The (Exact) Renormalization Group, Holography and Higher Spin Theory

(fluids are somewhat in the future)

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Introduction

- An appealing aspect of holography is its interpretation in terms of the renormalization group of quantum field theories — the 'radial coordinate' is a geometrization of the renormalization scale — Hamilton-Jacobi theory of the radial quantization is expected to play a central role.

  e.g., [de Boer, Verlinde^2 '99, Skenderis '02, Heemskerk & Polchinski '10, Faulkner, Liu & Rangamani '10 ...]
- usually this is studied from the bulk side, as the QFT is typically strongly coupled
- here, we will approach the problem directly from the field theory side, using the Wilson-Polchinski exact renormalization group around (initially free) field theories [Douglas, Mazzucato & Razamat '10]

Higher Spin Holography

- standard holography:

  strongly coupled QFT $\leftrightarrow$ weakly coupled (s)gravity

  (no direct QFT/RG methods)
- vector model holography:

  weakly coupled QFT $\leftrightarrow$ higher spin theory

  (direct (exact) QFT/RG methods)

  [Klebanov & Polyakov '02, Sezgin & Sundell '02, Leigh & **Petkou** '03] [Vasiliev '96, '99, '12] [de Mello Koch, et al '11], ...

- goals: obtain precise geometric characterization, study the outcomes of interacting RG flows, emergence of standard holography

Higher Spin Holography

- free field theories have an infinite set of conserved currents

$$j^{(s)}_{\mu_1 \ldots \mu_s}(x) \sim \phi_m(x)\partial_{\mu_1} \ldots \partial_{\mu_s}\phi^m(x)$$

- one expects that these are all holographically dual to gauge fields (of appropriate spin)
- the corresponding gauge group is enormous!
  - characterize this group?
  - organize the hs theory in terms of connections on some bundle?
  - understand the origins of holography in interacting theories?
- note that gravity (stress tensor $\leftrightarrow$ bulk metric) is just one of these modes

Punch Lines

- We will first study free field theories perturbed by arbitrary bi-local ‘single-trace’ operators.
  - This is a ‘consistent truncation’ of the full RG system.
  - No approximations allowed!
  - It corresponds to the ‘unbroken phase’ of a HS theory.
- We identify a formulation in which the operator sources correspond (amongst other things) manifestly to a connection on a *really big* principal bundle (i.e., a so-called infinite jet bundle).
- The ‘gauge group’ can be understood directly in terms of field redefinitions in the path integral
  - consequently there are exact Ward identities that correspond to the ERG equations,
  - these subsequently define/determine the bulk theory exactly.

More Punch Lines

- The space-time structure extends in a natural way (governed by ERG) to a geometric structure over a space-time of one higher dimension, and AdS_{d+1} emerges as a geometry corresponding to the (relativistic) free fixed point, encoded in a flat connection.
- the ERG equations are the first-order equations of motion of a bulk phase space structure (corresponding to 'radial quantization')
- from the bulk point of view, these are equivalent to the equations of motion of a higher spin gauge theory
- identifying this Hamilton-Jacobi structure gives us an action for the higher spin theory, which, as is usual in holography, if taken on-shell encodes all of the correlation functions of the field theory

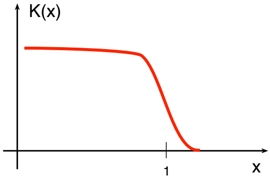
More Punch Lines

- all of the correlation functions of the free fixed point can be calculated exactly and have a holographic interpretation (corresponding to 'Witten diagrams')
- free fixed points with other symmetries also have duals with unbroken higher spin symmetries (with a corresponding background geometry (flat connection))
  - e.g., the ERG of $z = 2$ non-relativistic free theory and its corresponding holographic HS theory can be constructed via DLCQ
  - the bulk theory remembers the choice of superselection sector (value of the 'mass' n) via a holonomy of the flat connection
  - this flat connection equivalent to the Schrödinger geometry

The Exact Renormalization Group (ERG)

- Polchinski '84: formulated field theory path integral by introducing a regulator given by a cutoff function accompanying the fixed point action (i.e., the kinetic term).

$$Z = \int [d\phi] e^{-\int \phi K_F^{-1}(-\Box/M^2)\Box\phi - S_{int}[\phi]}$$



$$M\frac{\partial S_{int}}{\partial M} = -\frac{1}{2}\int M\frac{\partial K_F}{\partial M}\Box^{-1}\left[\frac{\delta S_{int}}{\delta\phi}\frac{\delta S_{int}}{\delta\phi} + \frac{\delta^2 S_{int}}{\delta\phi^2}\right]$$

- this equation describes how the couplings must depend on the RG scale in order that the partition function be independent of the cutoff.
- can apply similar methods to correlation functions, and thus obtain exact Callan-Symanzik equations as well

The ERG and Holography

- in this form, the ERG equations will be inconvenient — instead of moving the cutoff, we would like to fix the cutoff ($\sim M$) and move a renormalization scale (z)
- the ERG equations are first order equations, while bulk EOM are often second order
- solutions of such equations though are interpreted in terms of sources and vevs — the expected H-J structure implies that these should be thought of as canonically conjugate in radial quantization
- thus, we anticipate that the ERG equations for sources and vevs should be thought of as first-order Hamilton equations in the bulk

Locality is Over-Rated

- unbroken higher spin symmetry implies an infinite number of conserved currents — one can hardly expect to find a local theory
- indeed, free field theories have a huge non-local symmetry
- e.g., N Majoranas in $2+1$

$$S_0 = \int_{x,y} \widetilde{\psi}^m(x)\gamma^\mu P_{F;\mu}(x,y)\psi^m(y) \equiv \int \widetilde{\psi}^m \cdot \gamma^\mu P_{F;\mu} \cdot \psi^m$$

$$P_{F;\mu}(x,y) = \partial_\mu^{(x)}\delta(x-y)$$

- we include sources for the identity operator and all ‘single-trace’ operators

$$S_{int} = U + \frac{1}{2}\int_{x,y} \widetilde{\psi}^m(x)\Big(A(x,y) + \gamma^\mu W_\mu(x,y)\Big)\psi^m(y)$$

The $O(L_2(\mathbb{R}^d))$ Symmetry

- Bi-local sources collect together infinite sets of local operators, obtained by expanding near $x \to y$

$$A(x,y) = \sum_{s=0}^{\infty} A^{a_1 \cdots a_s}(x) \partial_{a_1}^{(x)} \cdots \partial_{a_s}^{(x)} \delta(x-y)$$

- Now we consider the following bi-local map of elementary fields

$$\psi^m(x) \mapsto \int_y \mathcal{L}(x,y) \psi^m(y) = \mathcal{L} \cdot \psi^m(x)$$

- We look at the action

$$\begin{aligned} S &\to \widetilde{\psi}^m \cdot \mathcal{L}^T \cdot \left[\gamma^\mu (P_{F;\mu} + W_\mu) + A \right] \cdot \mathcal{L} \cdot \psi^m \\ &= \widetilde{\psi}^m \cdot \gamma^\mu \mathcal{L}^T \cdot \mathcal{L} \cdot P_{F;\mu} \cdot \psi^m \\ &+ \widetilde{\psi}^m \cdot \left[\gamma^\mu (\mathcal{L}^T \cdot [P_{F;\mu}, \mathcal{L}] + \mathcal{L}^T \cdot W_\mu \cdot \mathcal{L}) + \mathcal{L}^T \cdot A \cdot \mathcal{L} \right] \cdot \psi^m \end{aligned}$$

The $O(L_2)$ Symmetry

- Thus, if we take $\mathcal{L}$ to be orthogonal,

$$\mathcal{L}^T \cdot \mathcal{L}(x,y) = \int_z \mathcal{L}(z,x)\mathcal{L}(z,y) = \delta(x,y),$$

the kinetic term is invariant, while the sources transform as

$$\begin{aligned} W_\mu &\mapsto \mathcal{L}^{-1} \cdot W_\mu \cdot \mathcal{L} + \mathcal{L}^{-1} \cdot \left[P_{F;\mu}, \mathcal{L}\right] \\ A &\mapsto \mathcal{L}^{-1} \cdot A \cdot \mathcal{L} \end{aligned}$$

- We interpret this to mean that the source $W_\mu(x,y)$ is the $O(L_2)$ connection, while A transforms tensorially under $O(L_2)$

The $O(L_2)$ Symmetry

- Punch line: the $O(L_2)$ transformation leaves the fixed point action invariant. $D_\mu = P_{F;\mu} + W_\mu$ plays the role of covariant derivative.
- More precisely, the free fixed point corresponds to any configuration

$$(A, W_\mu) = (0, W_\mu^{(0)})$$

  where $W^{(0)}$ is any flat connection, $dW^{(0)} + W^{(0)} \wedge W^{(0)} = 0$
- It is therefore useful to split the full connection as

$$W_\mu = W_\mu^{(0)} + \widehat{W}_\mu$$

  - $W^{(0)}$ is a flat connection associated with the fixed point
  - A, $\widehat{W}$ are operator sources, transforming tensorially under $O(L_2)$

The $O(L_2)$ Connection

- $D_\mu^{(0)} = P_{F;\mu} + W_\mu^{(0)}$ is the background covariant derivative
- to employ the ERG, we must introduce a regulator. A simple modification allows us to do that:

$$P_{F,\mu}(x,y) \mapsto K_F^{-1}(-D^{(0)2}/M^2)\partial_\mu^{(x)}\delta(x-y)$$

- this is covariant under $O(L_2)$
- the content of ERG will be the statement that the partition function is independent of M, and we will construct this statement as an exact Ward identity

The $O(L_2)$ Ward Identity

- Indeed, the $O(L_2)$ transformation is a trivial operation from the path integral point of view (change of integration variable), and so we conclude that there is an exact Ward identity

$$Z[M, g_{(0)}, W_\mu, A] = Z[M, g_{(0)}, \mathcal{L}^{-1} \cdot W_\mu \cdot \mathcal{L} + \mathcal{L}^{-1} \cdot P_{F;\mu} \cdot \mathcal{L}, \mathcal{L}^{-1} \cdot A \cdot \mathcal{L}]$$

- this is the usual notion of a background symmetry: a transformation of the elementary fields is compensated by a change in background
- more generally, we can turn on sources for arbitrary multi-local multi-trace operators — the sources will generally transform tensorially under $O(L_2)$
  (see later, perhaps)

The $O(L_2(\mathbb{R}^d))$ Symmetry

- In fact, the subgroup of $O(L_2)$ leaving $W^{(0)}$ invariant is $O(2,d)$, the conformal group of the boundary theory
- Thus the quasi-local expansion that we previously wrote

$$A(x,y) = \sum_{s=0}^{\infty} A^{a_1\cdots a_s}(x)\partial^{(x)}_{a_1}\cdots\partial^{(x)}_{a_s}\delta(x-y)$$

  should best be reformulated as a sum over conformal modules (the representation of $O(L_2(\mathbb{R}^d))$ being reducible as a direct sum of $O(2,d)$ irreps)
- soon, $W^{(0)}$ will be extended to a corresponding (Cartan) connection in the bulk, and we will identify it with that corresponding to AdS geometry

The $CO(L_2)$ symmetry

- We can generalize the $O(L_2)$ condition to include scale transformations

$$\int_z \mathcal{L}(z,x)\mathcal{L}(z,y) = \lambda(x)^{2\Delta_\psi}\delta(x-y)$$

- This is a symmetry (in the previous sense) provided we also transform the metric, the cutoff and the sources

$$g_{(0)} \mapsto \lambda^2 g_{(0)}, \quad M \mapsto \lambda^{-1} M$$

$$A \mapsto \mathcal{L}^{-1}\cdot A \cdot \mathcal{L}$$

$$W_\mu \mapsto \mathcal{L}^{-1}\cdot W_\mu \cdot \mathcal{L} + \mathcal{L}^{-1}\cdot \left[P_{F;\mu}, \mathcal{L}\right].$$

- A convenient way to keep track of the scale is to introduce the conformal factor $g_{(0)} = \frac{1}{z^2}\eta$. Then $z \mapsto \lambda^{-1} z$. This z should be thought of as the renormalization scale.

The Renormalization group

- To study RG systematically, we proceed in two steps:

  Step 1: Lower the cutoff $M \mapsto \lambda M$, by integrating out the "fast modes"

  $$Z[M, z, A, W] = Z[\lambda M, z, \widetilde{A}, \widetilde{W}] \qquad (\textit{Polchinski})$$

  Step 2: Perform a $CO(L_2)$ transformation to bring the cutoff back to M, but in the process changing $z \mapsto \lambda^{-1} z$

  $$Z[\lambda M, z, \widetilde{A}, \widetilde{W}] = Z[M, \lambda^{-1} z, \mathcal{L}^{-1} \cdot \widetilde{A} \cdot \mathcal{L}, \mathcal{L}^{-1} \cdot \widetilde{W} \cdot \mathcal{L} + \mathcal{L}^{-1} \cdot [P_F, \mathcal{L}]]$$

- We can now compare the sources at the same cutoff, but different z. Thus, z becomes the natural flow parameter, and we can think of the sources as being z-dependent.
  - Thus we have the Polchinski formalism extended to include both a cutoff and an RG scale — required for a holographic interpretation.

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Infinitesimal version: RG equations

- Infinitesimally, we parametrize the $CO(L_2)$ transformation as

$$\mathcal{L} \simeq \mathbf{1} + \varepsilon z W_z$$

- think of as the z-component of the connection.
- The RG equations become

$$A(z+\varepsilon z) = A(z) + \varepsilon z\,[W_z, A] + \varepsilon z \beta^{(A)} + O(\varepsilon^2)$$

$$W_\mu(z+\varepsilon z) = W_\mu(z) + \varepsilon z\left[P_{F;\mu} + W_\mu, W_z\right] + \varepsilon z \beta^{(W)}_\mu + O(\varepsilon^2)$$

- The beta functions are *tensorial*, and quadratic in A and $\widehat{W}$.
- The flat connection $W^{(0)}$ also satisfies a "pure-gauge" RG equation

$$W^{(0)}_\mu(z+\varepsilon z) = W^{(0)}_\mu(z) + \varepsilon z\left[P_{F;\mu} + W^{(0)}_\mu, W^{(0)}_z\right] + O(\varepsilon^2)$$

RG equations

- Thus, RG extends the sources A and W_μ to bulk fields $\mathcal{A}$ and $\mathcal{W}_I$.
- Comparing terms linear in ε gives

$$\partial_z \mathcal{W}^{(0)}_\mu - [P_{F;\mu}, \mathcal{W}^{(0)}_z] + [\mathcal{W}^{(0)}_z, \mathcal{W}^{(0)}_\mu] = 0$$

$$\partial_z \mathcal{A} + [\mathcal{W}_z, \mathcal{A}] = \beta^{(\mathcal{A})}$$

$$\partial_z \mathcal{W}_\mu - [P_{F;\mu}, \mathcal{W}_z] + [\mathcal{W}_z, \mathcal{W}_\mu] = \beta^{(\mathcal{W})}_\mu$$

- These equations are naturally thought of as being part of fully covariant equations (e.g., the first is the $z\mu$ component of a bulk 2-form equation, where $d \equiv dx^\mu P_{F,\mu} + dz\partial_z$.)

$$d\mathcal{W}^{(0)} + \mathcal{W}^{(0)} \wedge \mathcal{W}^{(0)} = 0$$

$$d\mathcal{A} + [\mathcal{W}, \mathcal{A}] = \boldsymbol{\beta}^{(\mathcal{A})}$$

$$d\mathcal{W} + \mathcal{W} \wedge \mathcal{W} = \boldsymbol{\beta}^{(\mathcal{W})}$$

- The resulting equations are then diff invariant in the bulk.

Hamilton-Jacobi Structure

- Similarly, one can extract exact Callan-Symanzik equations for the z-dependence of $\Pi(x, y) = \langle\tilde{\psi}(x)\psi(y)\rangle$, $\Pi^\mu(x, y) = \langle\tilde{\psi}(x)\gamma^\mu\psi(y)\rangle$. These extend to bulk fields $\mathcal{P}, \mathcal{P}^A$.
- The full set of equations then give rise to a phase space formulation of a dynamical system — $(\mathcal{A}, \mathcal{P})$ and $(\mathcal{W}_A, \mathcal{P}^A)$ are canonically conjugate pairs from the point of view of the bulk.
- If we identify $Z = e^{iS_{HJ}}$, then a fundamental relation in H-J theory is
$$\frac{\partial}{\partial z} S_{HJ} = -\mathcal{H}$$
- We can thus read off this Hamiltonian — it can be thought of as the output of the ERG analysis
- there is a corresponding action S_{HJ} for this higher spin theory, written in terms of phase space variables

Hamilton-Jacobi Structure

- We interpret this phase space theory as the higher spin gauge theory
- this theory is written as a gauge theory on a spacetime, topology $\sim \mathbb{R}^d \times \mathbb{R}^+$
- a flat connection $\mathcal{W}^{(0)}$ representing the free fixed point, in suitable coordinates, might be written

$$\mathcal{W}^{(0)}(x, y) = -\frac{dz}{z} D(x, y) + \frac{dx^\mu}{z} P_\mu(x, y)$$

  where $P_\mu(x, y) = \partial_\mu^{(x)} \delta(x - y)$, etc.
- This (Cartan) connection is equivalent to the vielbein and spin connection of AdS_{d+1}.

Bosonic Relativistic Free Fixed Point

- Another example consists of N complex scalar fields. In this case, we formulate the single-trace deformations in terms of the $CU(L_2)$ connection.

$$S = \int \phi_m^* \cdot \left(\left[D_{F;\mu} + W_\mu \right]^2 + B \right) \cdot \phi^m$$

- The ERG equations give rise to an 'A-model' in any dimension.
- Here though there is an extra background symmetry

$$Z[M, z, B, W_\mu^{(0)}, \widehat{W}_\mu + \Lambda_\mu] = Z[M, z, B + \{\Lambda^\mu, D_\mu\} + \Lambda_\mu \cdot \Lambda^\mu, W_\mu^{(0)}, \widehat{W}_\mu]$$

- this background symmetry allows for fixing $W_\mu \to W_\mu^{(0)}$, and the corresponding transformed B sources all single-trace currents.

[This was the starting point of Douglas, et al, and so geometry was not manifest.]

The Bulk Action and Correlation Functions

- For the bosonic theory, the bulk phase space action is

$$I = \int dz\, \mathrm{Tr}\left\{\mathcal{P}^I \cdot \left(\mathcal{D}_I \mathfrak{B} - \beta_I^{(\mathfrak{B})}\right) + \mathcal{P}^{IJ} \cdot \mathcal{F}_{IJ}^{(0)} + N\, \Delta_B \cdot \mathfrak{B}\right\}$$

- Here Δ_B is a derivative with respect to M of the cutoff function.
- As in any holographic theory, we solve the bulk equations of motion in terms of boundary data, and obtain the on-shell action, which encodes the correlation functions of the field theory.
- It is straightforward to carry this out exactly for the free fixed point.
- Here we have

$$I_{o.s.} = N \int \Delta_B \cdot \mathfrak{B}$$

where now $\mathfrak{B}$ is the bulk solution

The Bulk Action and Correlation Functions

- The RG equation

$$\left[\mathcal{D}_z^{(0)}, \mathfrak{B}\right] = \beta_z^{(\mathfrak{B})} = \mathfrak{B} \cdot \Delta_B \cdot \mathfrak{B}$$

can be solved iteratively

$$\mathfrak{B} = \alpha \mathfrak{B}_{(1)} + \alpha^2 \mathfrak{B}_{(2)} + \ldots,$$

$$\begin{aligned} \left[\mathcal{D}_z^{(0)}, \mathfrak{B}_{(1)}\right] &= 0 \\ \left[\mathcal{D}_z^{(0)}, \mathfrak{B}_{(2)}\right] &= \mathfrak{B}_{(1)} \cdot \Delta_B \cdot \mathfrak{B}_{(1)} \\ \left[\mathcal{D}_z^{(0)}, \mathfrak{B}_{(3)}\right] &= \mathfrak{B}_{(2)} \cdot \Delta_B \cdot \mathfrak{B}_{(1)} + \mathfrak{B}_{(1)} \cdot \Delta_B \cdot \mathfrak{B}_{(2)} \\ &\vdots \end{aligned}$$

The Bulk Action and Correlation Functions

- The first equation is homogeneous and has the solution

$$\mathfrak{B}_{(1)}(z;x,y)=\int_{x',y'}K^{-1}(z;x,x')b_{(0)}(x',y')K(z;y',y)$$

where we have defined the boundary-to-bulk Wilson line

$$K(z)=P.\exp\int_{\epsilon}^{z}dz'\,\mathcal{W}_z^{(0)}(z')$$

with the boundary being placed at $z=\epsilon$. (UV cutoff $\sim M/\epsilon$)

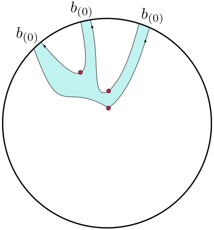
- $b_{(0)}$ has the interpretation of a boundary source
- this can then be inserted into the second order equation and the whole system solved iteratively

The Bulk Action and Correlation Functions

- At k^{th} order, one finds a contribution to the on-shell action

$$I^{(k)}_{o.s.} = N\int_\epsilon^\infty dz_1 \int_\epsilon^{z_1} dz_2 \ldots \int_\epsilon^{z_{k-1}} dz_k \\ \times \mathrm{Tr}\, H(z_1)\cdot b_{(0)}\cdot H(z_2)\cdot b_{(0)}\cdot \ldots \cdot H(z_k)\cdot b_{(0)} \\ + permutations$$

where $H(z) \equiv K^{-1}(z)\cdot\Delta_B(z)\cdot K(z) = \partial_z g(z)$



The Witten diagram for the bulk on-shell action at third order.

The Bulk Action and Correlation Functions

- The z-integrals can be performed trivially, resulting in

$$I^{(k)}_{o.s.} = \frac{N}{k}\,\mathrm{Tr}\left(g_{(0)}\cdot b_{(0)}\right)^k$$

  where $g_{(0)} = g(\infty)$ is the boundary free scalar propagator
- This result admits a resummation, resulting in

$$Z[b_{(0)}] = \det{}^{-N}\left(1 - g_{(0)}b_{(0)}\right)$$

  which is the exact generating functional for the free fixed point.
- Thus, this holographic theory does everything that it can for us.

Localization and Fronsdal

- clearly we have a higher spin theory on AdS_{d+1}. It has been formulated holographically, we have a precise understanding of the 'gauge group', we have a detailed understanding of all of the bulk interactions
- but it is not clear what relationship it might have with the Vasiliev higher spin theory (if in fact it does)
- To investigate this, the first thing we can do (see 1503.xxxxx) is to isolate the individual propagating local bulk modes (turn off bulk interactions and expand quasi-locally into individual spin-s modes). These should satisfy the Fronsdal higher spin equations of motion.
- this is essentially guaranteed by the fact that we have the group theory well-organized, but perhaps it is helpful to see it carefully.

Localization and Fronsdal

- in the boundary we have currents $j_{a_1\ldots a_s}(0)$ that are the lowest weights of a conformal module $D(\Delta, s)$ (s labels an $O(1, d-1)$ irrep). Because they are conserved, the representation is short, with $\Delta = s + d - 2$.
- more relevant to the present discussion is the fact that the vevs

$$\Pi_{a_1\ldots a_s}(x) \in D(\Delta, s),$$

while the corresponding sources

$$B^{a_1\cdots a_2}(x) \in D(d-\Delta, s),$$

the dual shadow representation
- in the bulk, dynamical variables $\left(B^{a_1\cdots a_2}(z,x), \Pi_{a_1\ldots a_s}(z,x)\right)$ that together transform in $D(\Delta, s) \otimes D(d-\Delta, s)$

Localization and Fronsdal

- The Fronsdal equations are nothing but the statement that the quadratic Casimir of $O(2,d)$ for either of these representations is

$$C_2 = s(s+d-2) - \Delta(d-\Delta)$$

- representing $O(2,d)$ in the bulk, we find

$$C_2 = z^2\partial_z^2 + (2s-d+1)z\partial_z + s(s-d) + s(s+d-2) + z^2\Box$$

  which is indeed Fronsdal (in an appropriate 'Coulomb' gauge)
- so how to see this from our higher spin theory?

Localization and Fronsdal

- recall the bulk equations of motion

$$\begin{aligned} \mathcal{D}_z^{(0)}\mathfrak{B} &= \mathfrak{B}\cdot\Delta_B\cdot\mathfrak{B} \\ \mathcal{D}_z^{(0)}\mathcal{P} &= iN\Delta_B - \mathcal{P}\cdot\mathfrak{B}\cdot\Delta_B - \Delta_B\cdot\mathfrak{B}\cdot\mathcal{P} \end{aligned}$$

  In particular, we want to study the above equations upon linearizing about the background

$$\mathfrak{B} = 0, \quad \mathcal{P} = \mathcal{P}^{(0)}$$

  where $\mathcal{P}^{(0)}$ satisfies $\mathcal{D}_z^{(0)}\mathcal{P}^{(0)} = iN\Delta_B$, which corresponds to the free fixed point.
- the linearized equations are of the form

$$\begin{aligned} z\mathcal{D}_z^{(0)}\mathfrak{b}_1 &= (d+2)\mathfrak{b}_1 \\ z\mathcal{D}_z^{(0)}\mathfrak{p}_1 &= (d-2)\mathfrak{p}_1 - z^{-3}\left(\mathcal{P}^{(0)}\cdot\mathfrak{b}_1\cdot\Delta_B - \Delta_B\cdot\mathfrak{b}_1\cdot\mathcal{P}^{(0)}\right) \end{aligned}$$

Localization and Fronsdal

- the linearized equations are of the form

$$z\mathcal{D}_z^{(0)}\mathfrak{b}_1 = (d+2)\mathfrak{b}_1$$
$$z\mathcal{D}_z^{(0)}\mathfrak{p}_1 = (d-2)\mathfrak{p}_1 - z^{-3}\left(\mathcal{P}^{(0)}\cdot\mathfrak{b}_1\cdot\Delta_B - \Delta_B\cdot\mathfrak{b}_1\cdot\mathcal{P}^{(0)}\right)$$

- this is unusual only in the sense that there is no $\mathfrak{p}_1^2$ term in the Hamiltonian (and thus no $\mathfrak{p}_1$ term in the first equation), even though the symplectic form is standard

$$\mathbf{\Omega}(z) = \int \frac{d^dx}{z^d}\, \delta\phi(z,x)\wedge\delta\pi(z,x)$$

- this indicates that canonical transformations in the bulk can be used to put the equations in a more familiar (harmonic oscillator) form
- when one makes use of that, one finds the Fronsdal equations precisely

Remarks

- What of standard gravitational holography?
- A standard piece of higher spin lore is expected to kick in here — when interactions in the field theory are included, the higher spin symmetry of the bulk breaks spontaneously (the operators get anomalous dimensions, corresponding to masses in the bulk).
- it is plausible that a gap might open up and only a few modes (such as graviton, ...) survive, but the detailed form of such interactions will determine where the theory flows

Interactions

- e.g., can introduce arbitrary (non-local) multi-trace interactions
  - can study all of these on an equal footing by Hubbard-Stratanovich
  - at $N = \infty$, the bulk equations are easily solved
    - gap equations of the field theory are the modified boundary conditions of the higher spin fields of the bulk theory
  - beyond leading order, the bulk theory is quantum; the H-S fields become dynamical in the bulk, and $1/N$ plays the role of $\hbar$, as expected
- an old conjecture: double trace deformations involving higher spin fields lead to new critical points in $d = 3$ [Leigh & **Petkou** '04]
  - we expect to be able to compute, for example, all of the anomalous dimensions of currents at these fixed points, bulk interactions, etc.

Open Questions

- (interacting) matrix theories?
- Gauge interactions (various)?
- Geometry of global symmetries?
- Emergence of just gravity?
- Entanglement? MERA?
- Other spacetime topologies?
- Other states (e.g., finite temperature), corresponding geometries?