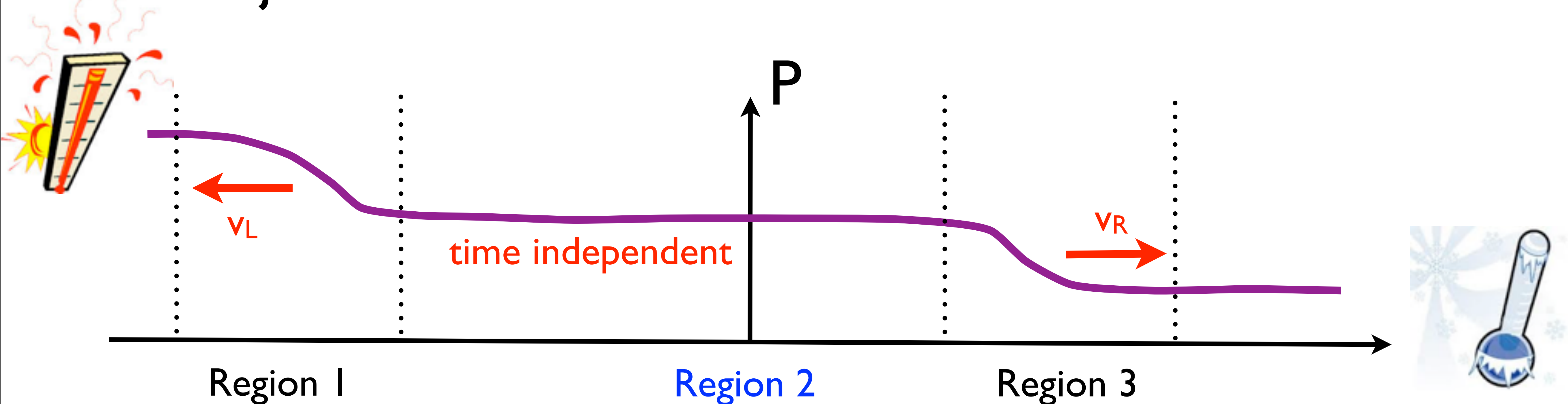

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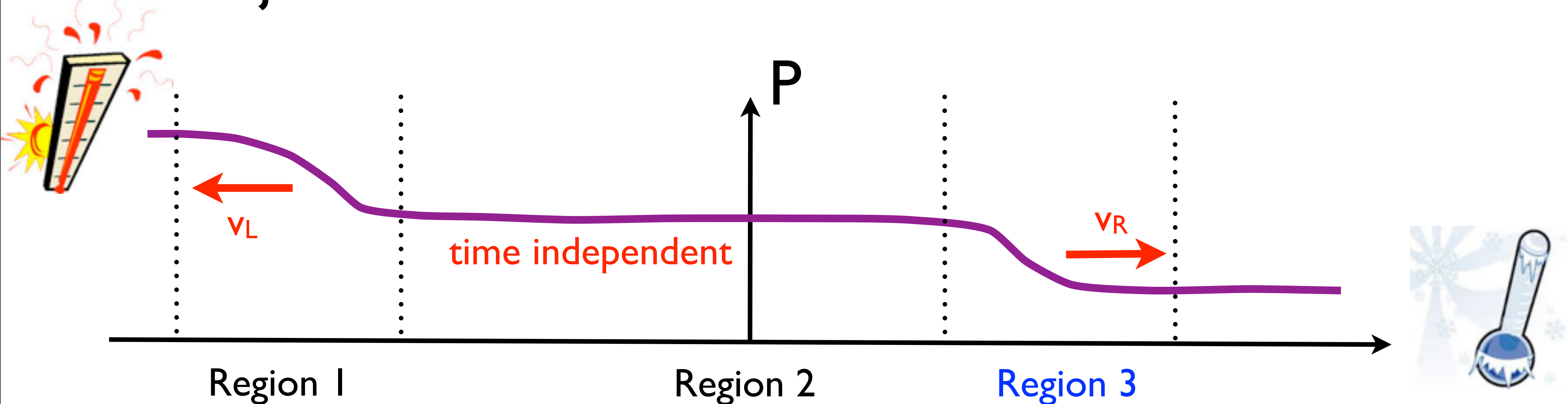

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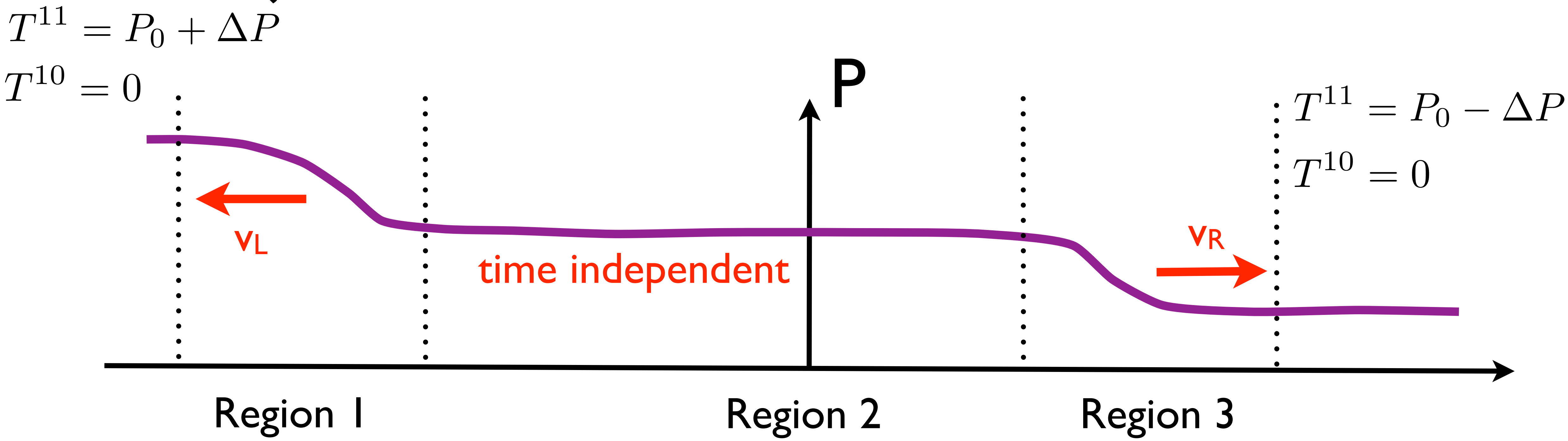
$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x - v_R t) & W_R(x - v_R t) \\ W_R(x - v_R t) & -v_R W_L(x - v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

Higher dimensions: the general case

We conjecture that:



Region 1

$$T_1^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x + v_L t) & W_L(x + v_L t) \\ W_L(x + v_L t) & -v_L W_L(x + v_L t) \end{pmatrix} + C_I^{\mu\nu}$$

Region 2

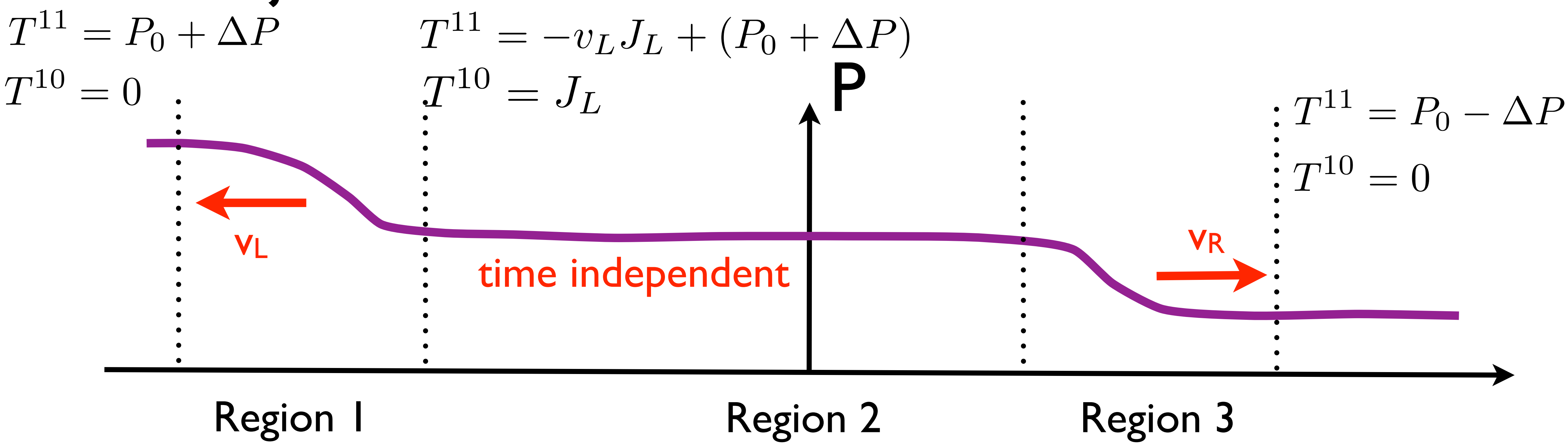
$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x - v_R t) & W_R(x - v_R t) \\ W_R(x - v_R t) & -v_R W_L(x - v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

Higher dimensions: the general case

We conjecture that:



Region 1

$$T_1^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x + v_L t) & W_L(x + v_L t) \\ W_L(x + v_L t) & -v_L W_L(x + v_L t) \end{pmatrix} + C_I^{\mu\nu}$$

Region 2

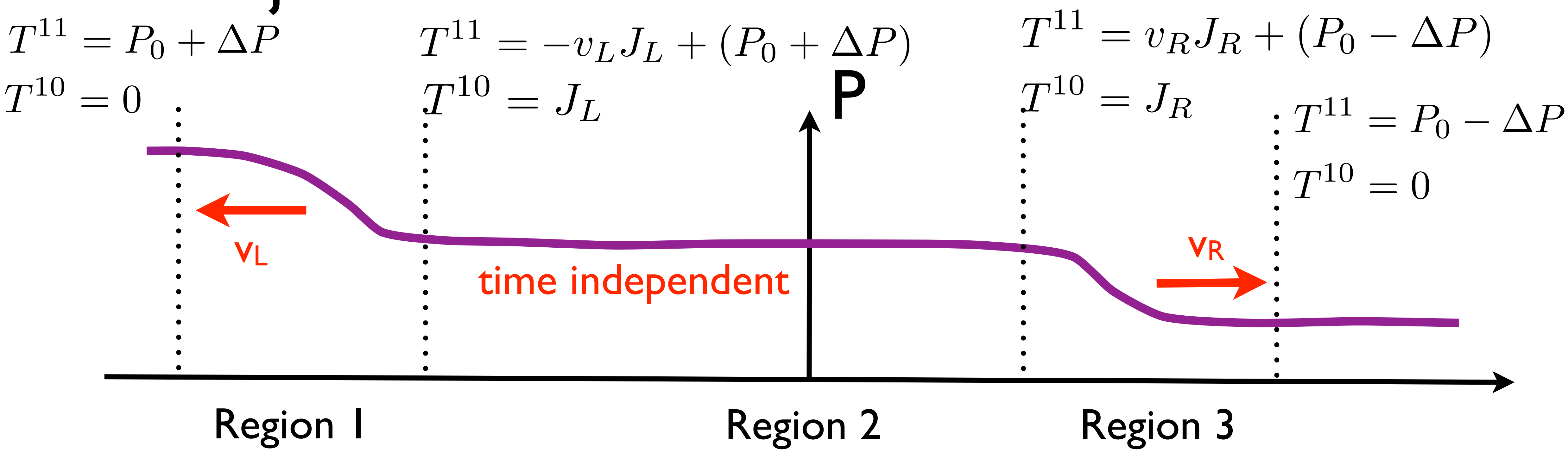
$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x - v_R t) & W_R(x - v_R t) \\ W_R(x - v_R t) & -v_R W_L(x - v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

Higher dimensions: the general case

We conjecture that:



Region 1

$$T_1^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x+v_L t) & W_L(x+v_L t) \\ W_L(x+v_L t) & -v_L W_L(x+v_L t) \end{pmatrix} + C_I^{\mu\nu}$$

Region 2

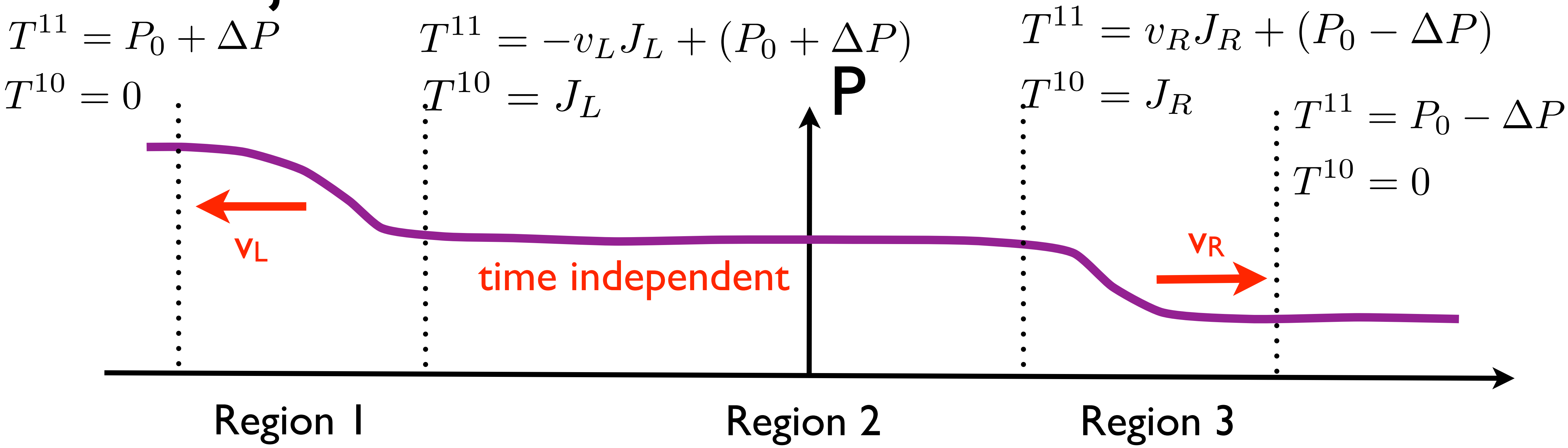
$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x-v_R t) & W_R(x-v_R t) \\ W_R(x-v_R t) & -v_R W_L(x-v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

Higher dimensions: the general case

We conjecture that:



Region 1

$$T_1^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x + v_L t) & W_L(x + v_L t) \\ W_L(x + v_L t) & -v_L W_L(x + v_L t) \end{pmatrix} + C_I^{\mu\nu}$$

Region 2

$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x - v_R t) & W_R(x - v_R t) \\ W_R(x - v_R t) & -v_R W_L(x - v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

Higher dimensions: the general case

We conjecture that:

$T^{11} = P_0 + \Delta P$

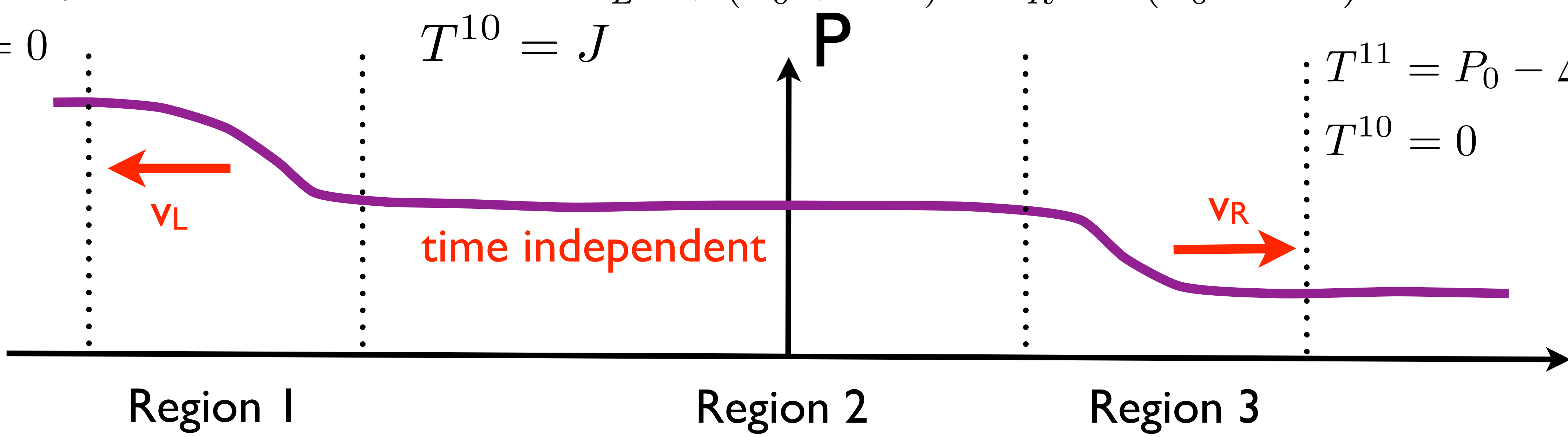
$T^{10} = 0$

$T^{11} = -v_L J + (P_0 + \Delta P) = v_R J + (P_0 - \Delta P)$

$T^{10} = J$

$T^{11} = P_0 - \Delta P$

$T^{10} = 0$



Region 1

$$T_1^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x+v_L t) & W_L(x+v_L t) \\ W_L(x+v_L t) & -v_L W_L(x+v_L t) \end{pmatrix} + C_I^{\mu\nu}$$

Region 2

$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x-v_R t) & W_R(x-v_R t) \\ W_R(x-v_R t) & -v_R W_L(x-v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

Higher dimensions: the general case

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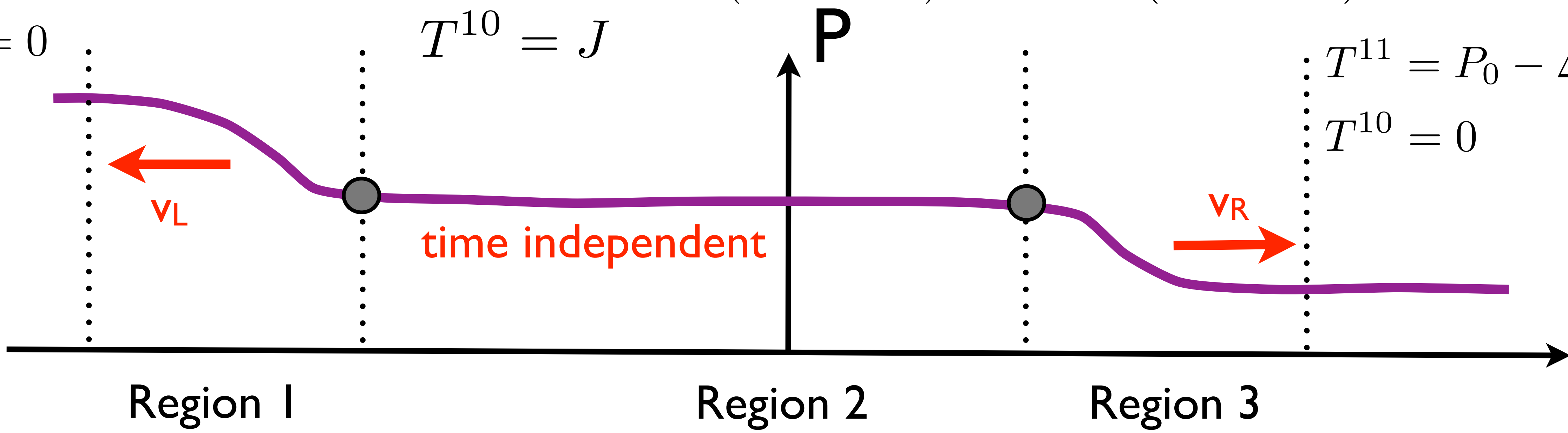

Region 1

$$T_1^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x + v_L t) & W_L(x + v_L t) \\ W_L(x + v_L t) & -v_L W_L(x + v_L t) \end{pmatrix} + C_I^{\mu\nu}$$

Region 2

$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x - v_R t) & W_R(x - v_R t) \\ W_R(x - v_R t) & -v_R W_L(x - v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

Higher dimensions: the general case

We conjecture that:

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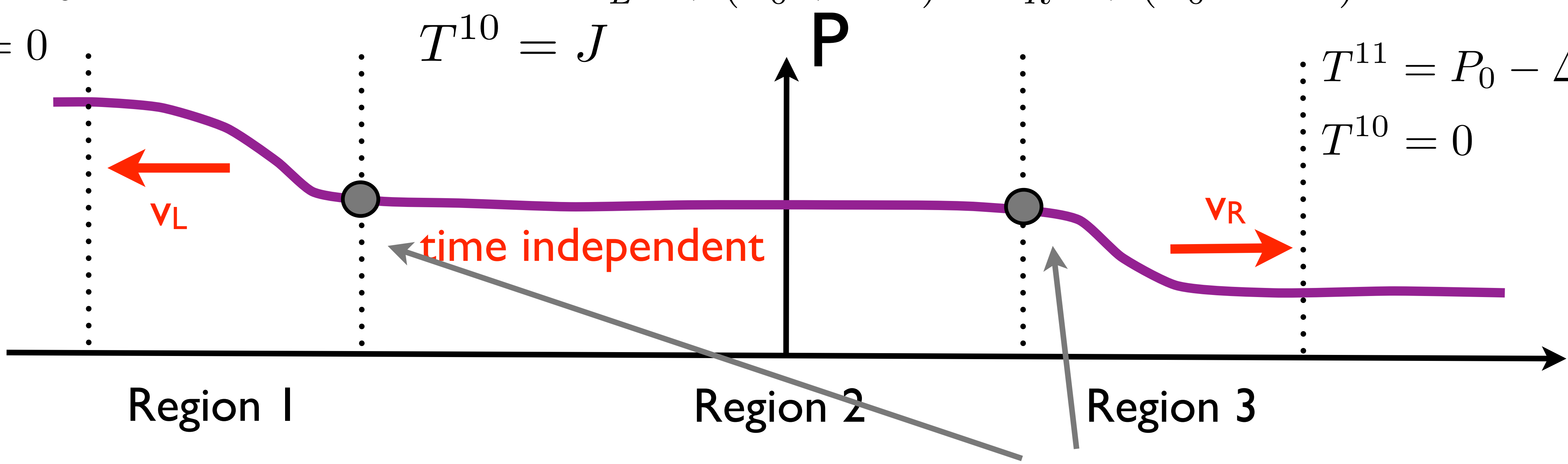

$T^{\mu\nu} = (d-1) P u^\mu u^\nu + (\eta^{\mu\nu} + u^\mu u^\nu) P$

Region 1

$$T_1^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x + v_L t) & W_L(x+v_L t) \\ W_L(x+v_L t) & -v_L W_L(x + v_L t) \end{pmatrix} + C_I^{\mu\nu}$$

Region 2

$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x - v_R t) & W_R(x-v_R t) \\ W_R(x-v_R t) & -v_R W_L(x - v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

Higher dimensions: the general case

We conjecture that:

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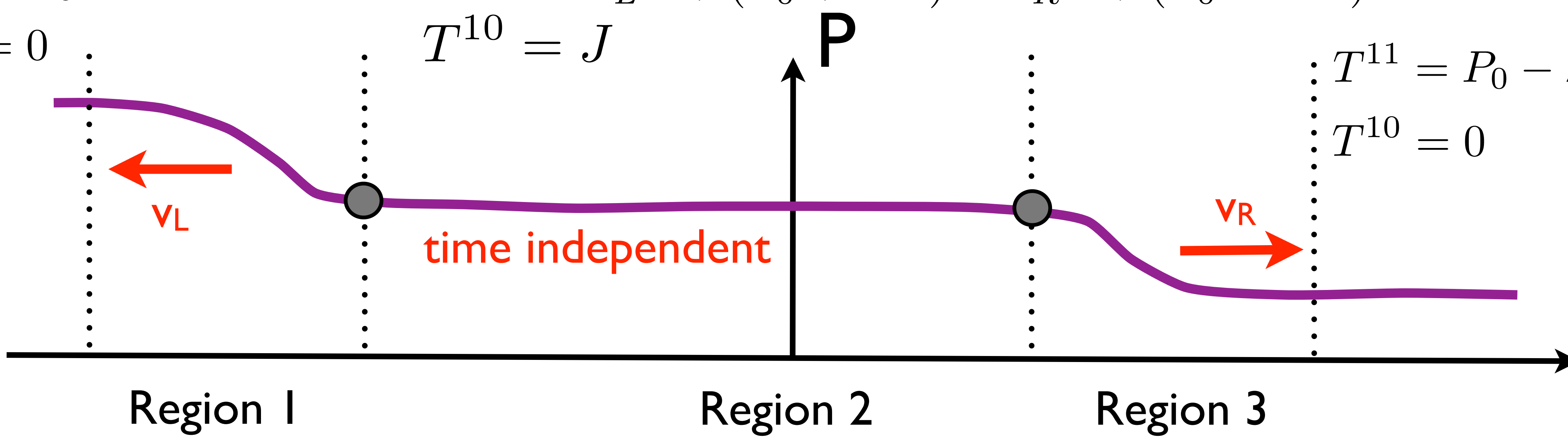


Region 1

$$T_1^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x+v_L t) & W_L(x+v_L t) \\ W_L(x+v_L t) & -v_L W_L(x+v_L t) \end{pmatrix} + C_I^{\mu\nu}$$

Region 2

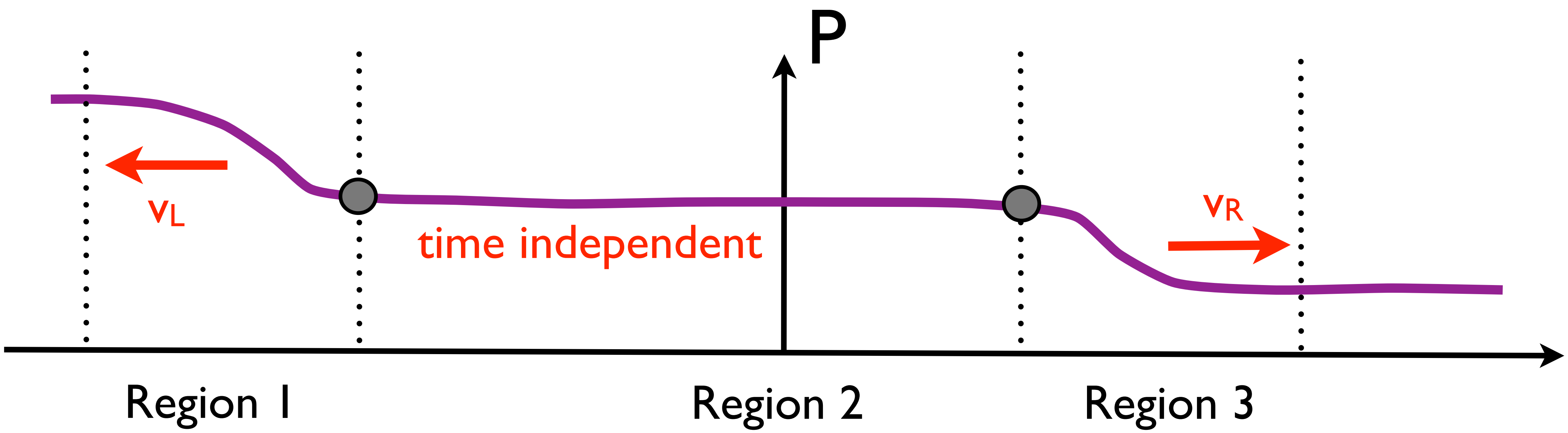
$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Region 3

$$T_3^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_R} W_R(x-v_R t) & W_R(x-v_R t) \\ W_R(x-v_R t) & -v_R W_L(x-v_R t) \end{pmatrix} + C_{III}^{\mu\nu}$$

Higher dimensions: the general case

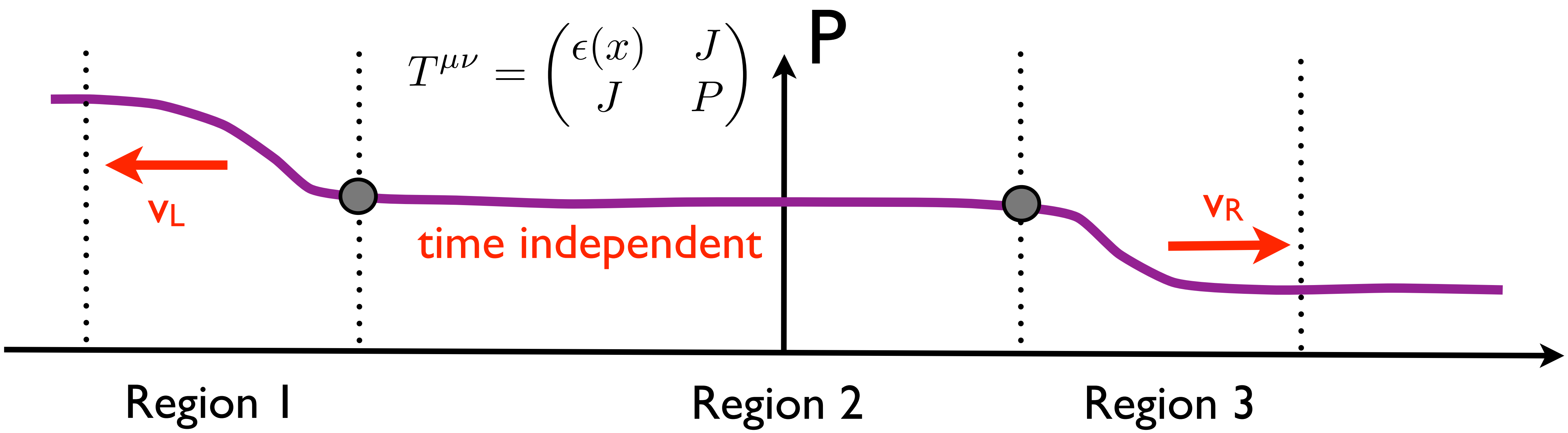
We conjecture that:



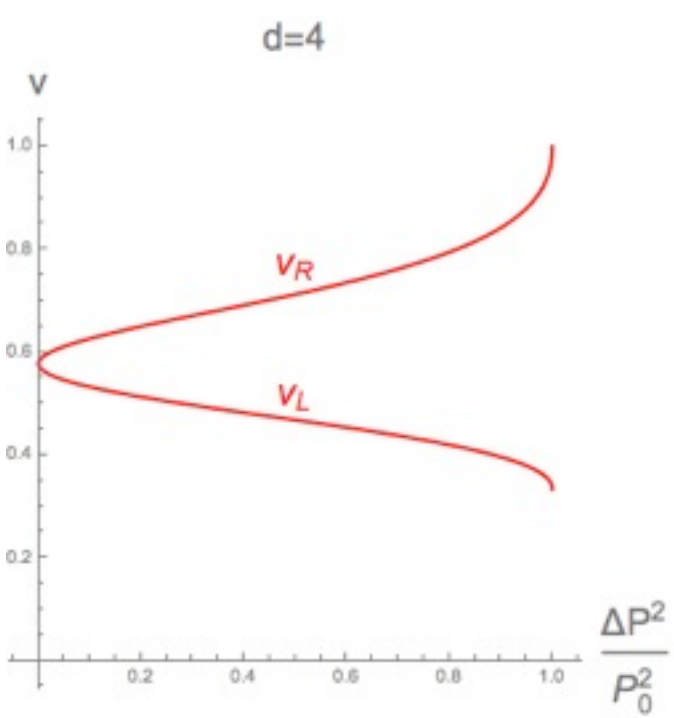
$$T^{\mu\nu} = \begin{pmatrix} \epsilon(x) & J \\ J & P \end{pmatrix}$$

Higher dimensions: the general case

We conjecture that:

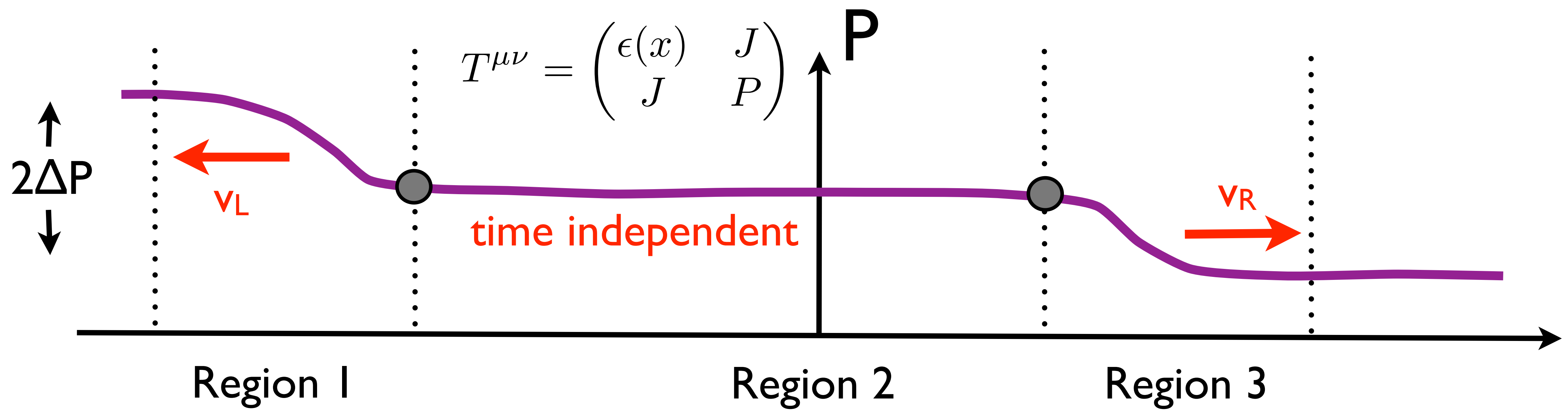


We find:

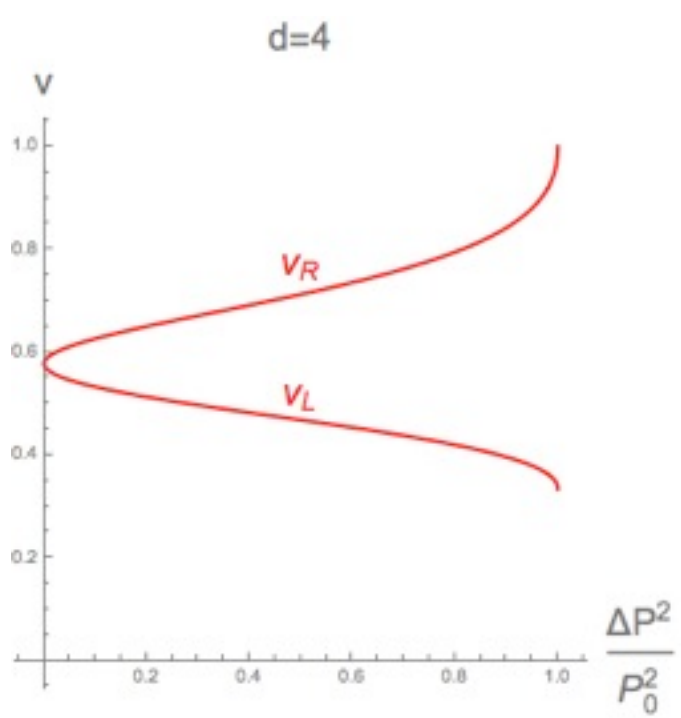


Higher dimensions: the general case

We conjecture that:

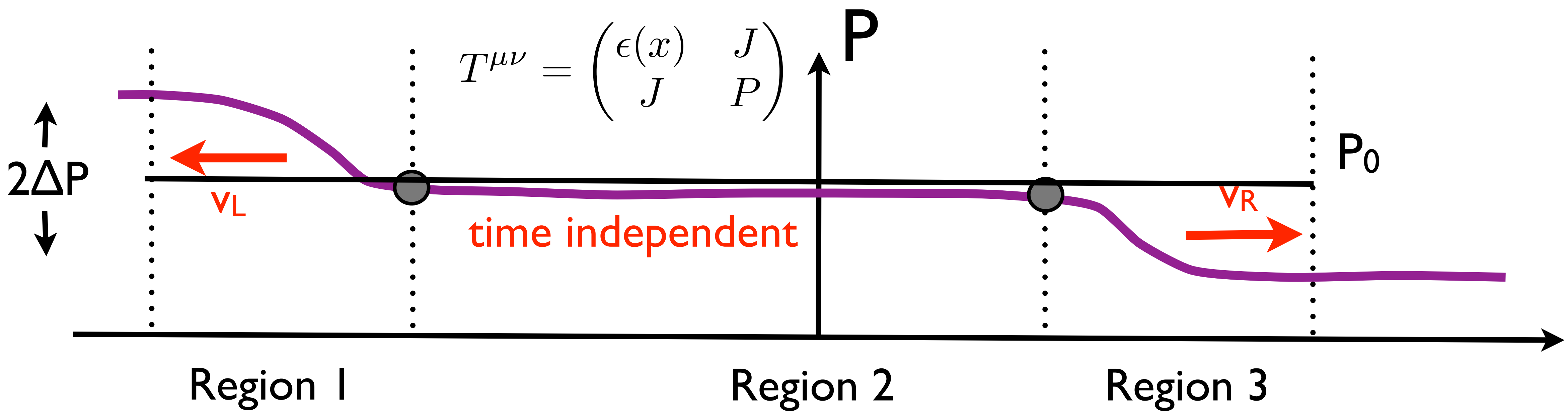


We find:



Higher dimensions: the general case

We conjecture that:



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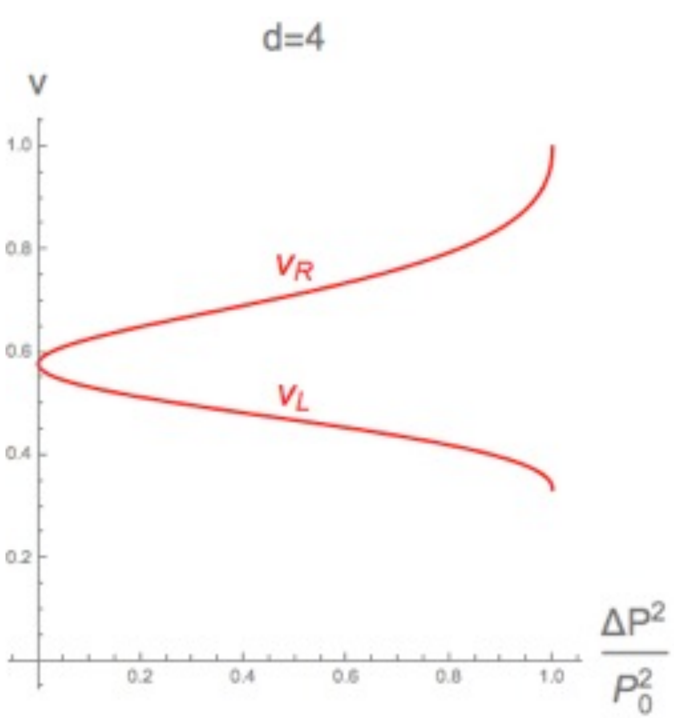
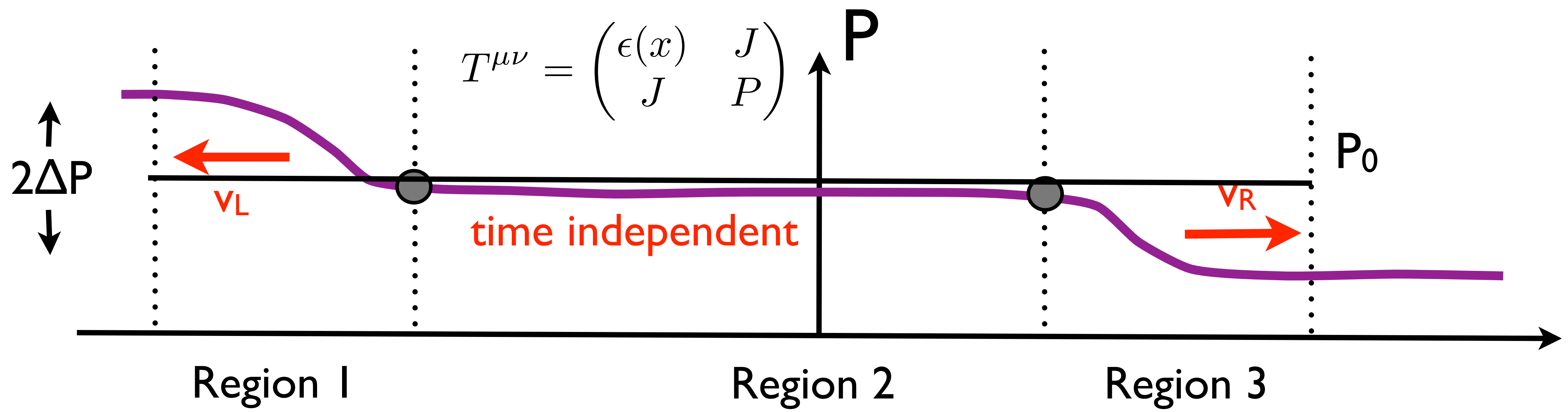
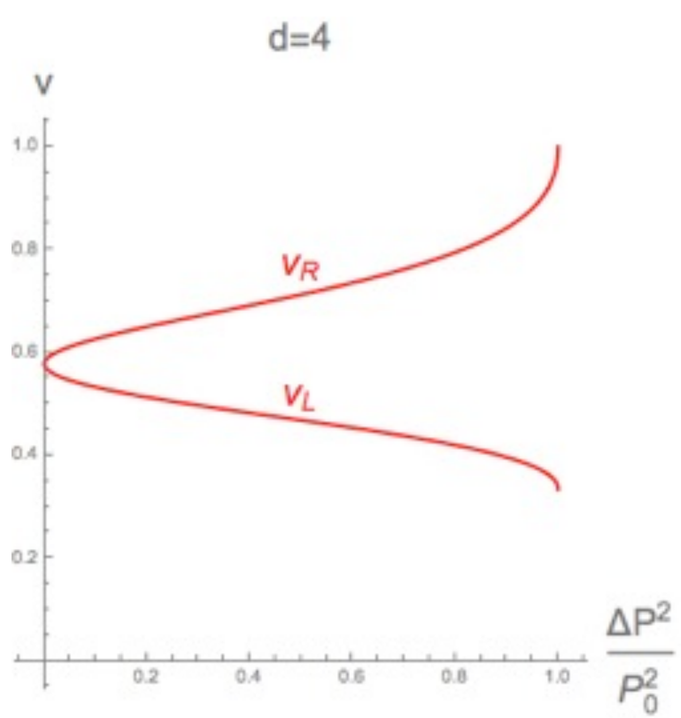


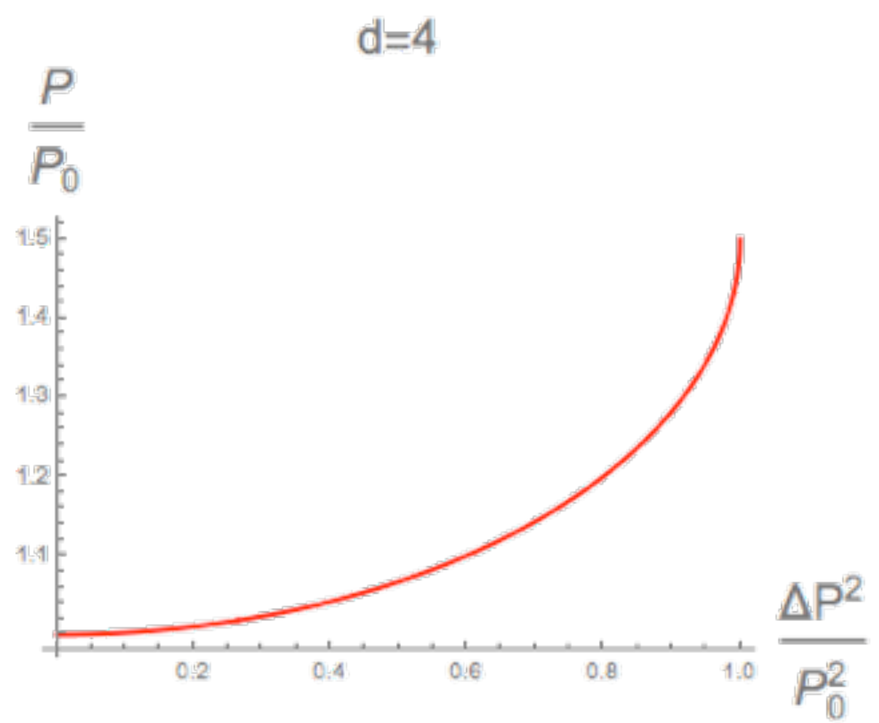
Higher dimensions: the general case

We conjecture that:



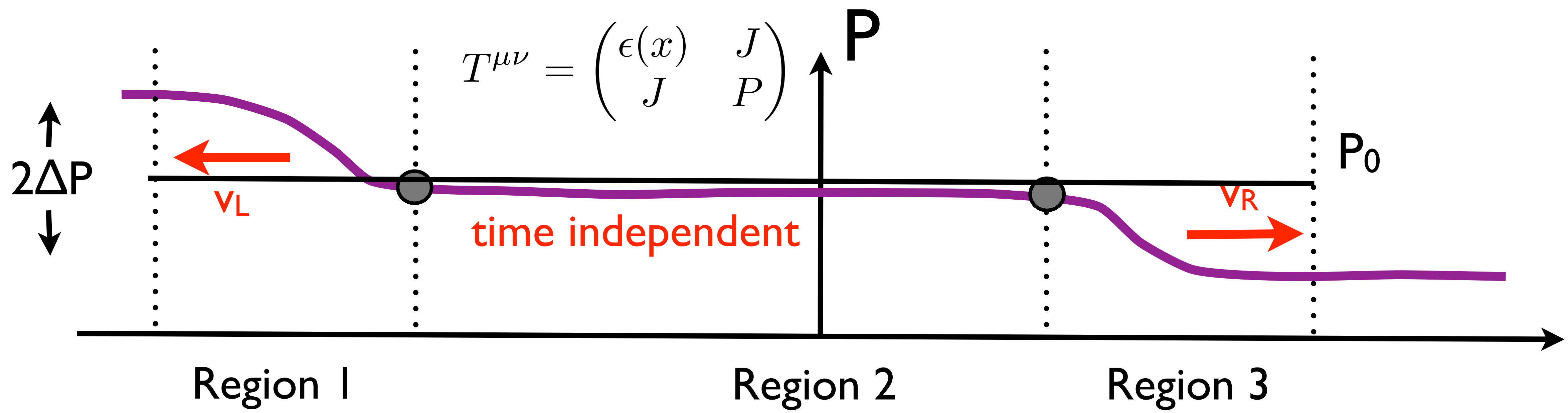
We find:



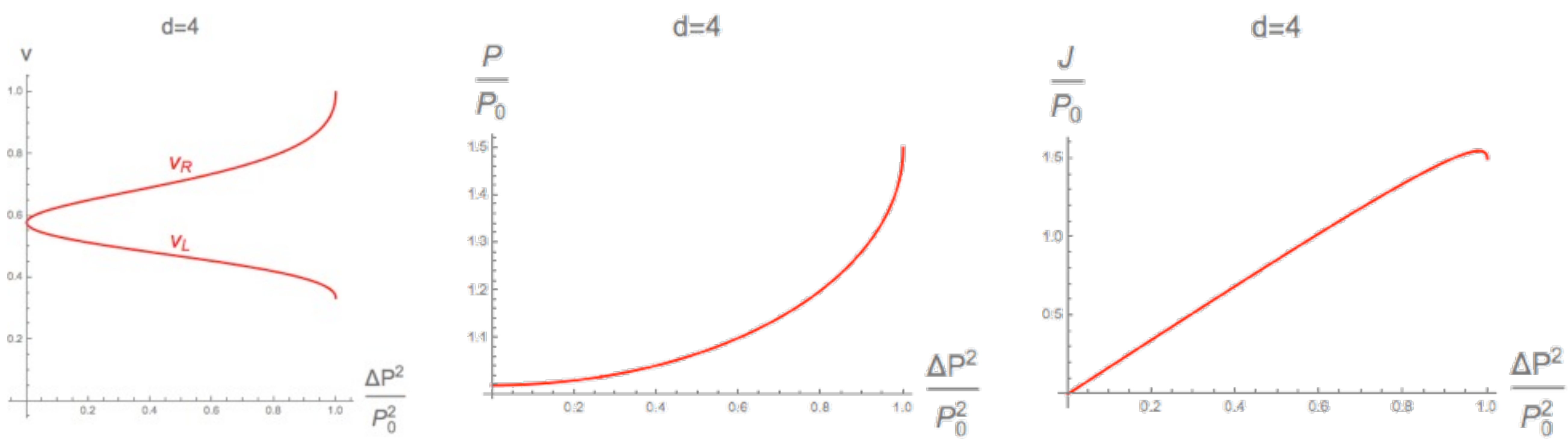


Higher dimensions: the general case

We conjecture that:

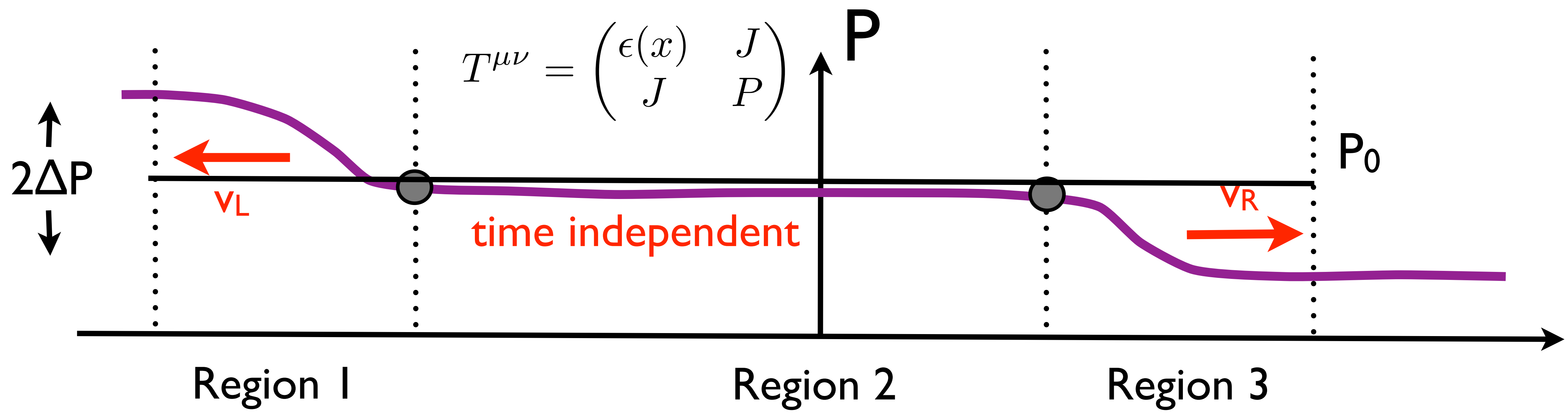


We find:

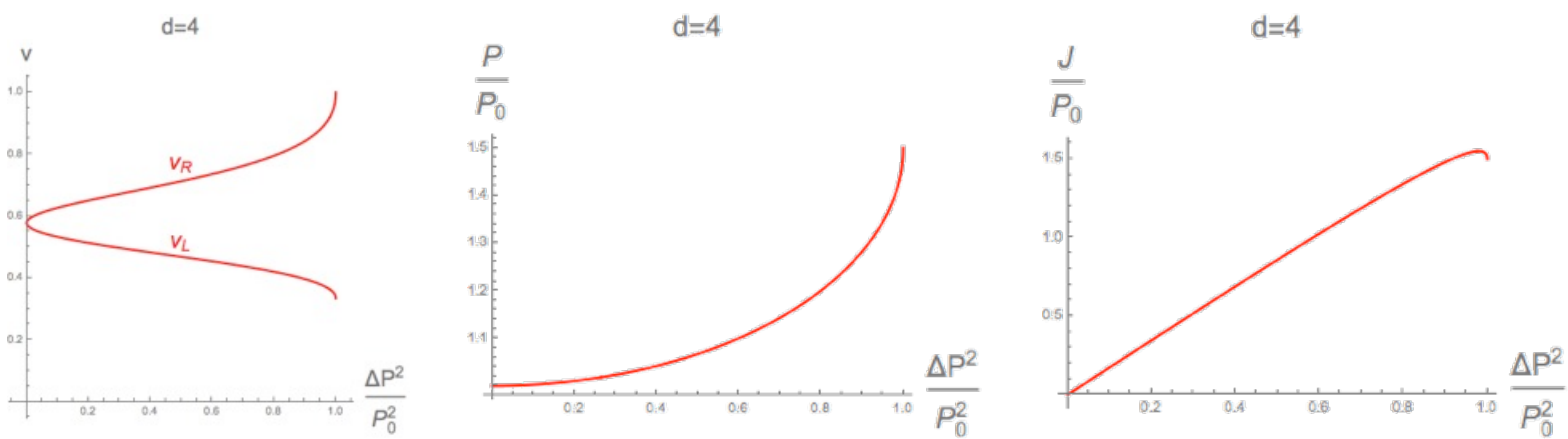


Higher dimensions: the general case

We conjecture that:



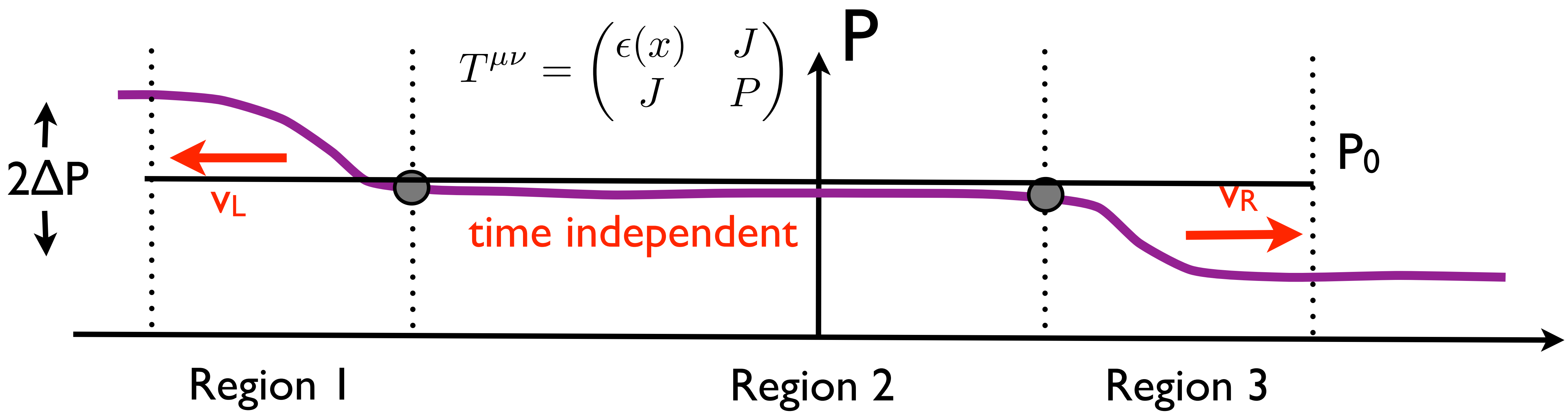
We find:



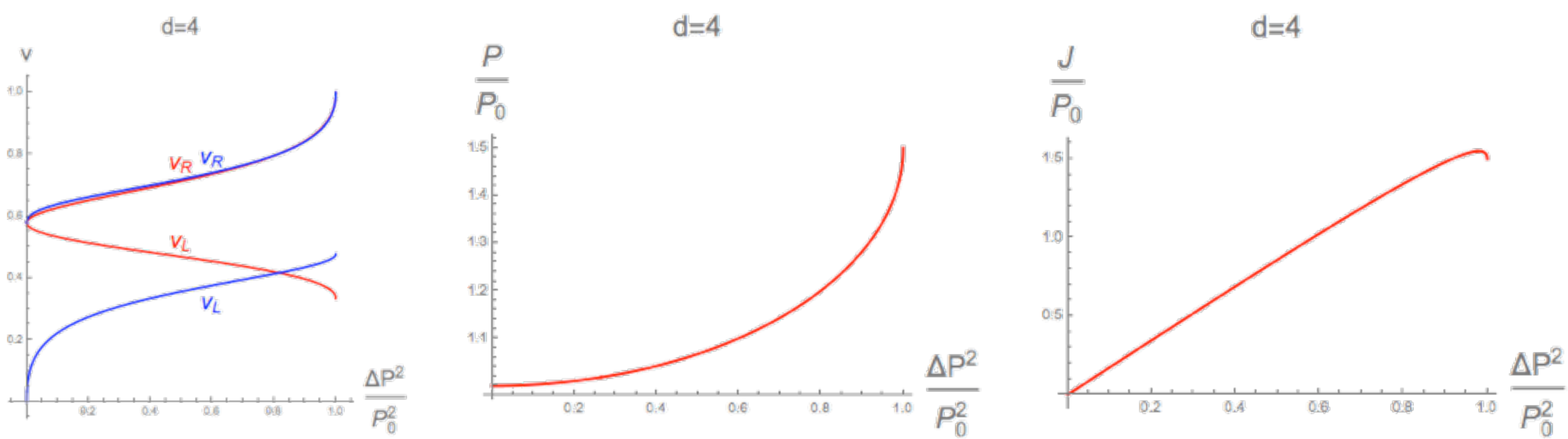
(See also Bhaseen et. al., 2013)

Higher dimensions: the general case

We conjecture that:

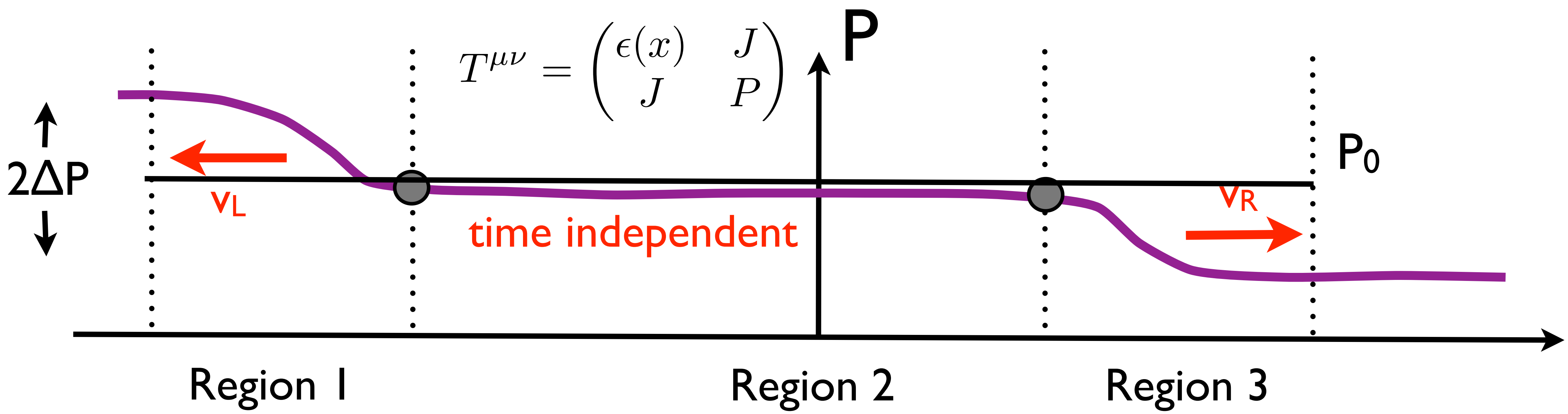


We find:

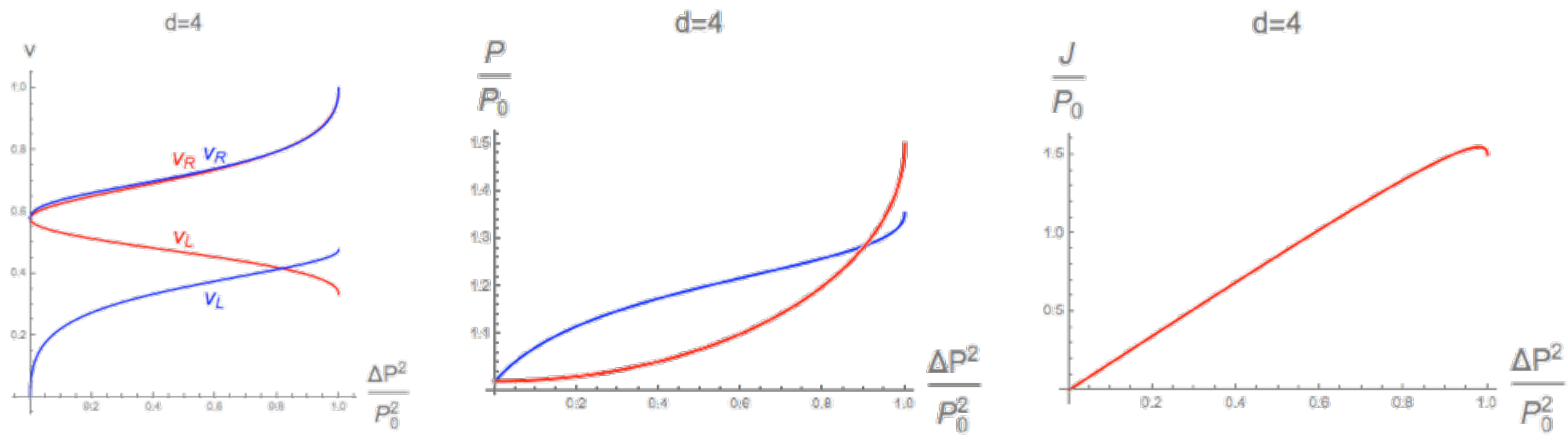


Higher dimensions: the general case

We conjecture that:

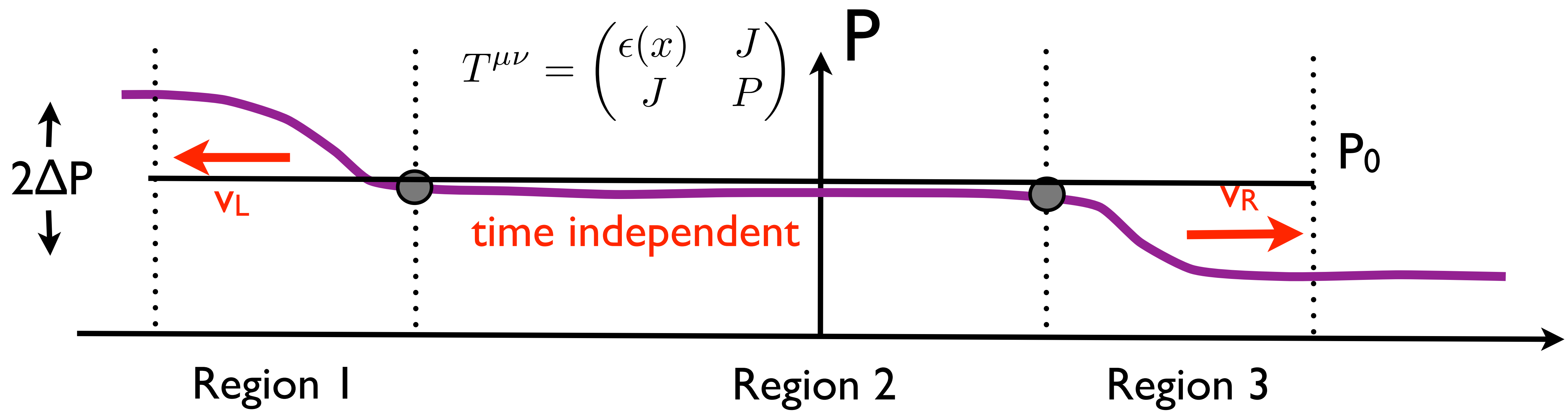


We find:

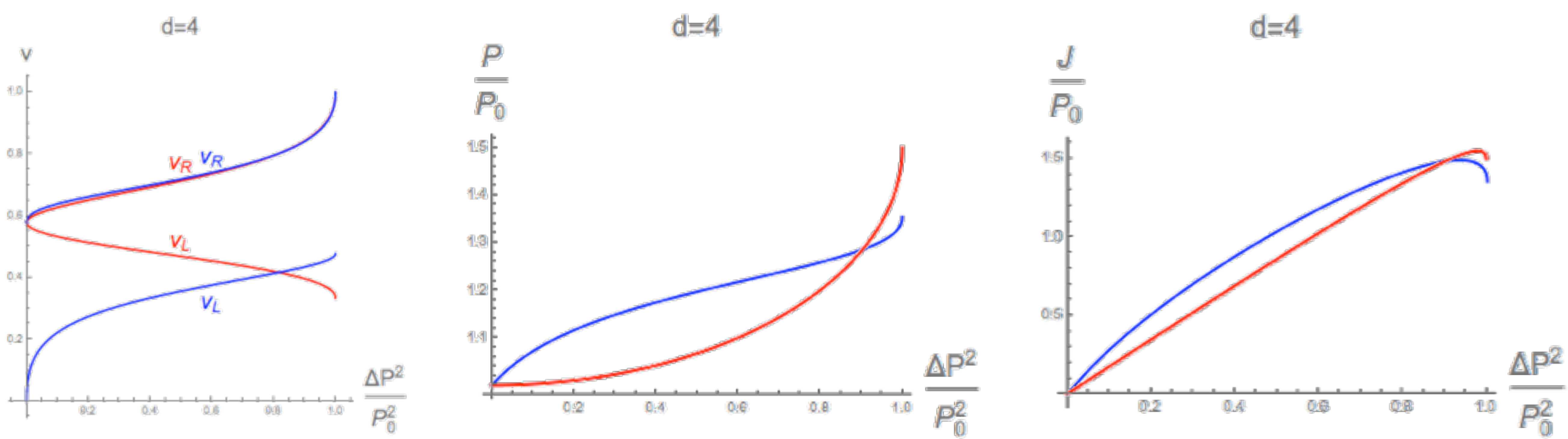


Higher dimensions: the general case

We conjecture that:

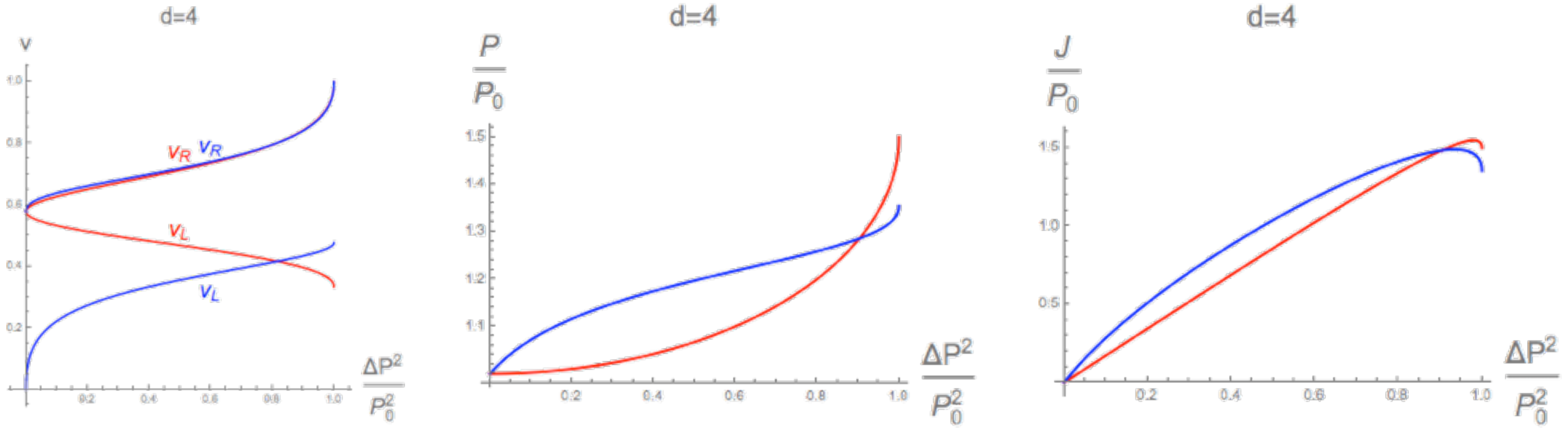


We find:



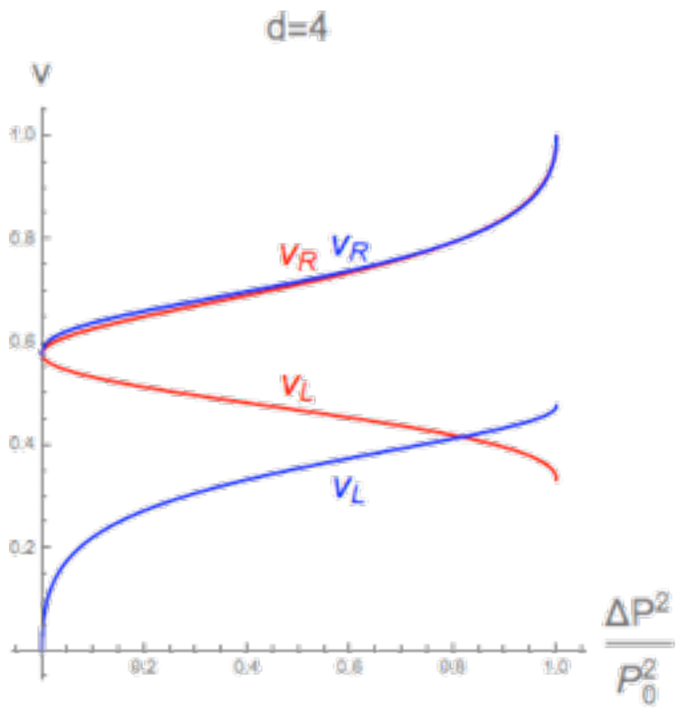
Higher dimensions: the general case

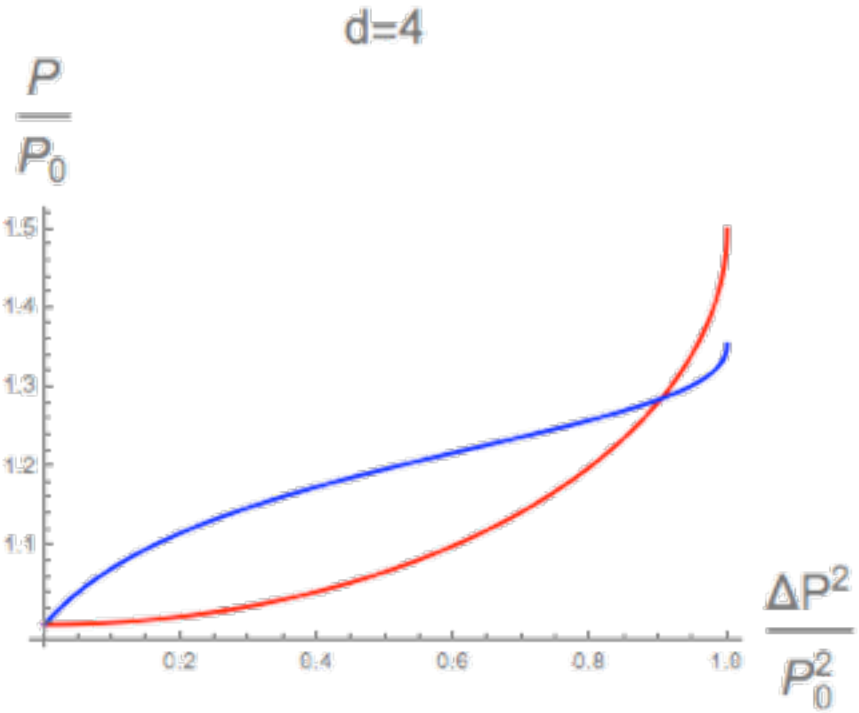
We find:

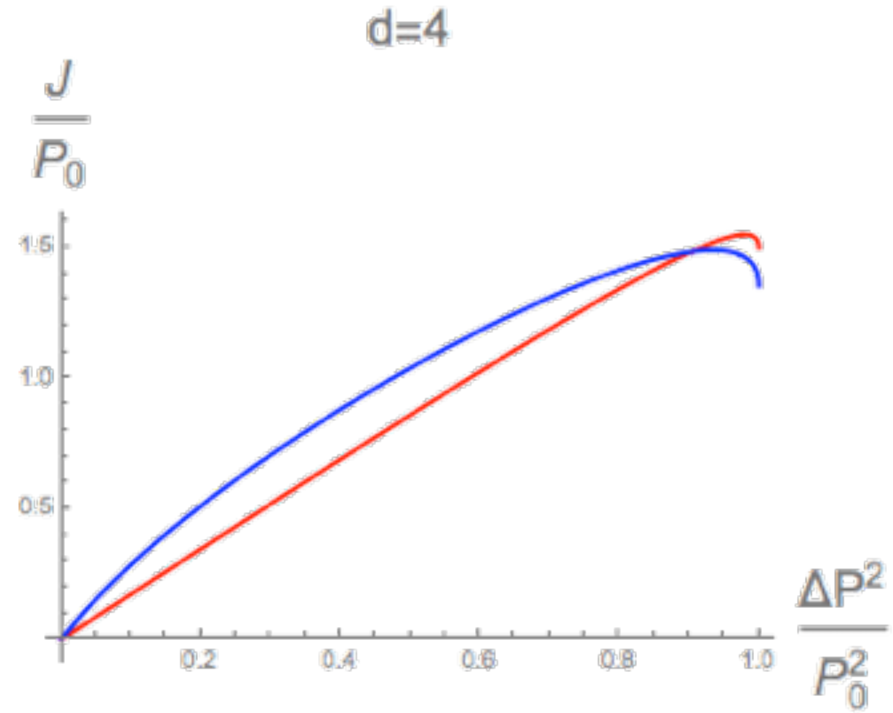


Higher dimensions: the general case

We find:





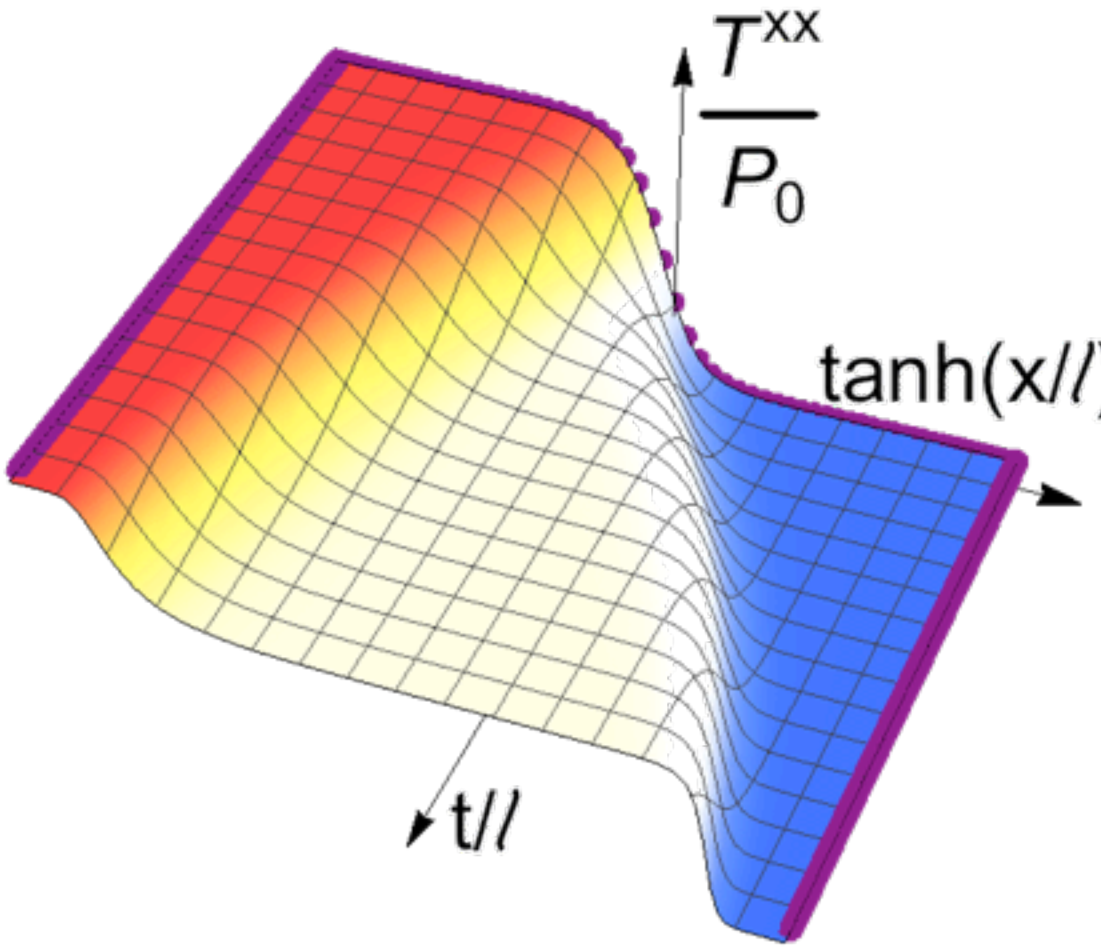


Test 1: nonlinear viscous hydrodynamics

Higher dimensions: viscous hydro

Higher dimensions: viscous hydro

We find ($d=3$, $\Delta P/P_0=0.8$)



Higher dimensions: viscous hydro

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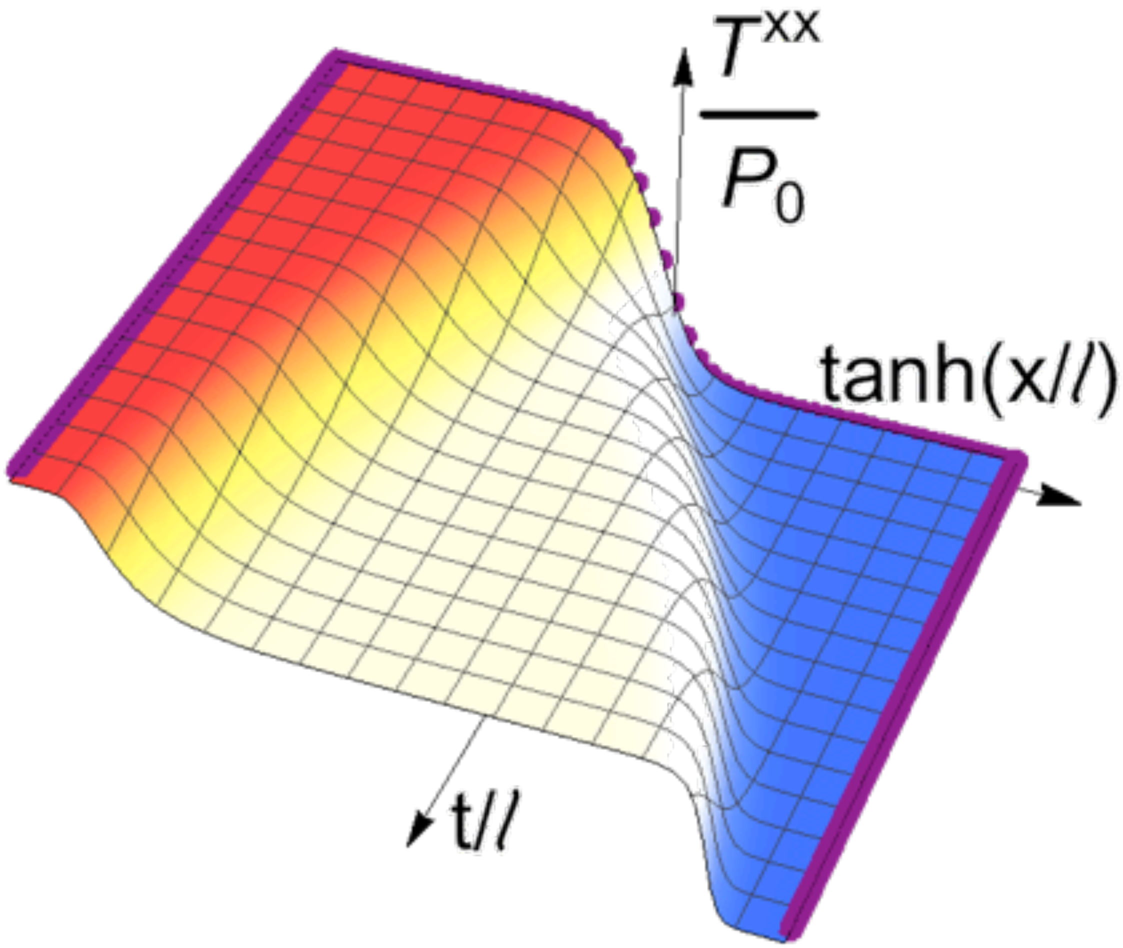
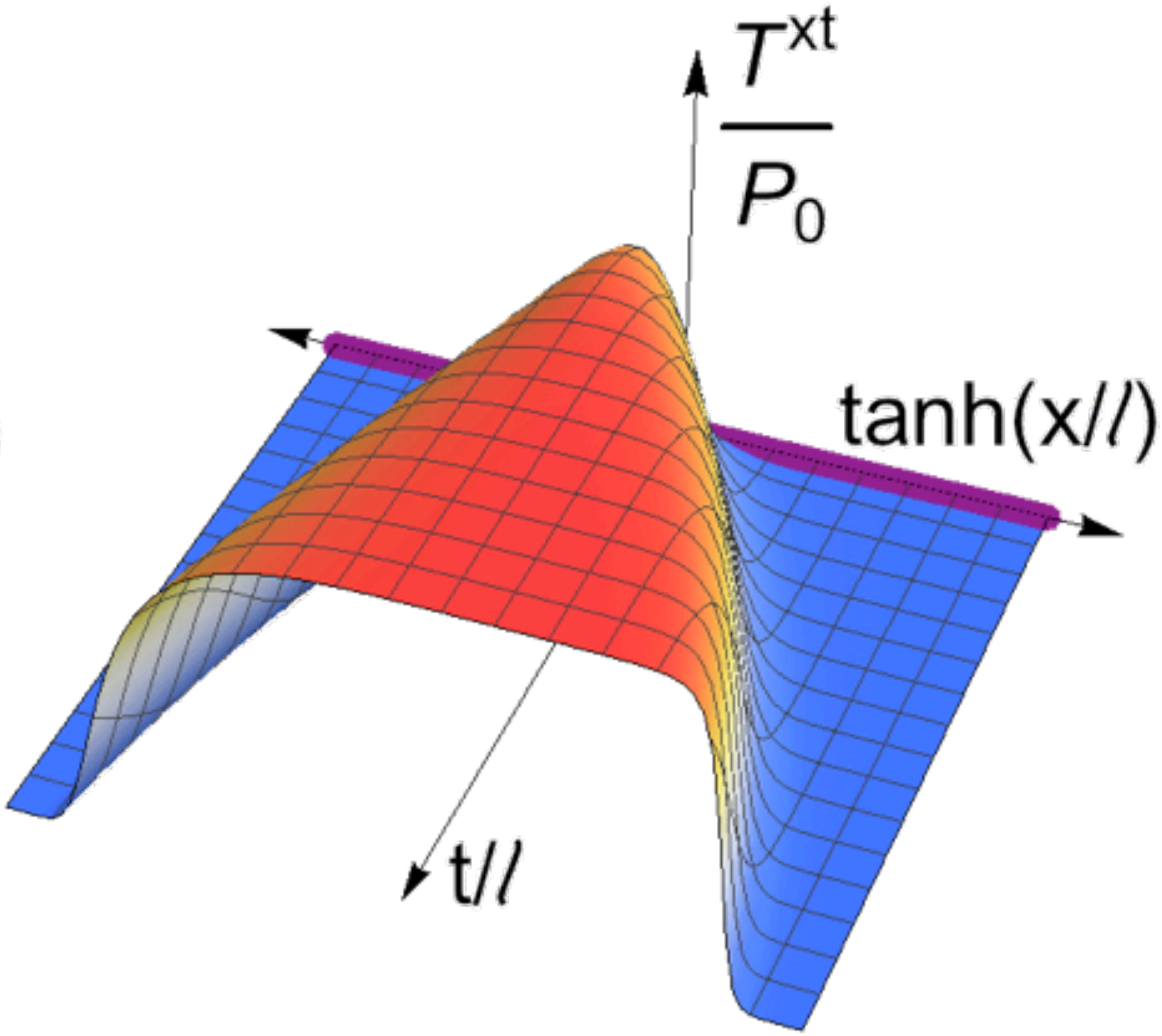
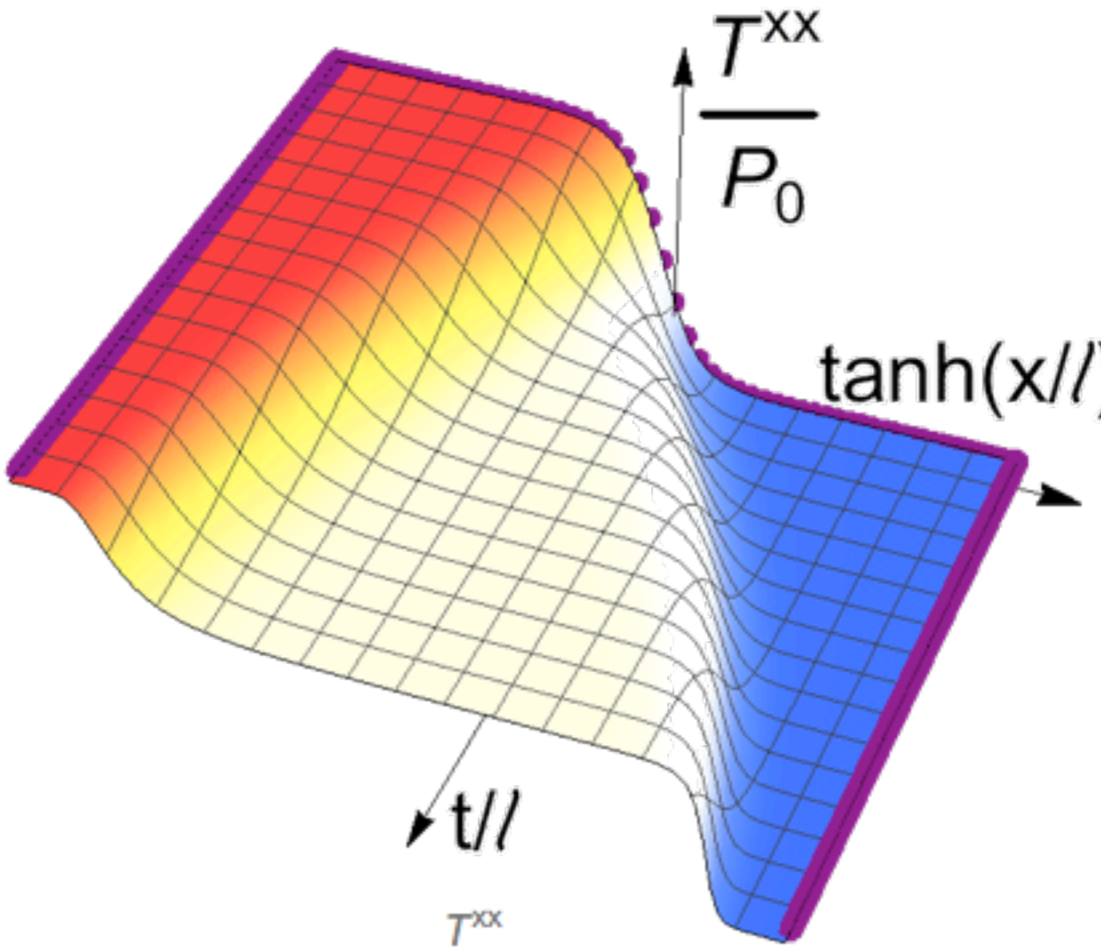
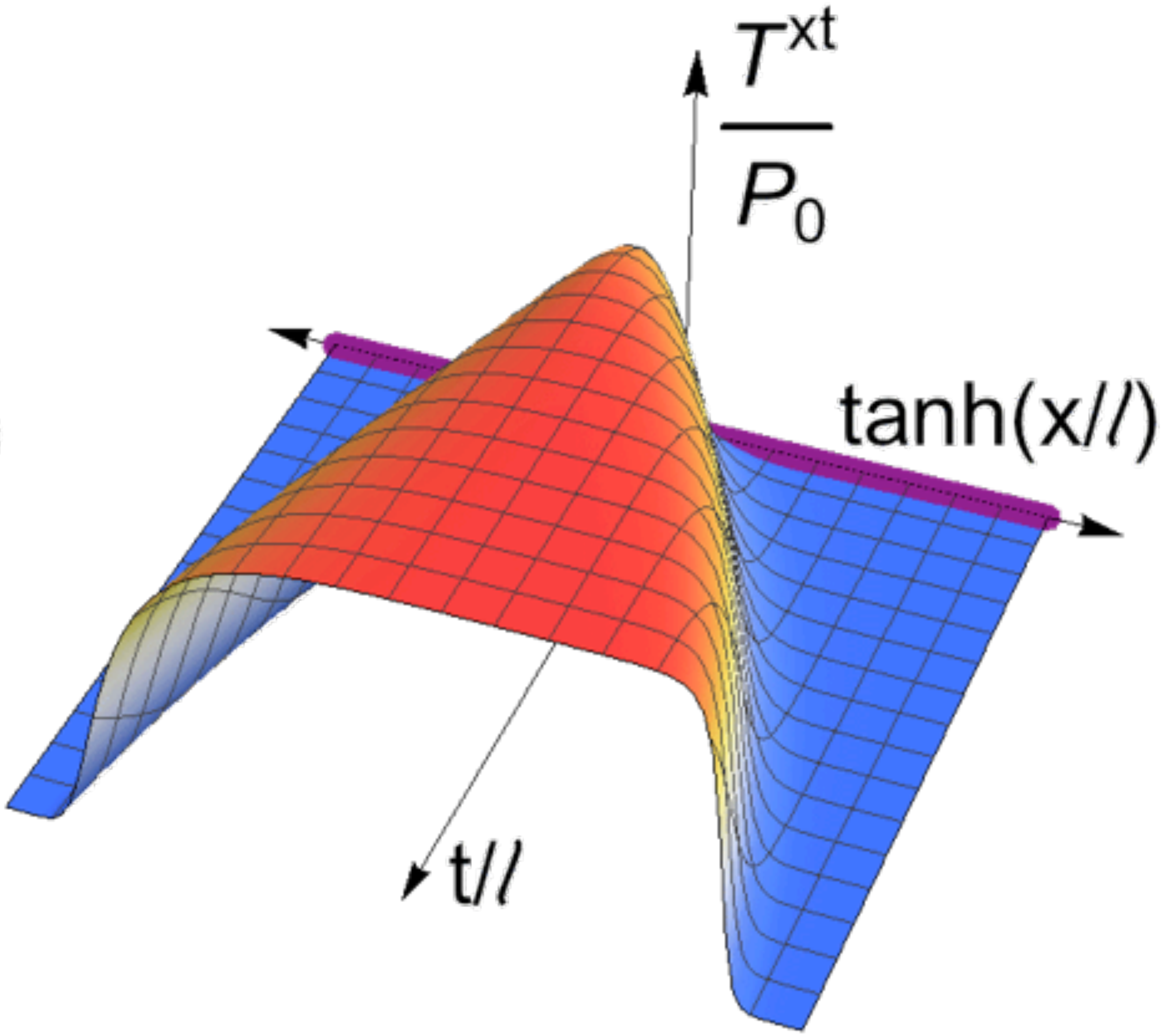


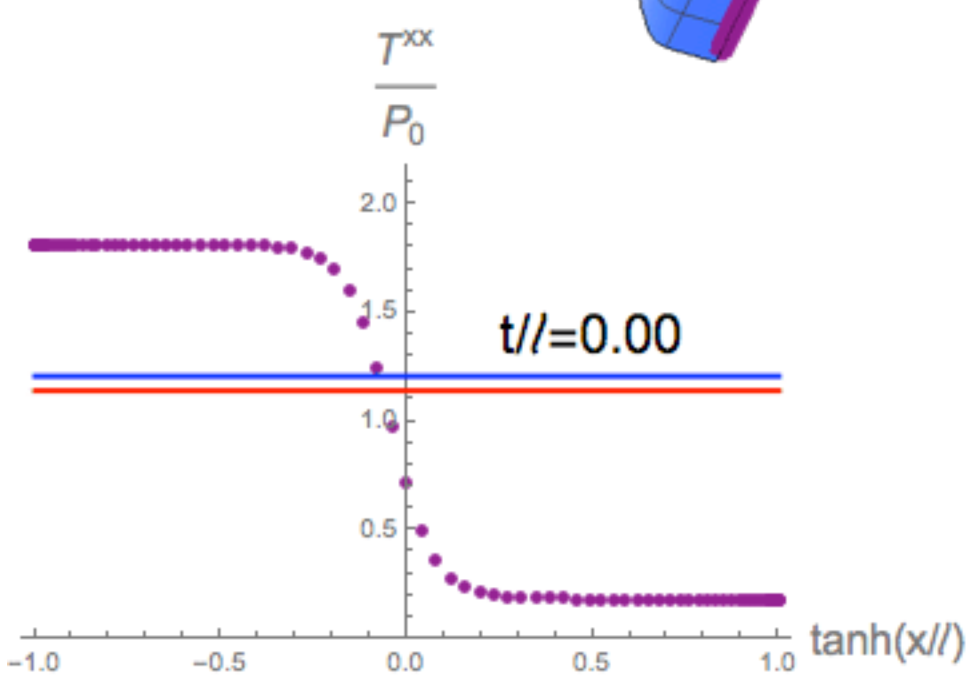


Higher dimensions: viscous hydro

We find (d=3, $\Delta P/P_0$=0.8)







Higher dimensions: viscous hydro

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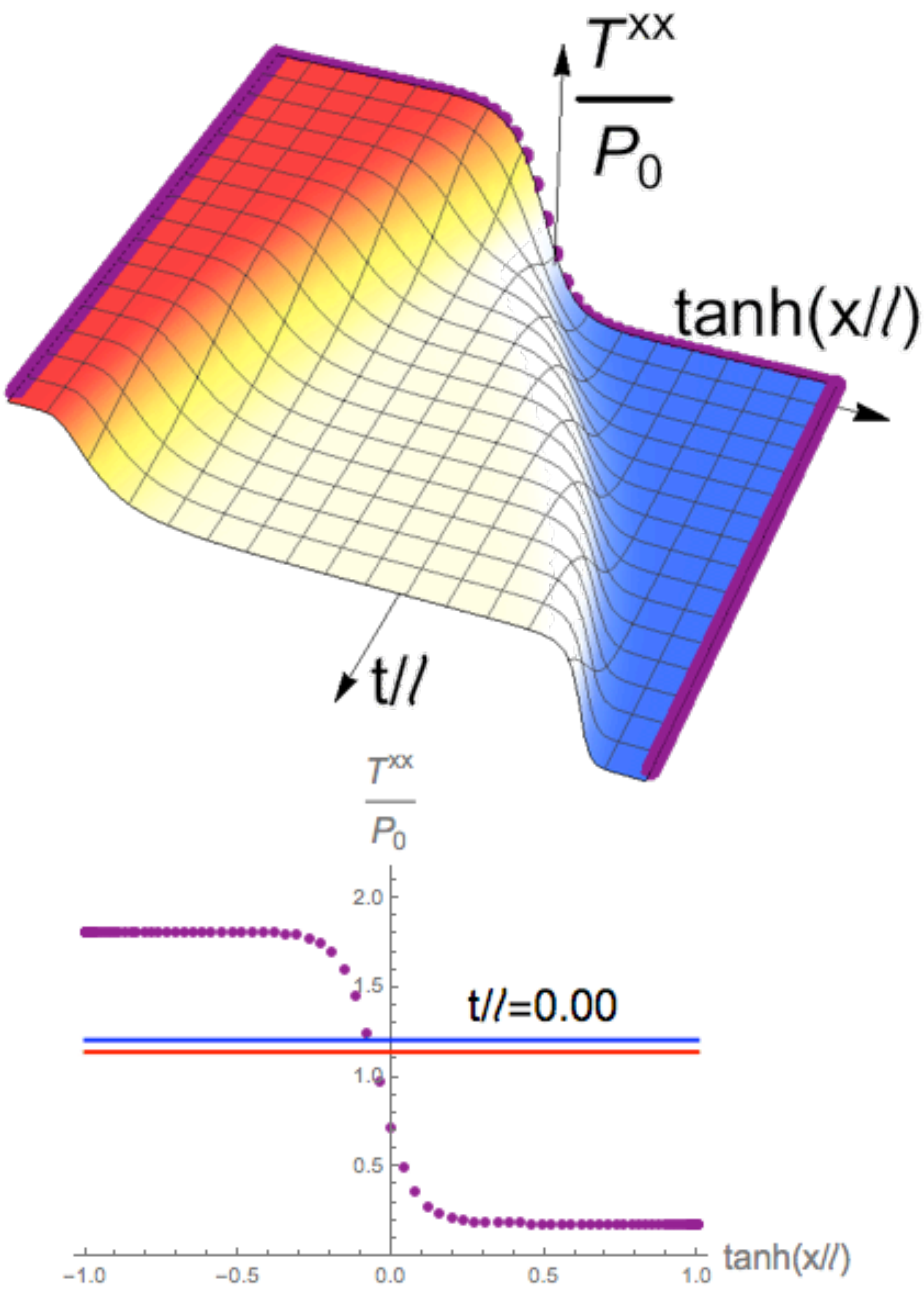
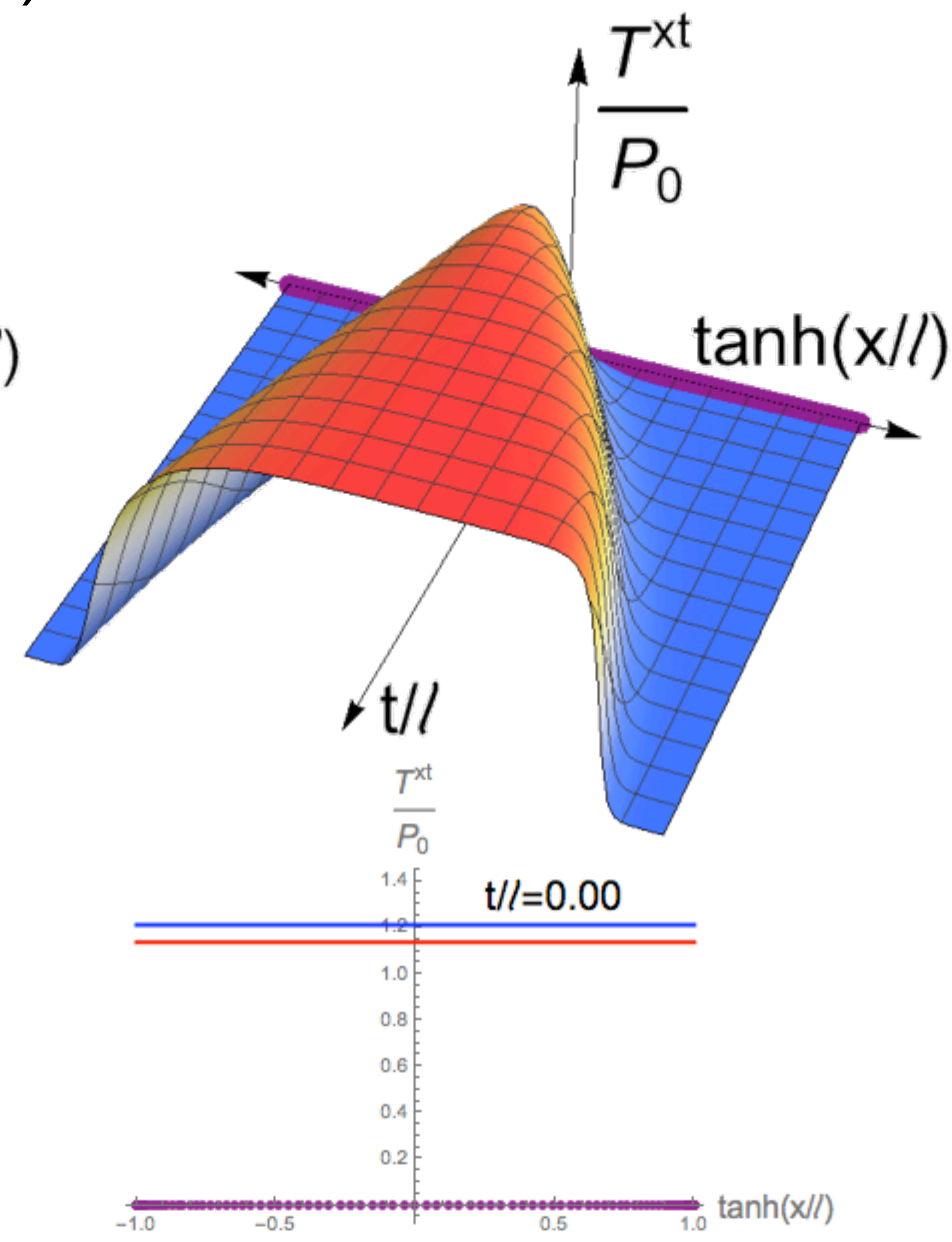
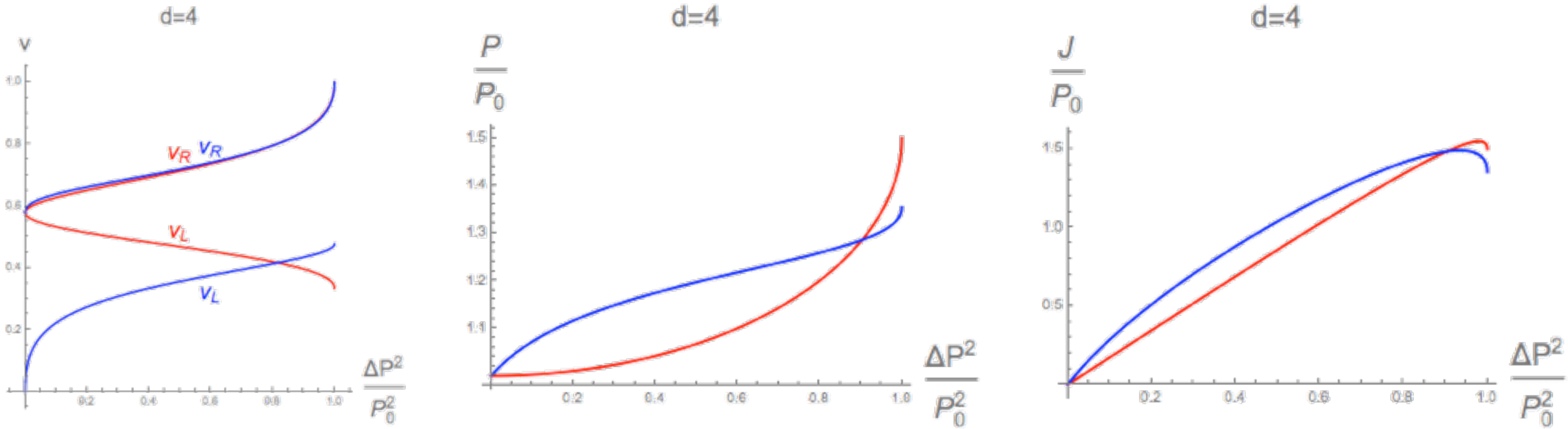




Higher dimensions: the general case

We find:

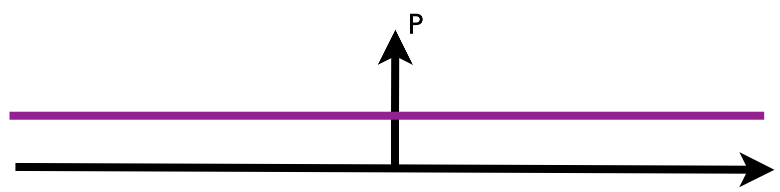


Test 1: nonlinear viscous hydrodynamics.

Test 2: Holography.

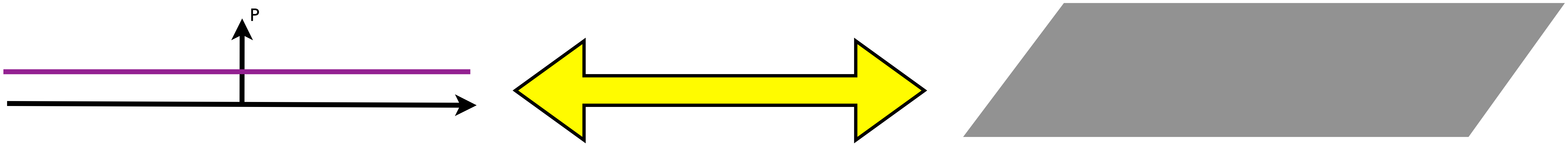
Holography

Let us start by considering an equilibrated configuration



Holography

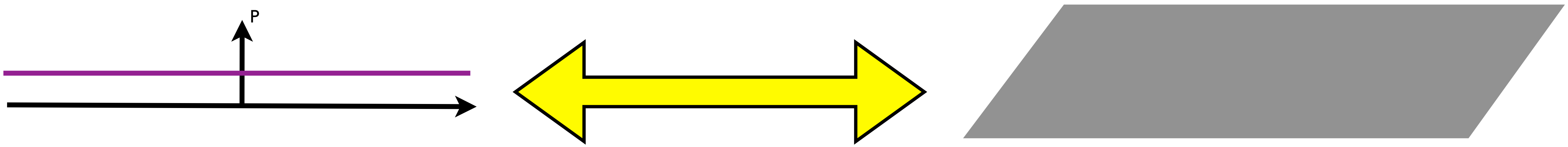
Let us start by considering an equilibrated configuration



A planar event horizon:

Holography

Let us start by considering an equilibrated configuration

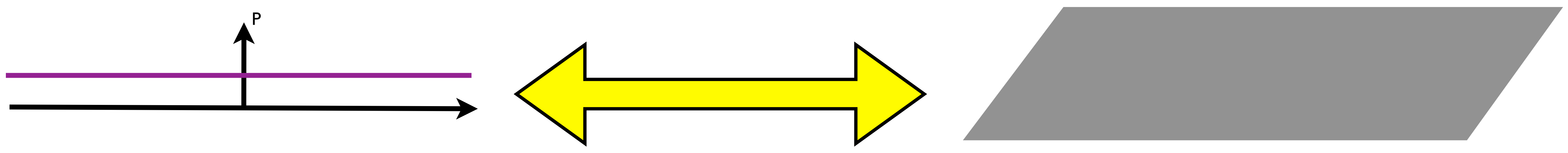


A planar event horizon:

$$ds^2 = 2dt\,(dr - A(r)dt) + r^2 d\vec{x}^2$$

Holography

Let us start by considering an equilibrated configuration



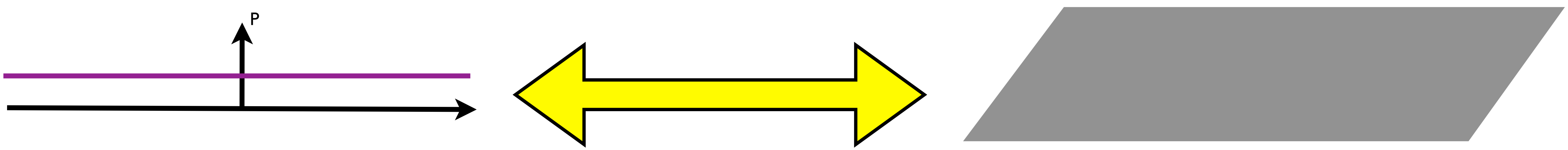
A planar event horizon:

$$ds^2 = 2dt\,(dr - A(r)dt) + r^2 d\vec{x}^2$$

$$A(r) = r^2 \left(1 - \left(\frac{4\pi T}{3r}\right)^3\right)$$

Holography

Let us start by considering an equilibrated configuration



A planar event horizon:

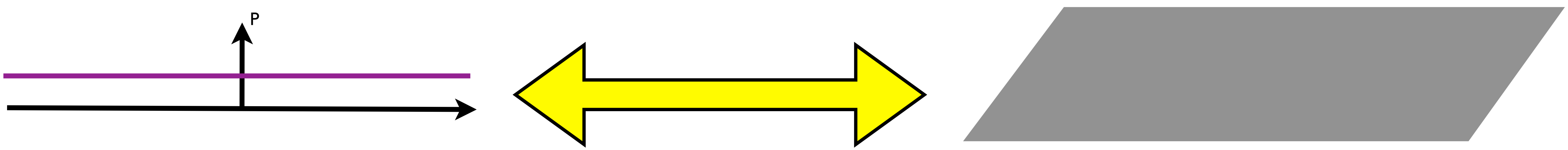
$$P(T) = p_0 \left(\frac{4\pi T}{3}\right)^3$$

$$ds^2 = 2dt\,(dr - A(r)dt) + r^2 d\vec{x}^2$$

$$A(r) = r^2 \left(1 - \left(\frac{4\pi T}{3r}\right)^3\right)$$

Holography

Let us start by considering an equilibrated configuration



A planar event horizon:

$$P(T) = p_0 \left(\frac{4\pi T}{3}\right)^3$$

$$ds^2 = 2dt\,(dr - A(r)dt) + r^2 d\vec{x}^2$$

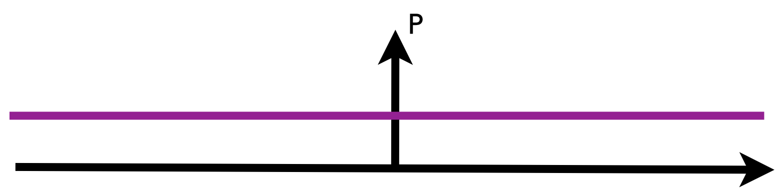
$$A(r) = r^2 \left(1 - \left(\frac{4\pi T}{3r}\right)^3\right)$$

e.g., in ABJM

$$p_0 = \frac{2N^2}{9\sqrt{2\lambda}} \qquad \lambda = \frac{N}{k}$$

Holography

Out of equilibrium we want to start with:



Holography

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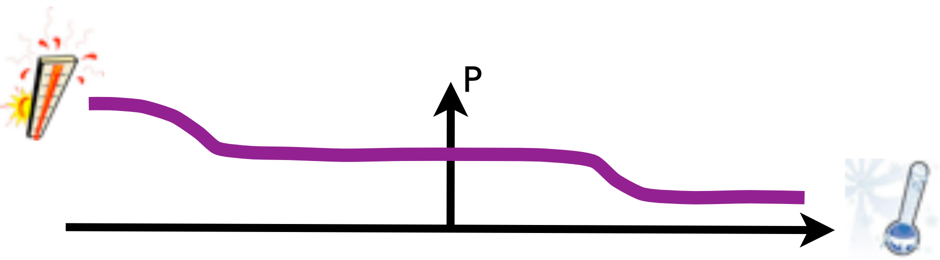

$$P(T_L) = p_0 \left(\frac{4\pi T_L}{3} \right)^3$$

$$P(T_R) = p_0 \left(\frac{4\pi T_R}{3} \right)^3$$

Holography

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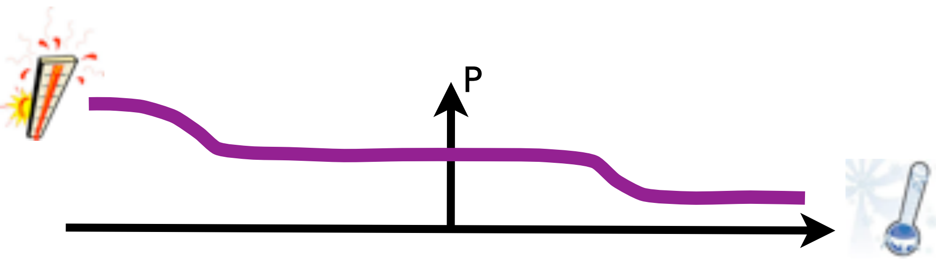

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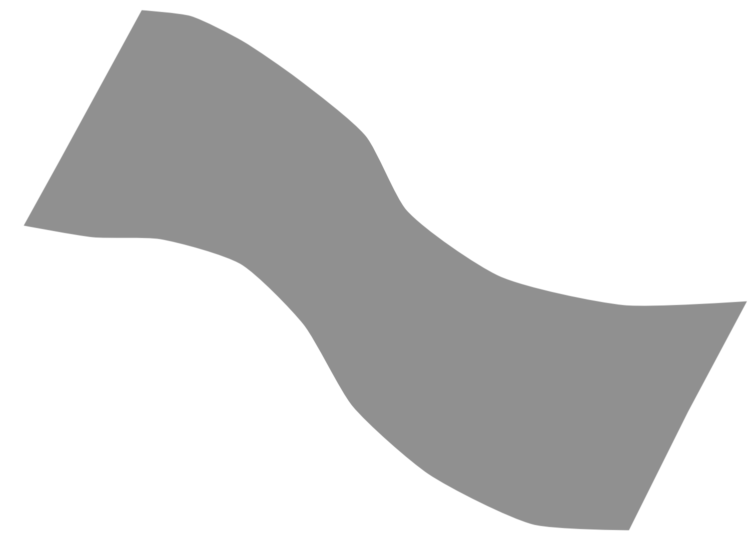
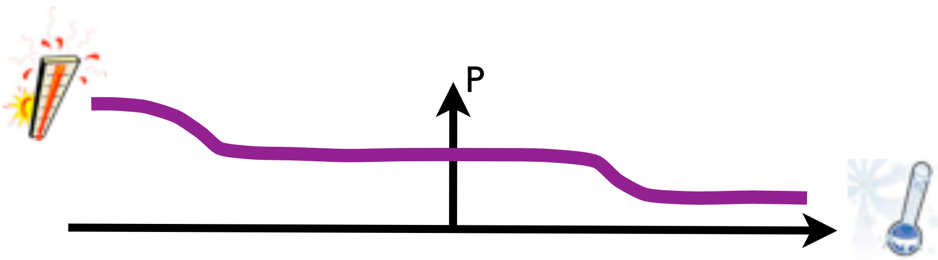


A planar event horizon:

Holography

Out of equilibrium we want to start with:



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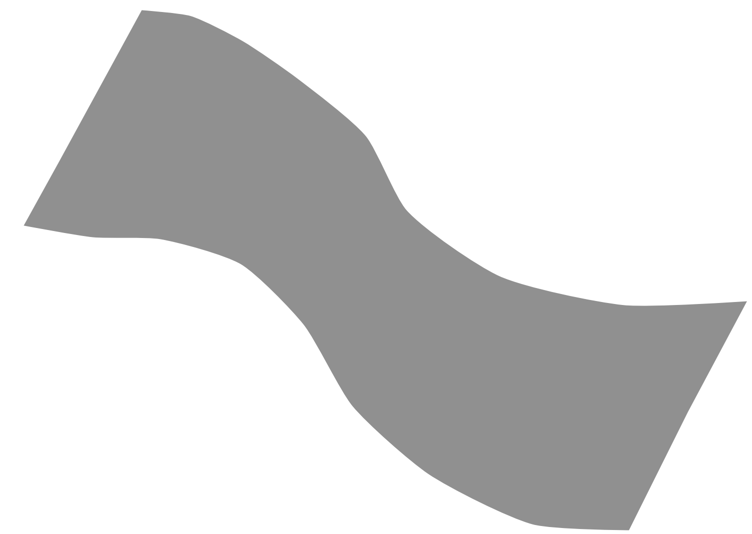

A planar event horizon:

$$ds^2 = 2dt\,(dr - A(r,z)dt) + r^2 d\vec{x}^2$$

$$A(r,z) = r^2 \left(1 - \left(\frac{a_1(z)}{3r} \right)^3 \right)$$

Holography

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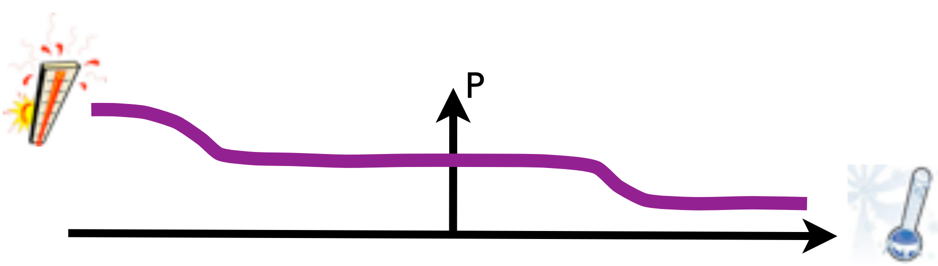

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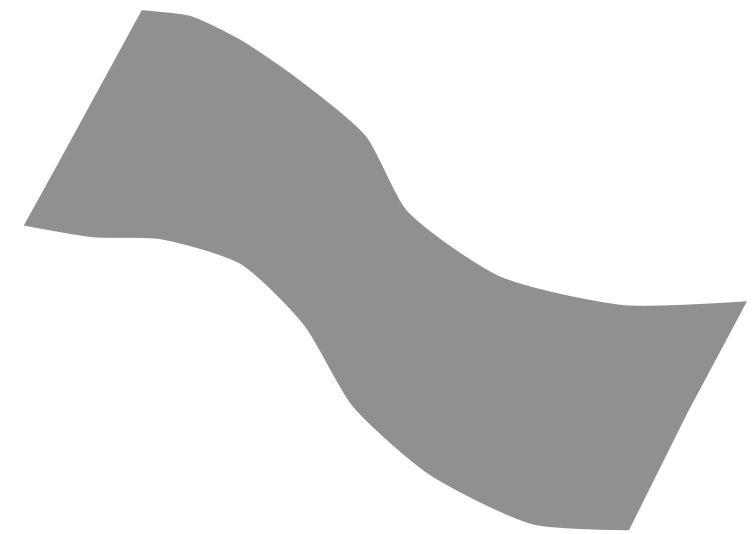

A planar event horizon:

$$ds^2 = 2dt\,(dr - A(r,z)dt) + r^2 d\vec{x}^2$$

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$$a_1 = -A_0 \left(1 - \alpha \tanh \left(\beta \tanh \left(\frac{z}{\lambda} \right) \right) \right)$$

$$a_1(-\infty) = \frac{4\pi T_L}{3} \qquad a_1(\infty) = \frac{4\pi T_R}{3}$$

Holography

Out of equilibrium we want to start with:

$$ds^2 = 2dt\,(dr - A(r,z)dt) + r^2 d\vec{x}^2$$

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and evolve it forward in time

Holography

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and evolve it forward in time. Using

$$ds^2 = 2dt(dr - A(t,z,r)dt - F(t,z,r)dz) + \Sigma^2(t,r,z)\left(e^{B(t,z,r)}dx_\perp^2 + e^{-B(t,z,r)}dz^2\right)$$

the Einstein equations reduce to a set of nested linear differential equations in the radial coordinate ‘r’.

Holography

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(Chesler, Yaffe, 2012)

Holography

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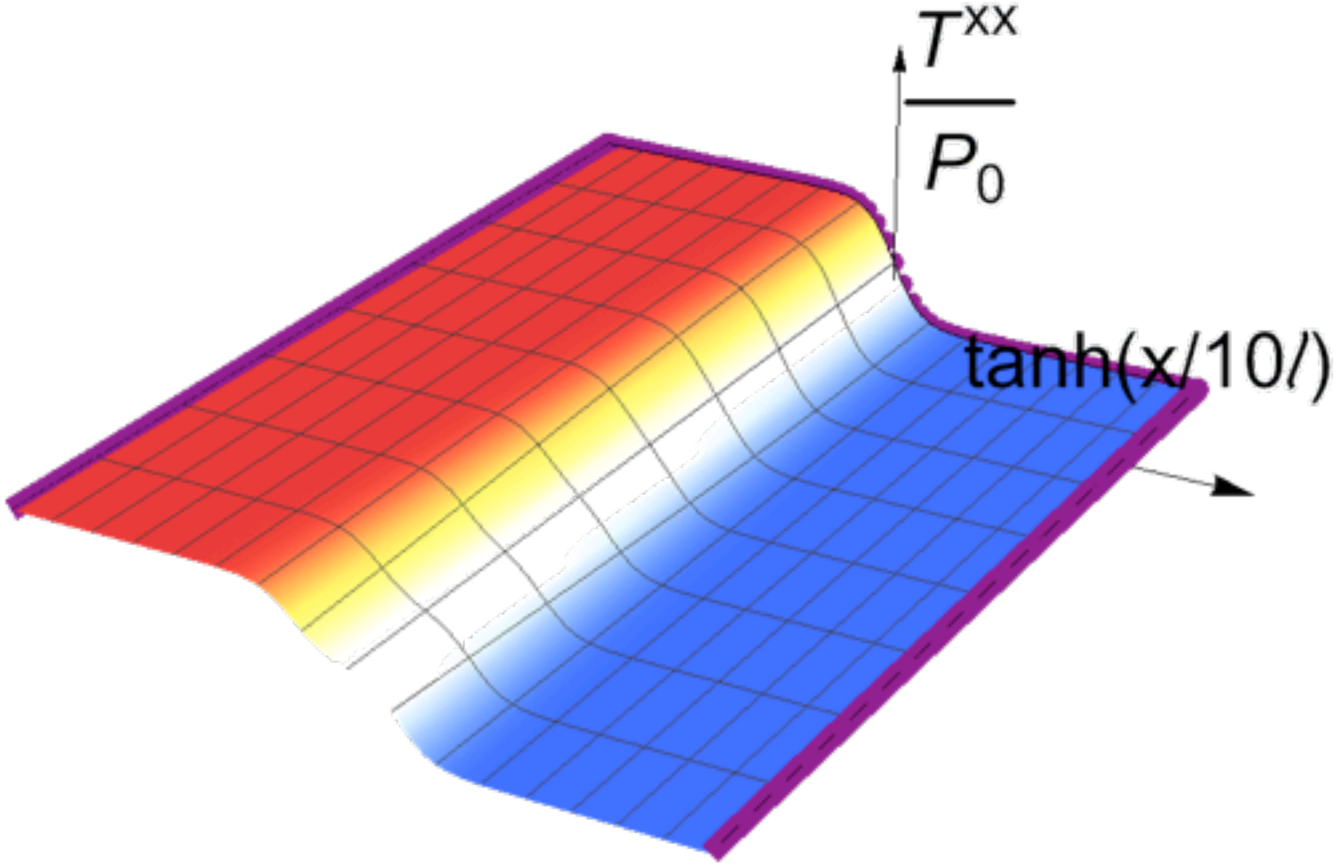
the Einstein equations reduce to a set of nested linear differential equations in the radial coordinate 'r'. We have solved these equations numerically.

Holography

Holography

We find ($d=3$, $\Delta P/P_0=0.4$)



Holography

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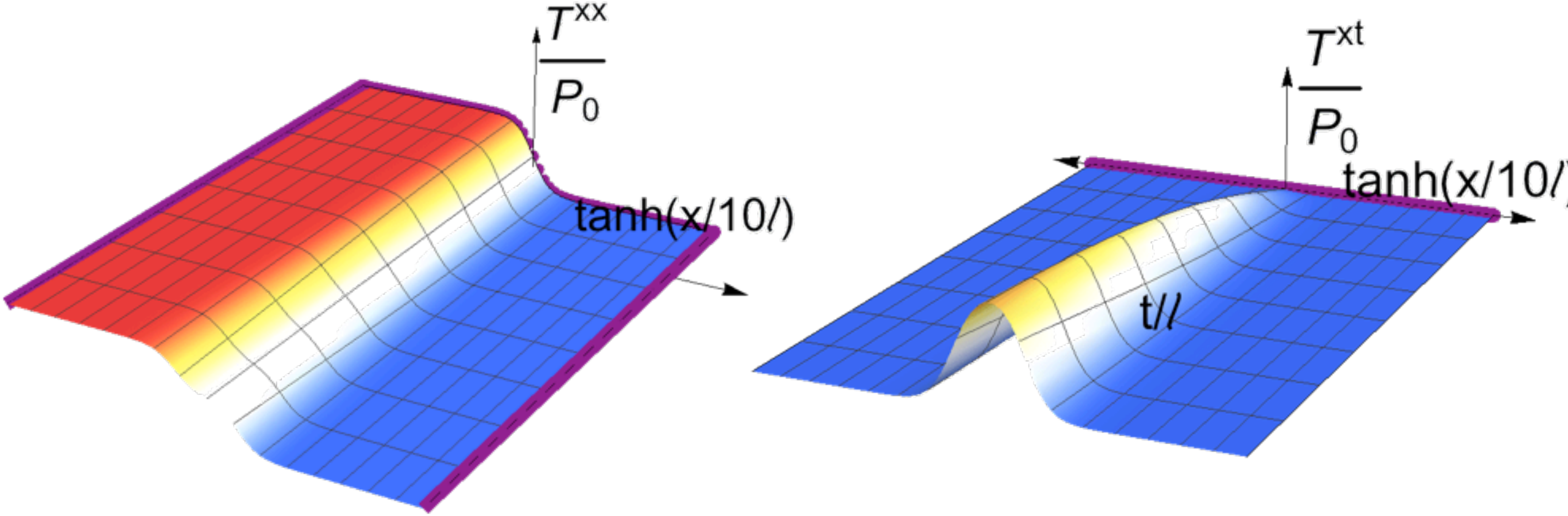

Holography

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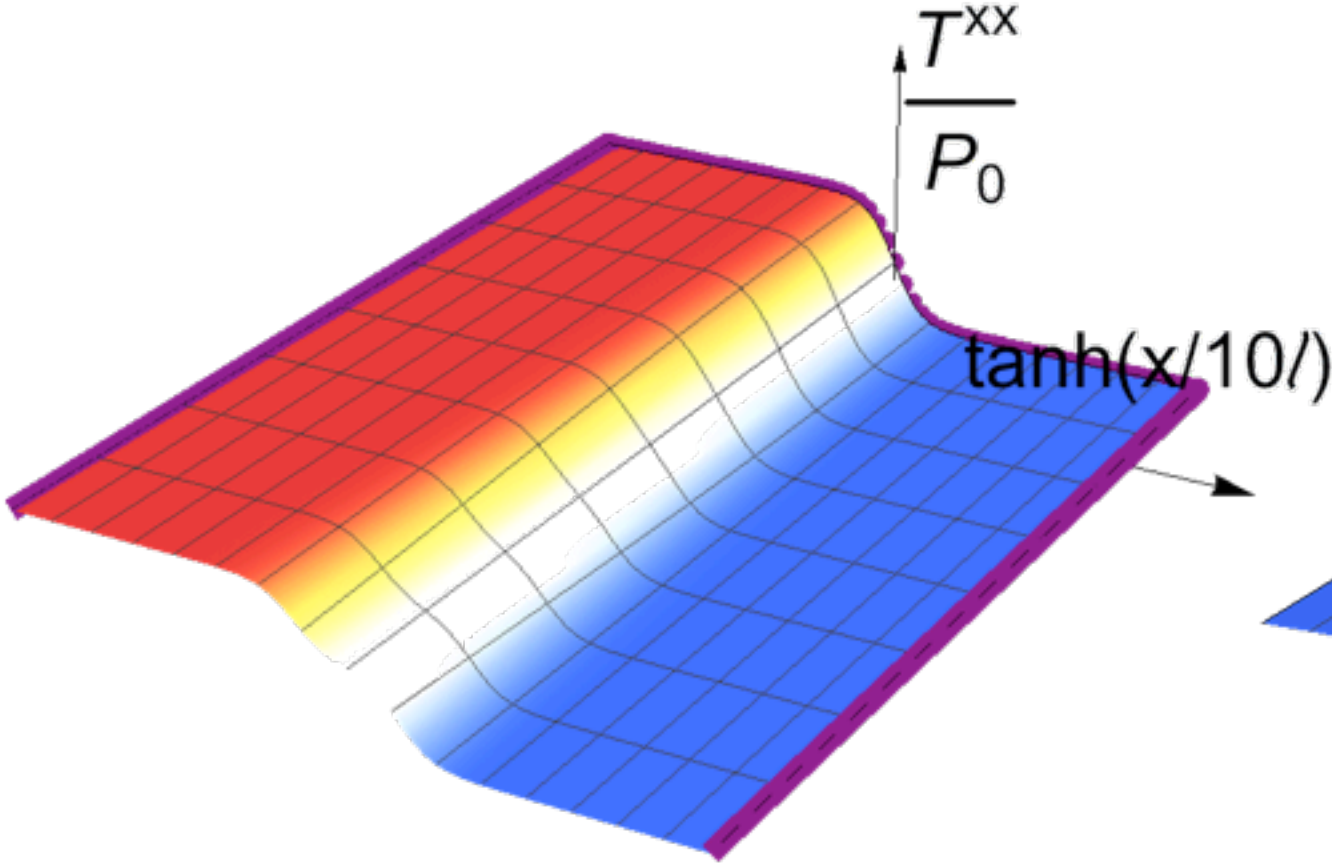

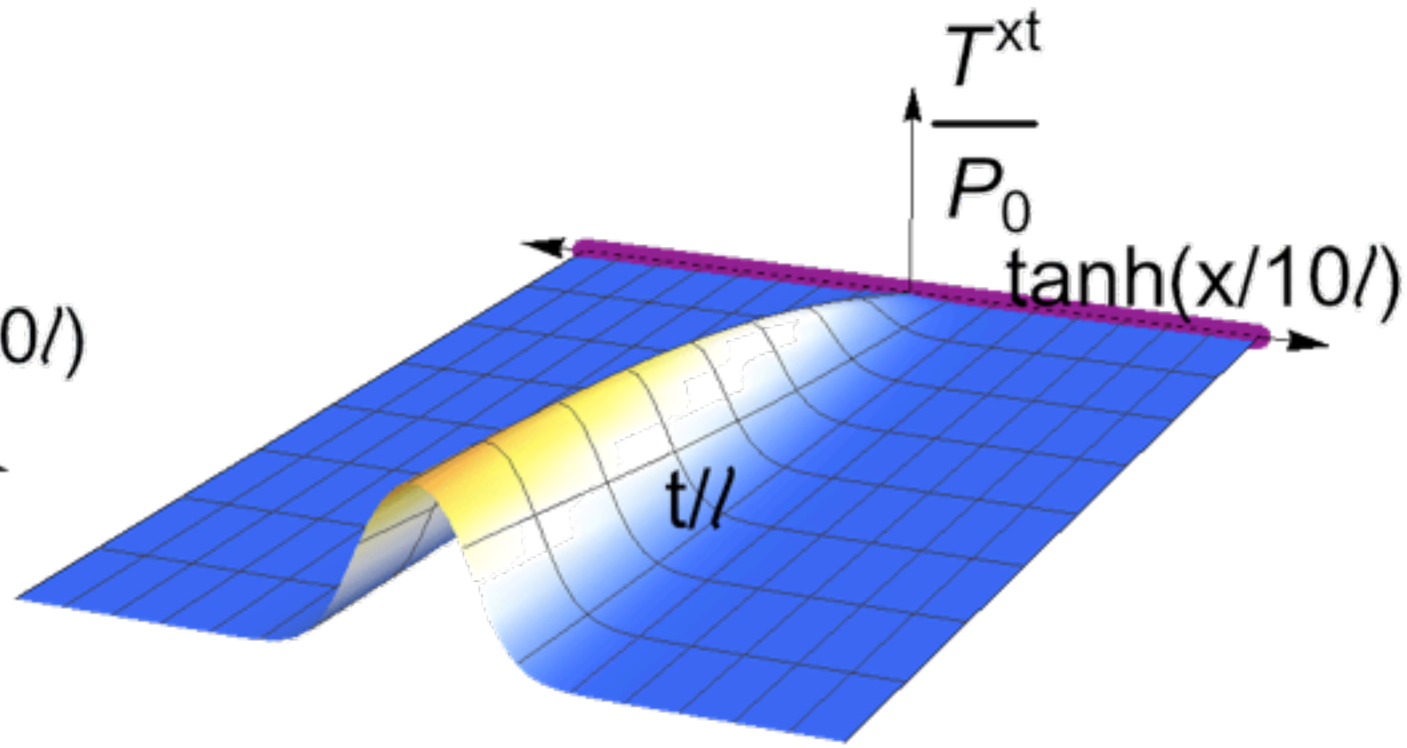

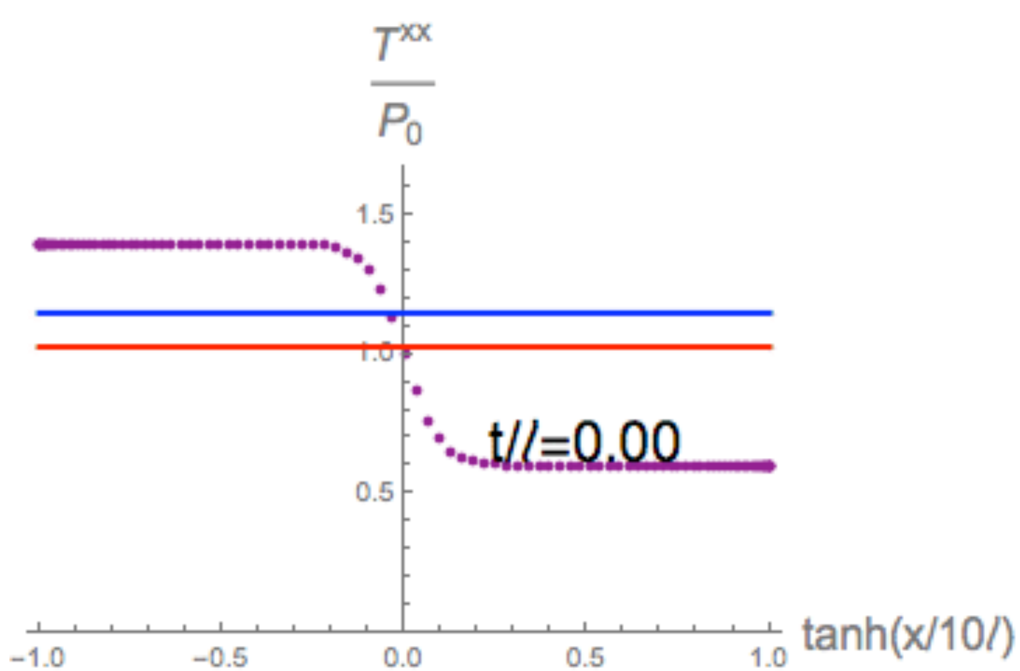

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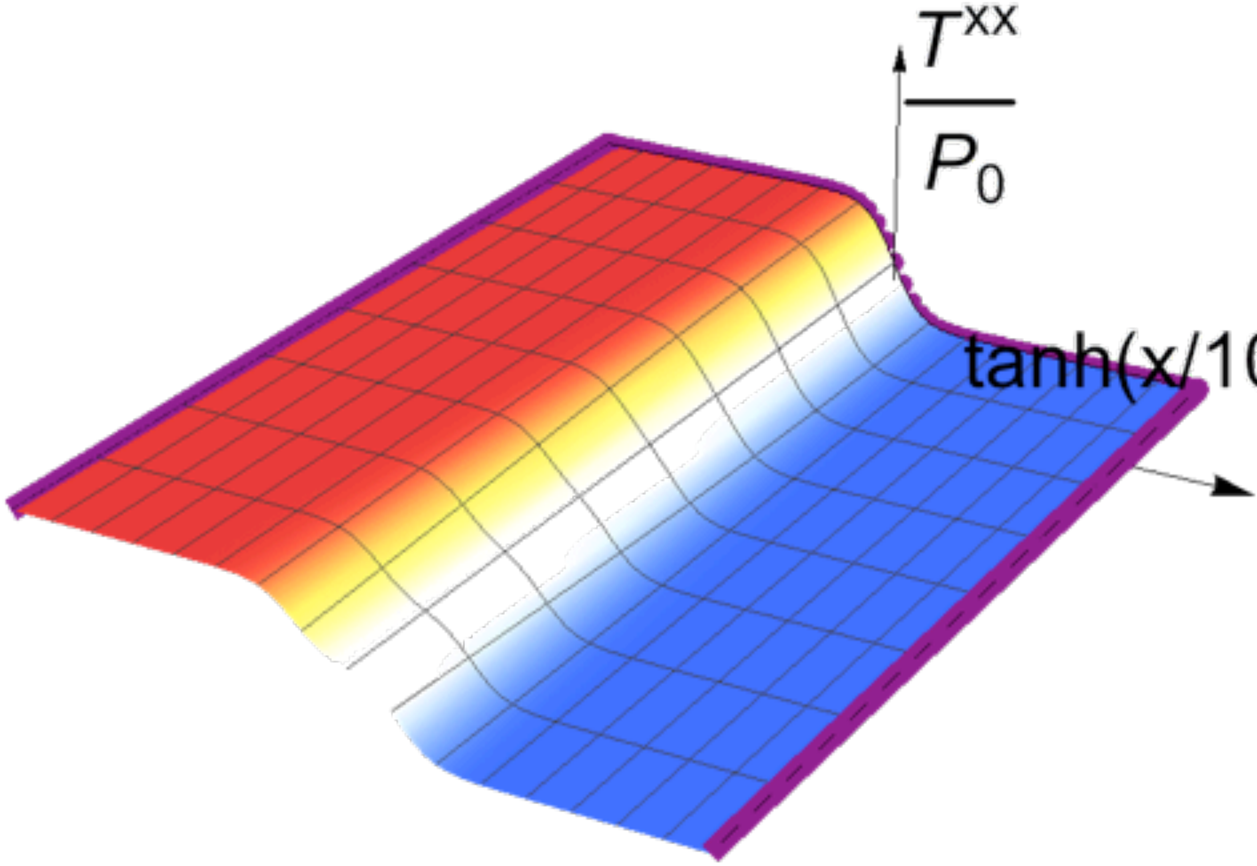

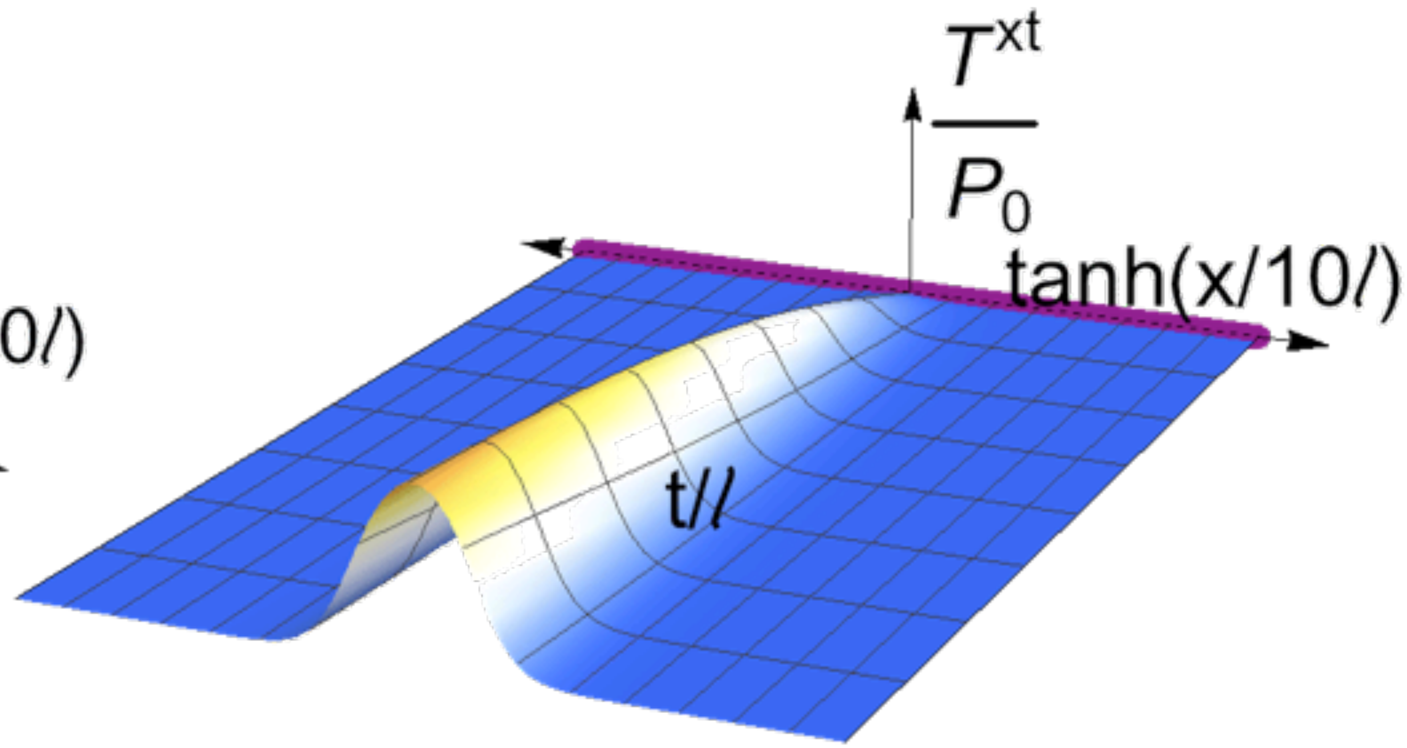

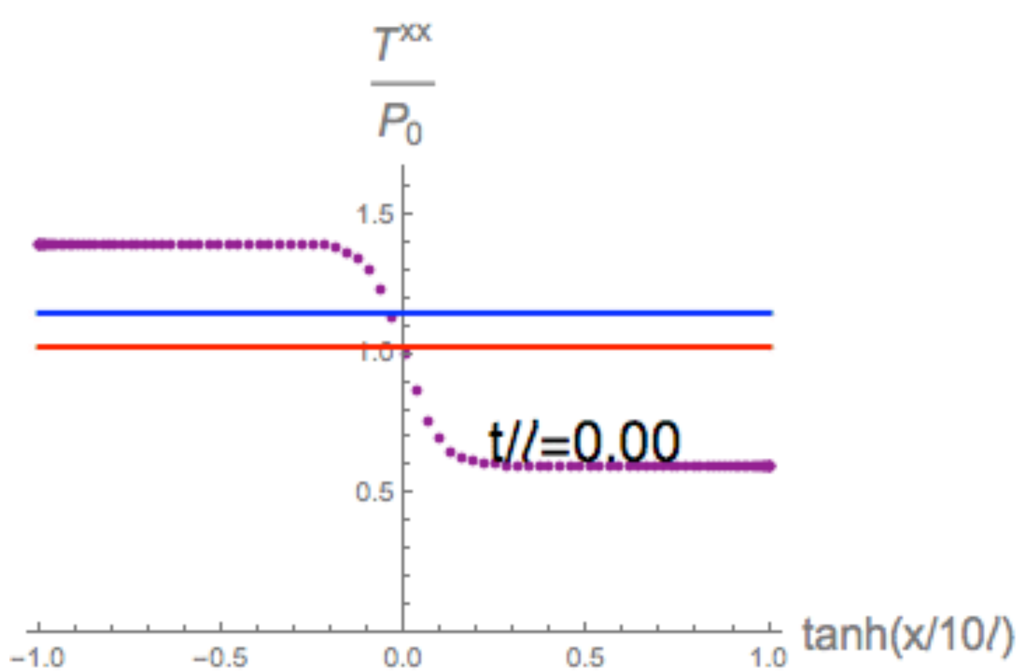

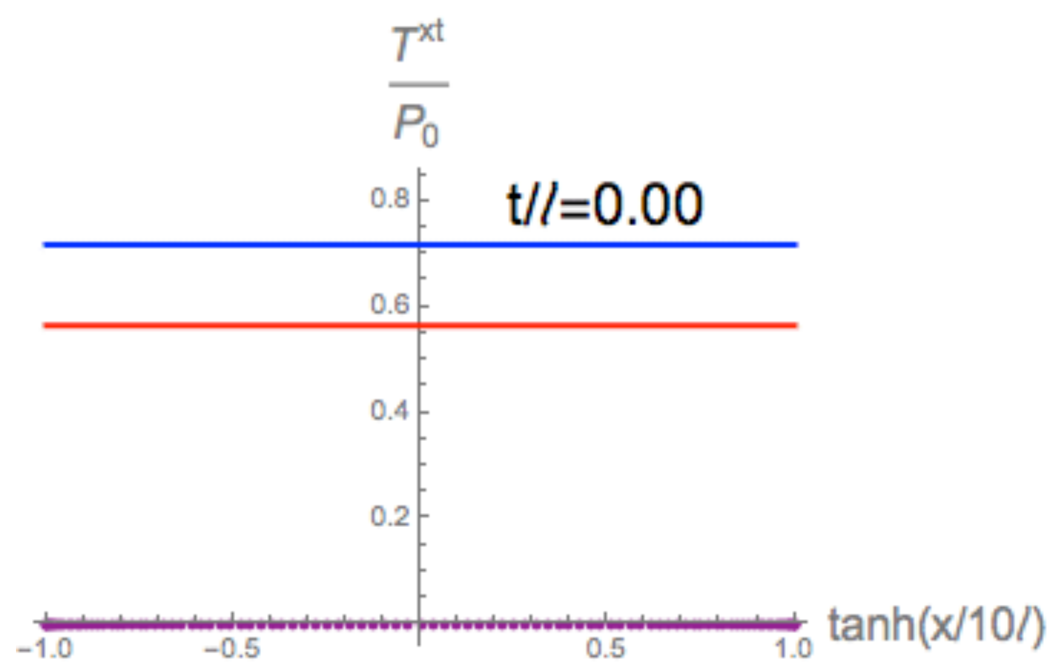

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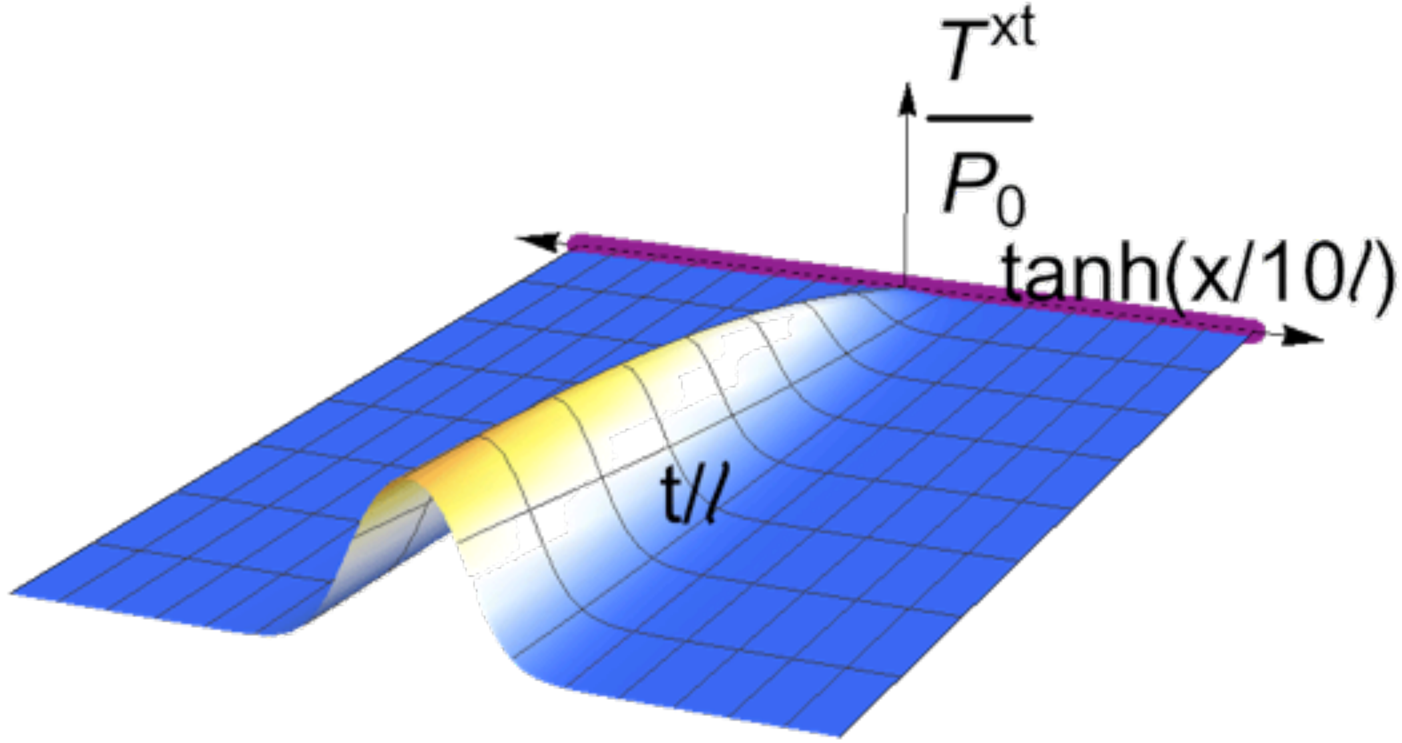

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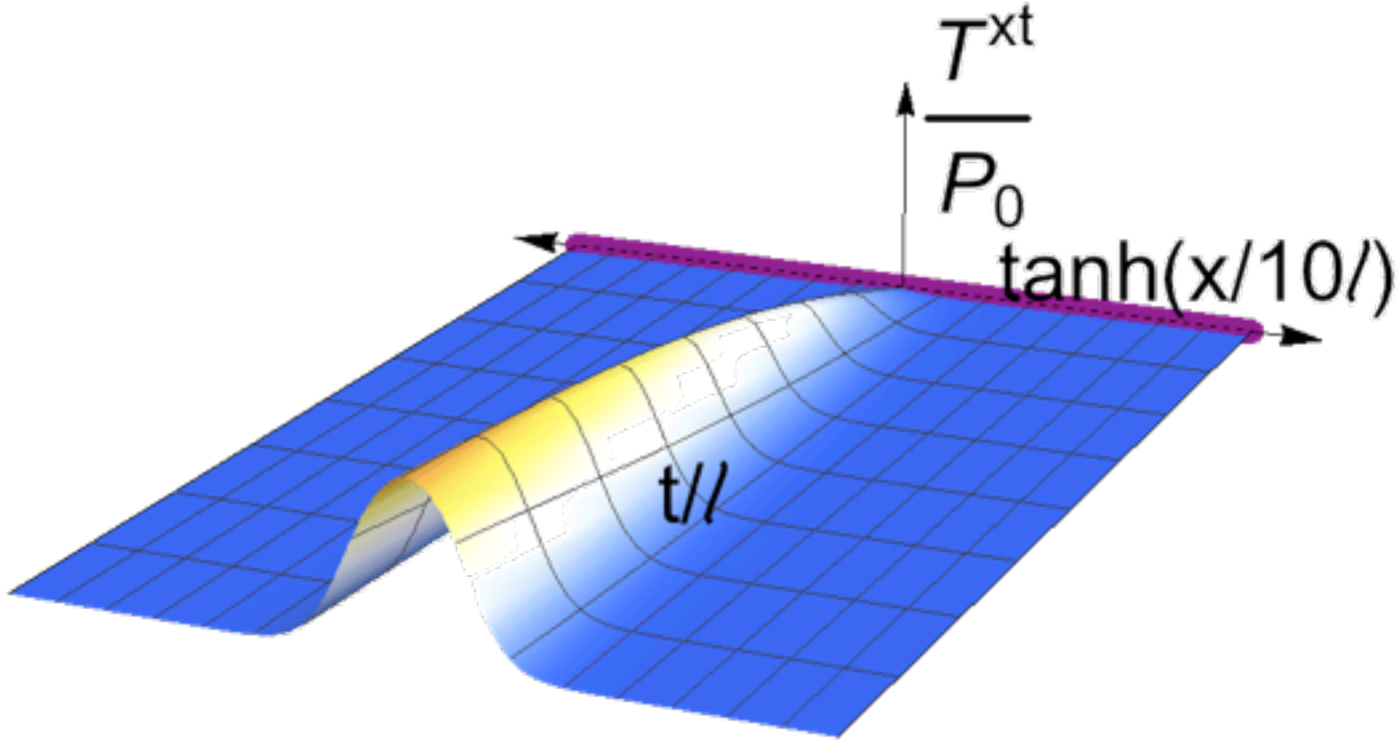

Holography

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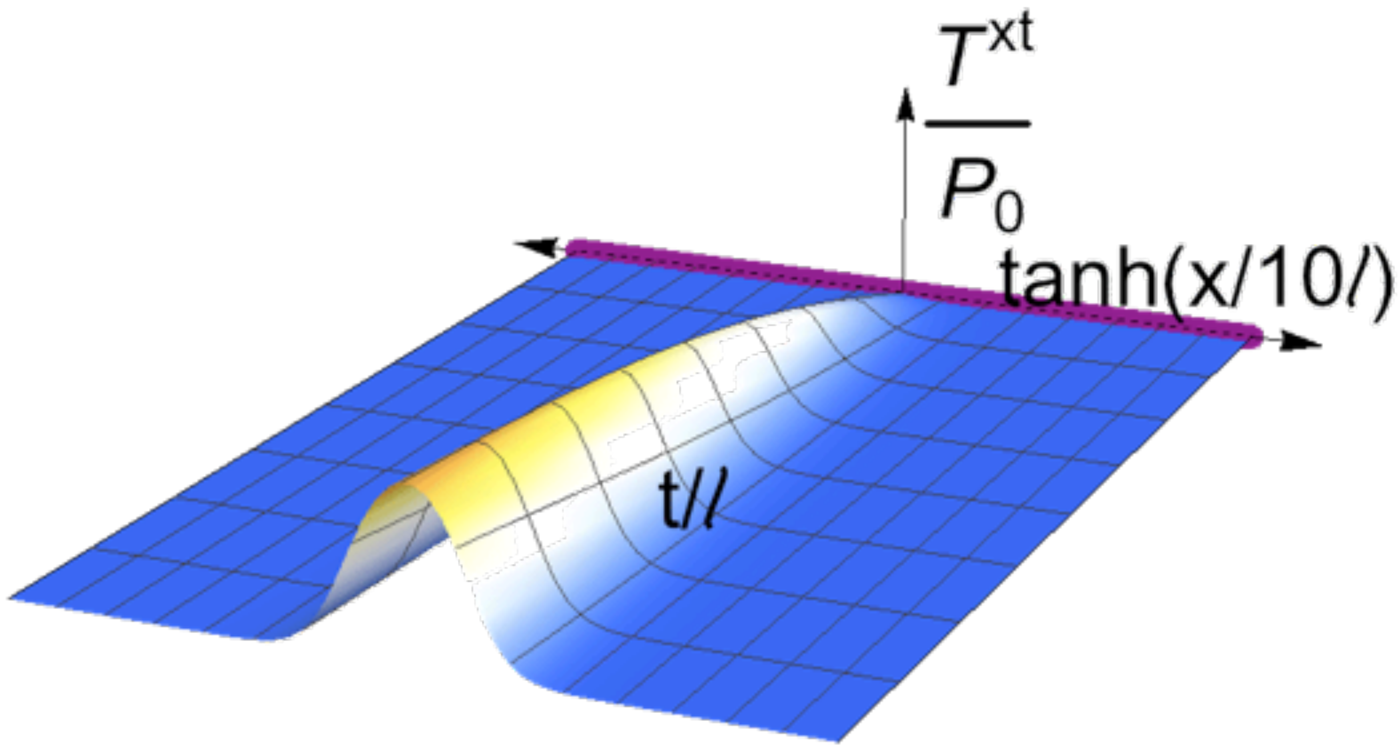

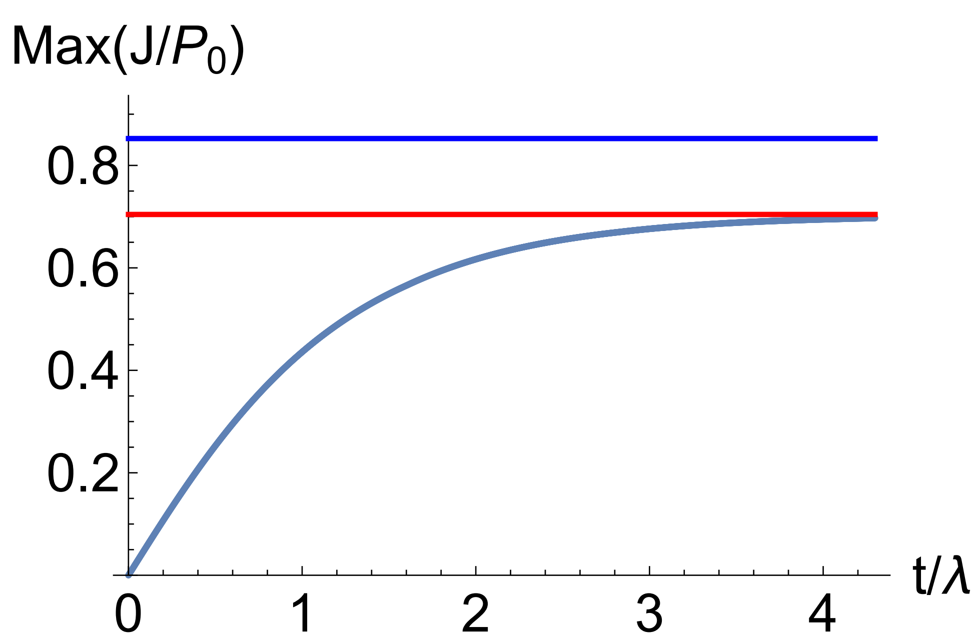

Holography

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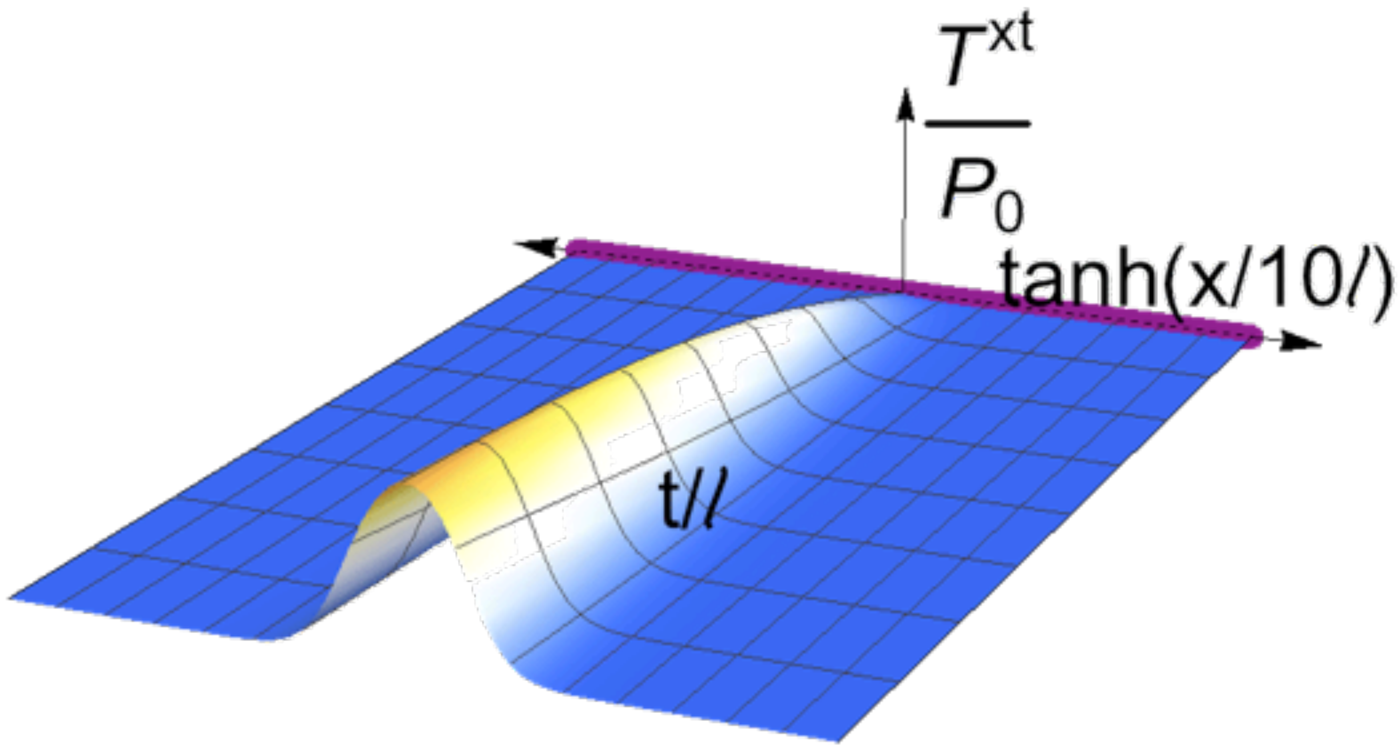
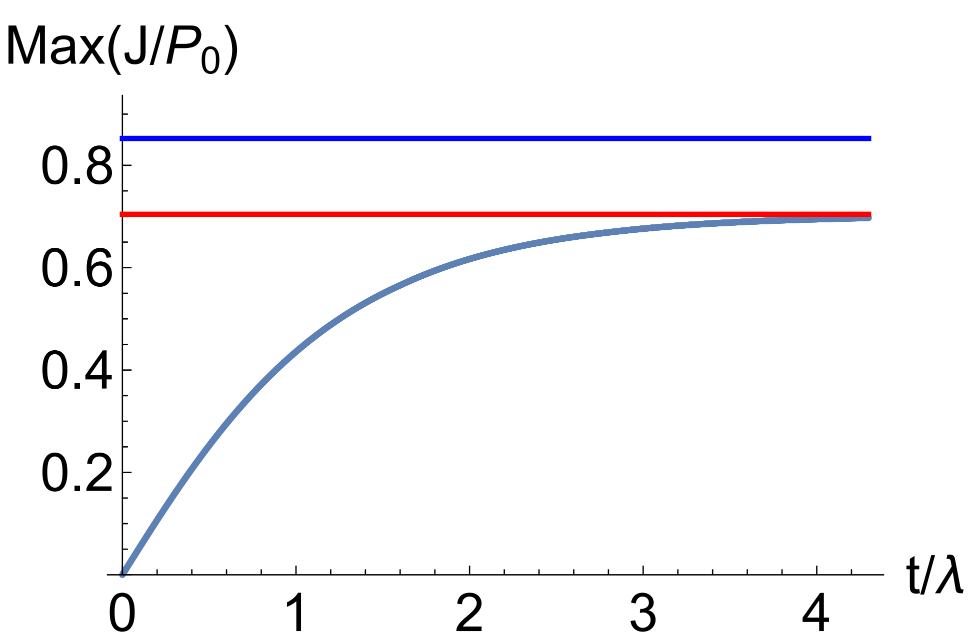
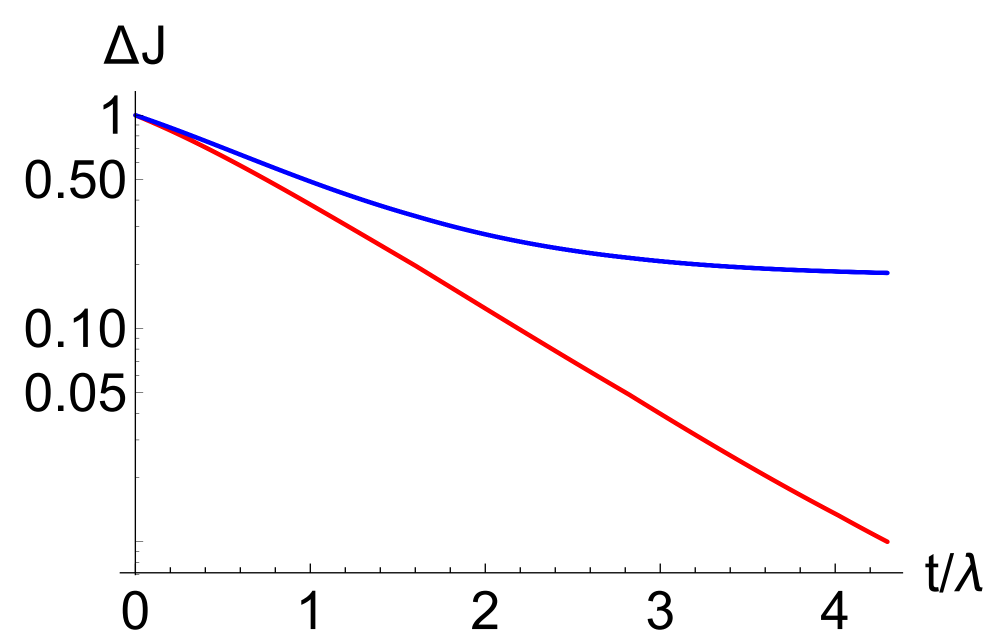
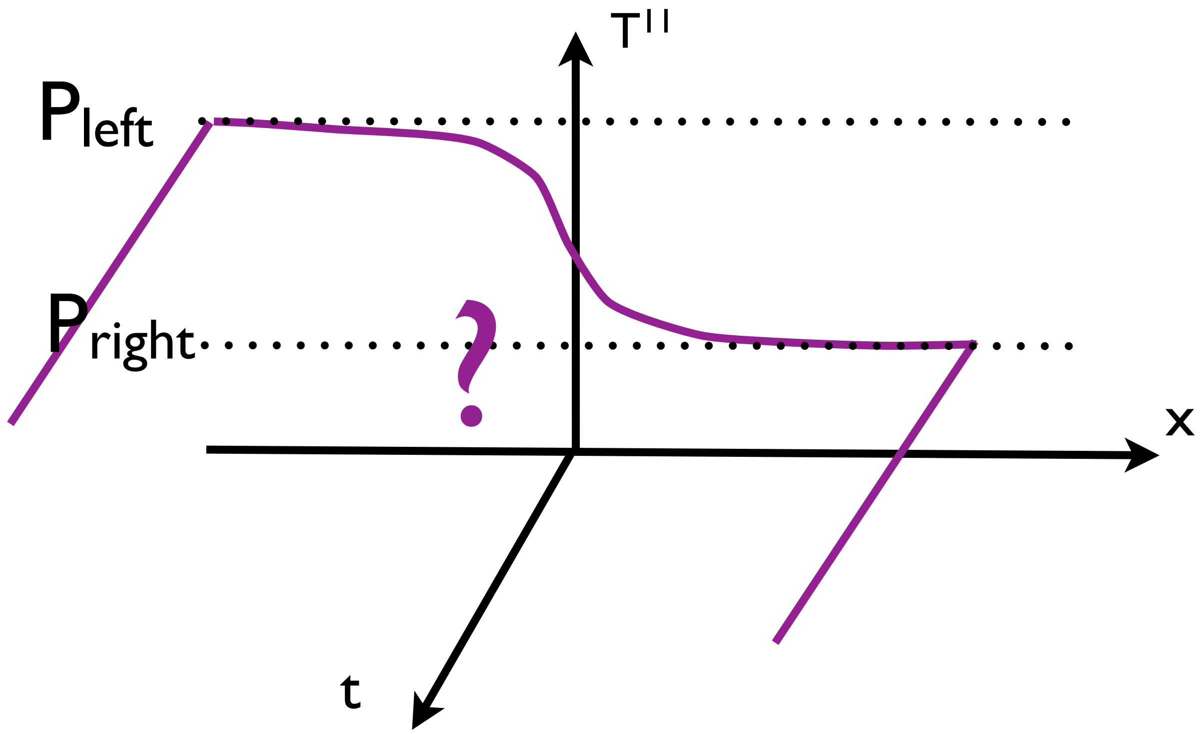
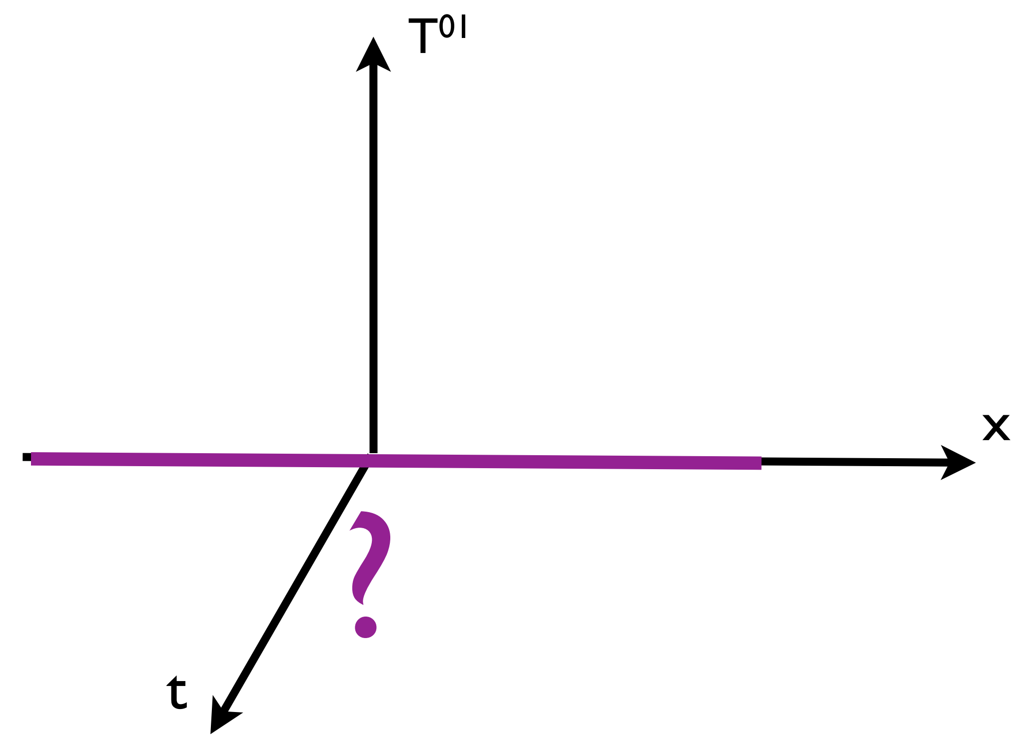




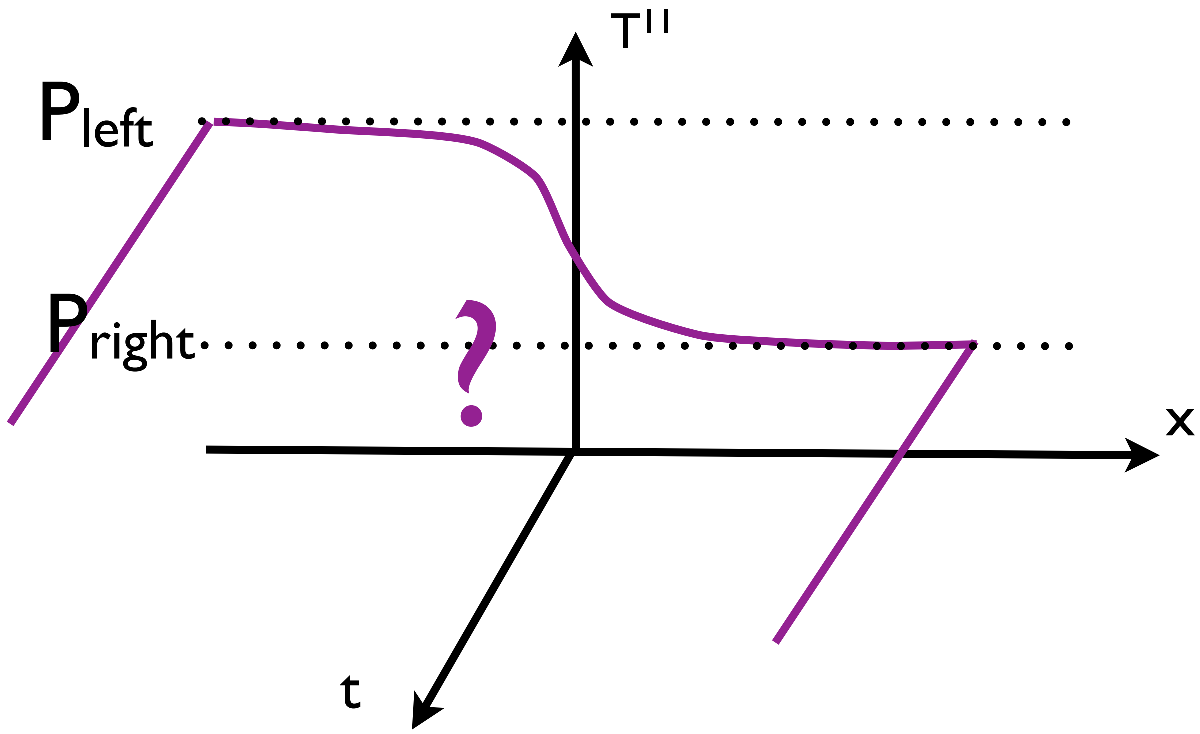


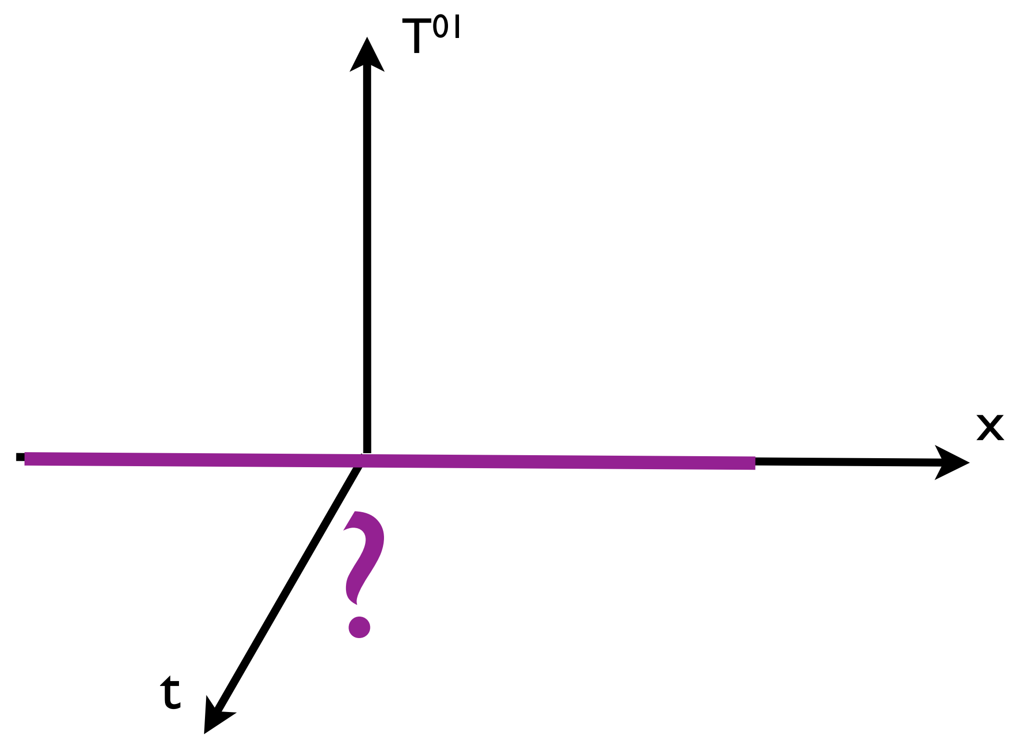
Summary





Summary



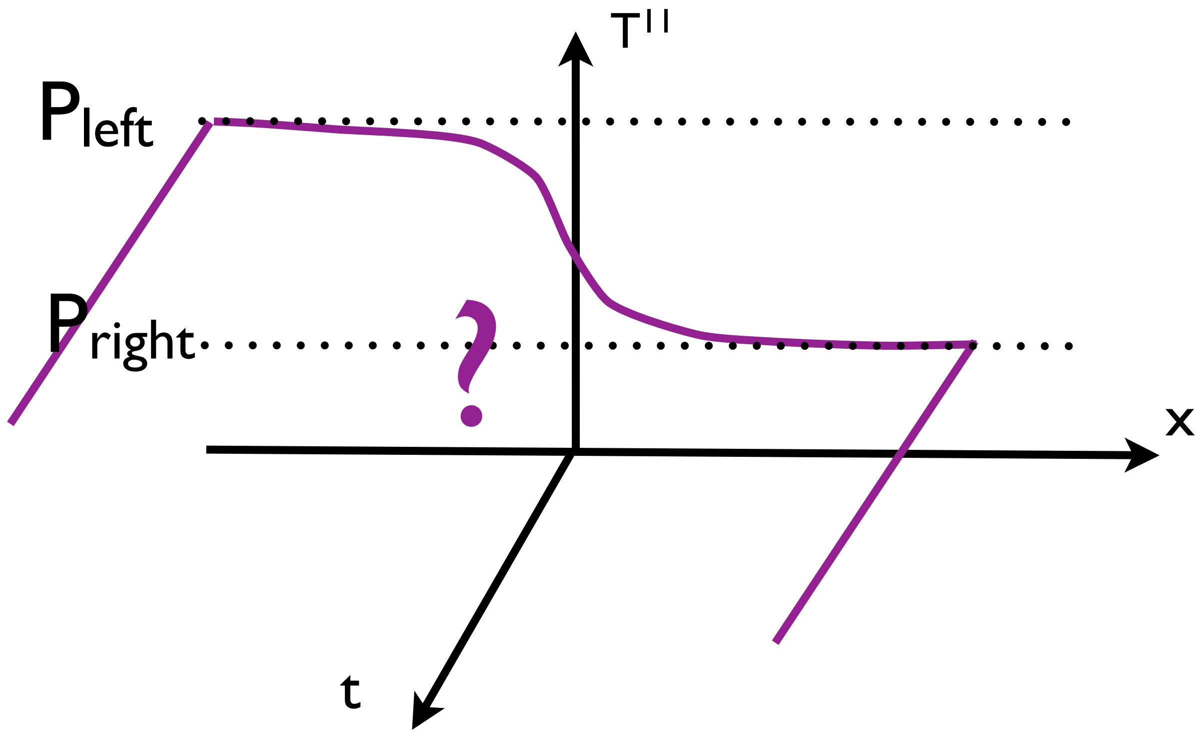


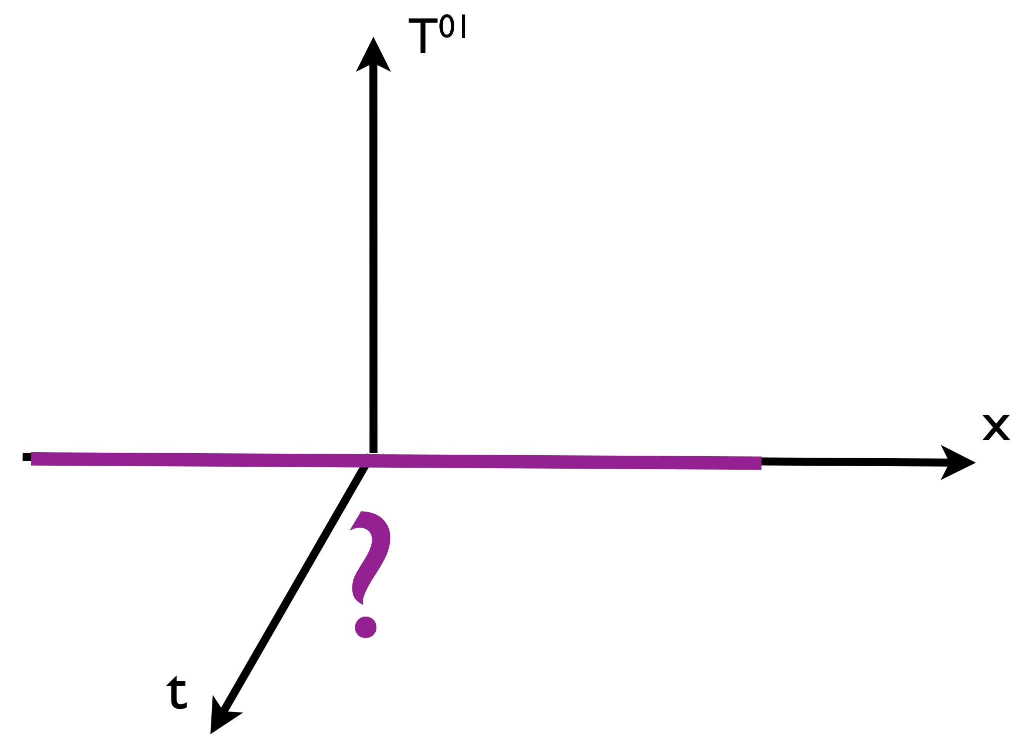
In a 2d CFT we find

$$T^{00} = T_+(\infty) + T_-(-\infty) = \frac{1}{2}\left(P_{\rm left} + P_{\rm right}\right) ,$$

$$T^{01} = T_-(-\infty) - T_+(\infty) = \frac{1}{2}\left(P_{\rm left} - P_{\rm right}\right)$$

Summary



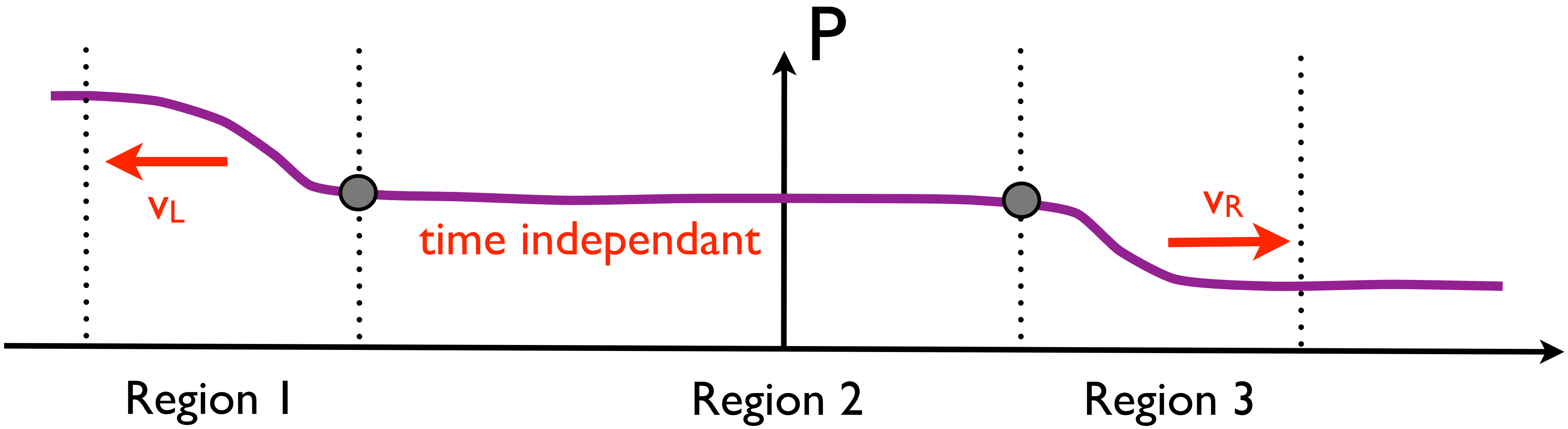


Also for linearized ideal fluids in d dimensions

$$T^{00}(t \to \infty) = (d-1)P_0\,, \quad T^{01}(t \to \infty) = \frac{\Delta P}{c_s}\,, \quad T^{11}(t \to \infty) = P_0$$

Summary

Otherwise, using the conjecture:



Summary

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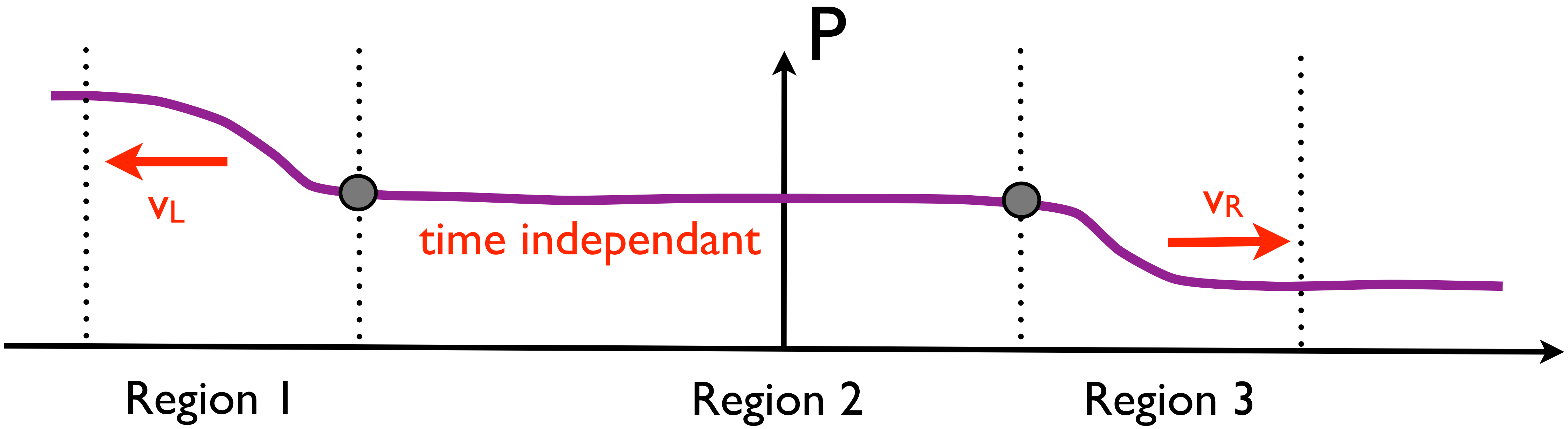

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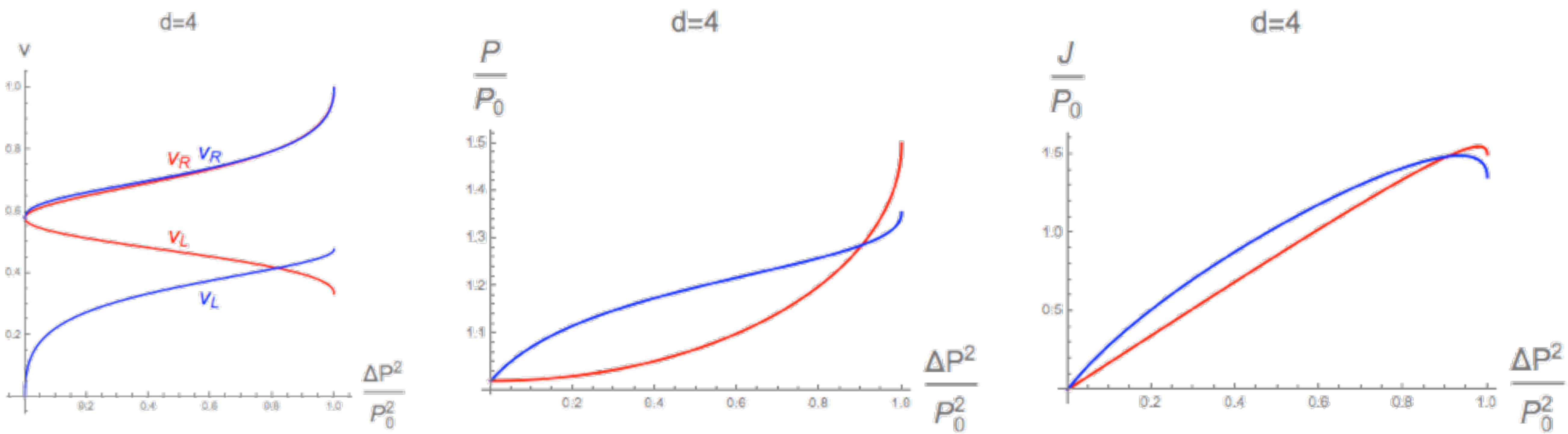

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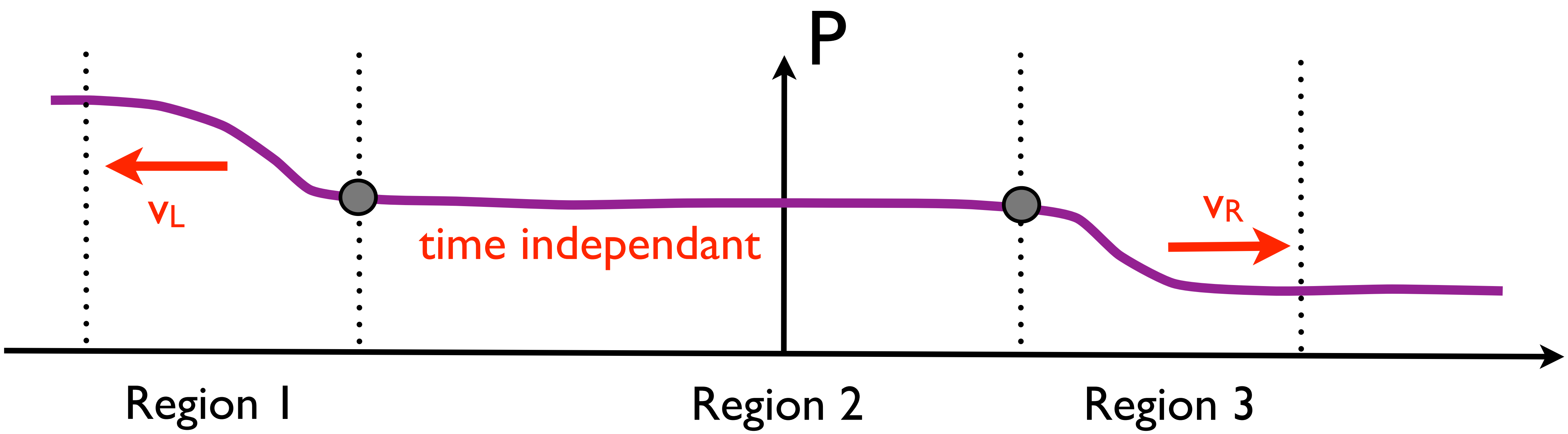
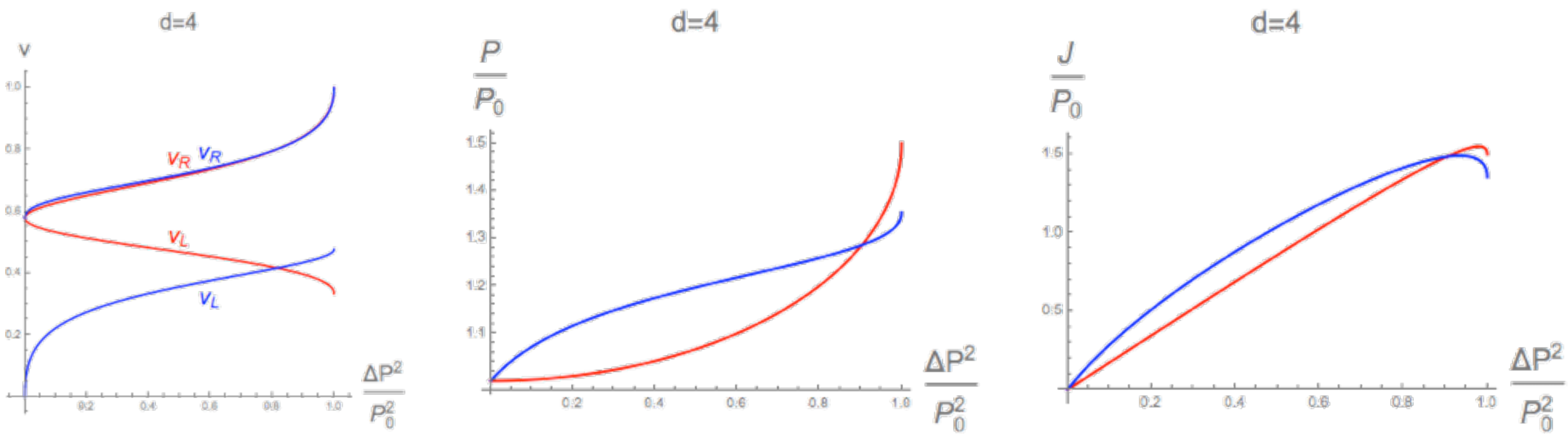


We find:



What about the blue branch?

Thank you