

This research has been co-financed by the European Union (European Social Fund, ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF), under the grants schemes "Funding of proposals that have received a positive evaluation in the 3rd and 4th Call of ERC Grant Schemes" and the program "Thales"

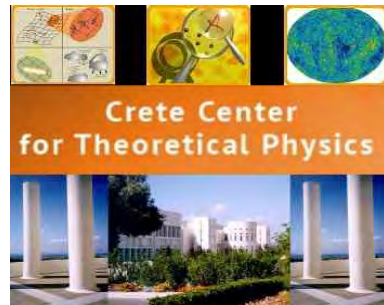


Thessaloniki Workshop

16 February 2015

Holographic conductivity: Insulators, Supersolids and power tails

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Bibliography

Ongoing work with

Jie Ren (Crete), Fransisco Peña-Benitez, (Crete)

and published work with

A. Donos (Durham), B. Gouteraux (Stanford) and E. Kiritsis (Crete) [arXiv:1406.6351](#) [hep-th]

B. S. Kim and C. Panagopoulos (Crete) [arXiv:1012.3464](#) [cond-mat.str-el]

C. Charmousis, B. Gouteraux (Orsay), B. S. Kim and R. Meyer (Crete)
[arXiv:1005.4690](#) [hep-th]

Holographic conductivity,

Elias Kiritsis

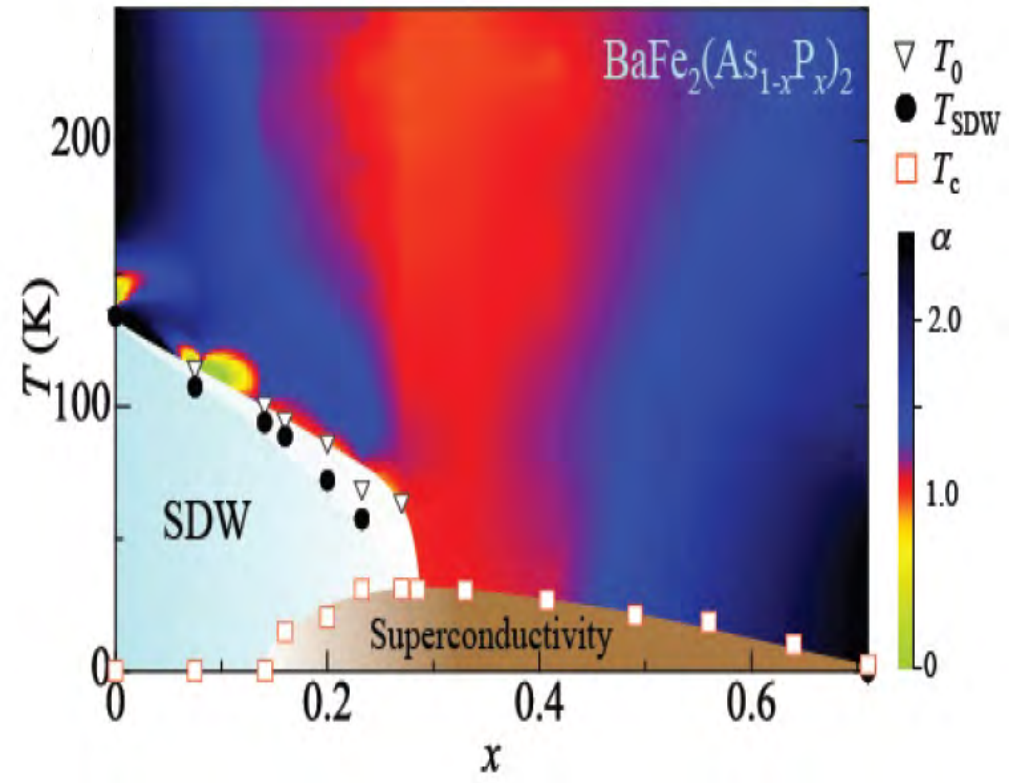
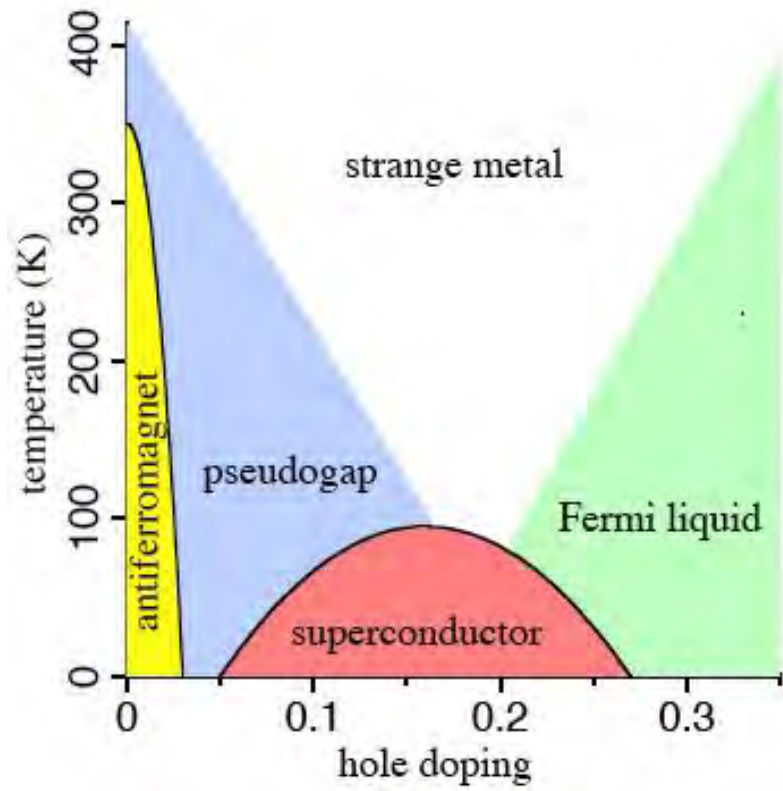
Introduction

- **Conductivity** is one of the most important observables in condensed matter systems
- It is relatively easy to define and measure.
- It tells us a lot about the dynamics of charge carriers in a medium
- For low enough voltages it is controlled by the retarded current two-point function.

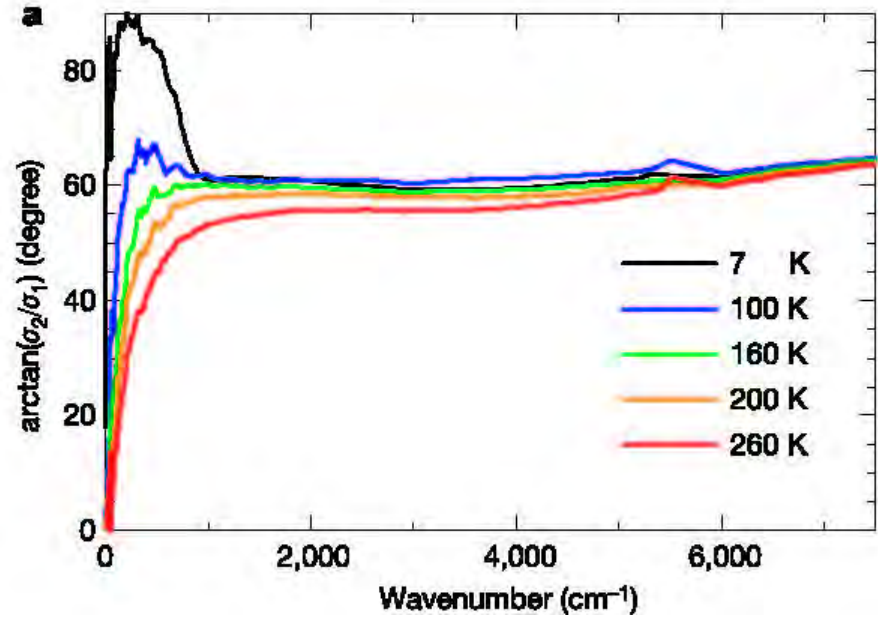
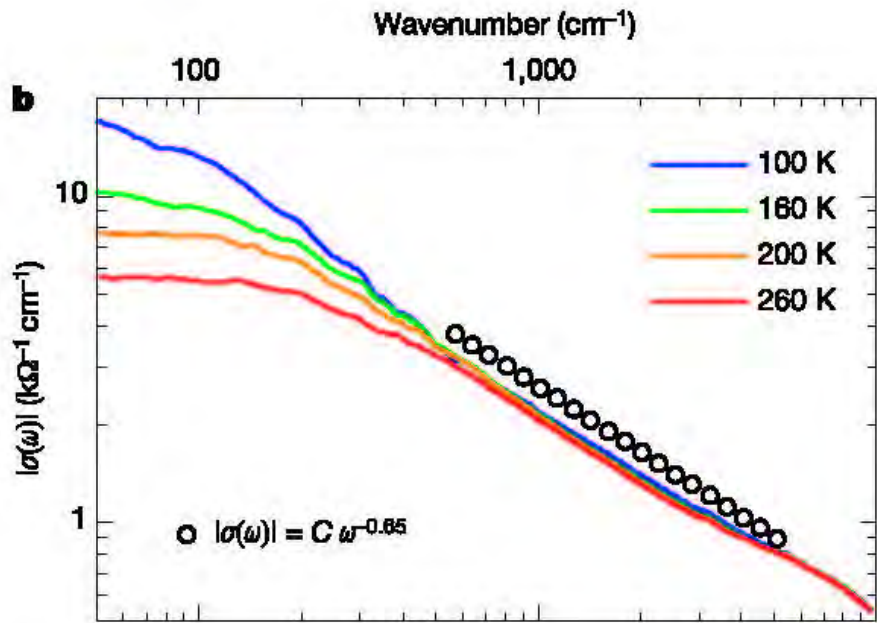
Strongly correlated electrons

- In metals, although bare electrons are strongly coupled, there are "dressed" (fermionic) **quasiparticles that are weakly-coupled** and behave as almost free electrons. They are responsible for transporting charge.
- In most materials that are on the border with magnetism (like the cuprates) there are **no weakly-coupled quasiparticles**.
- Such systems are **Mott insulators** in some part of their phase diagram.
- They have a benchmark **linear conductivity** in the area above their superconducting dome.
- In some of them the AC Conductivity shows a very simple scaling law,

$$\sigma(\omega) \sim \omega^{-\frac{2}{3}}$$



Scaling in AC conductivity



Van Der Marel et al.

Insulators

- There are several mechanisms for **insulating behavior** in condensed matter:

- 1) **Band gap insulators**, where the conduction band is empty, and there is therefore a gap that prevents current transport.

- 2) **Anderson localization** that is effective in two dimensions and where strong disorder inhibits conduction.

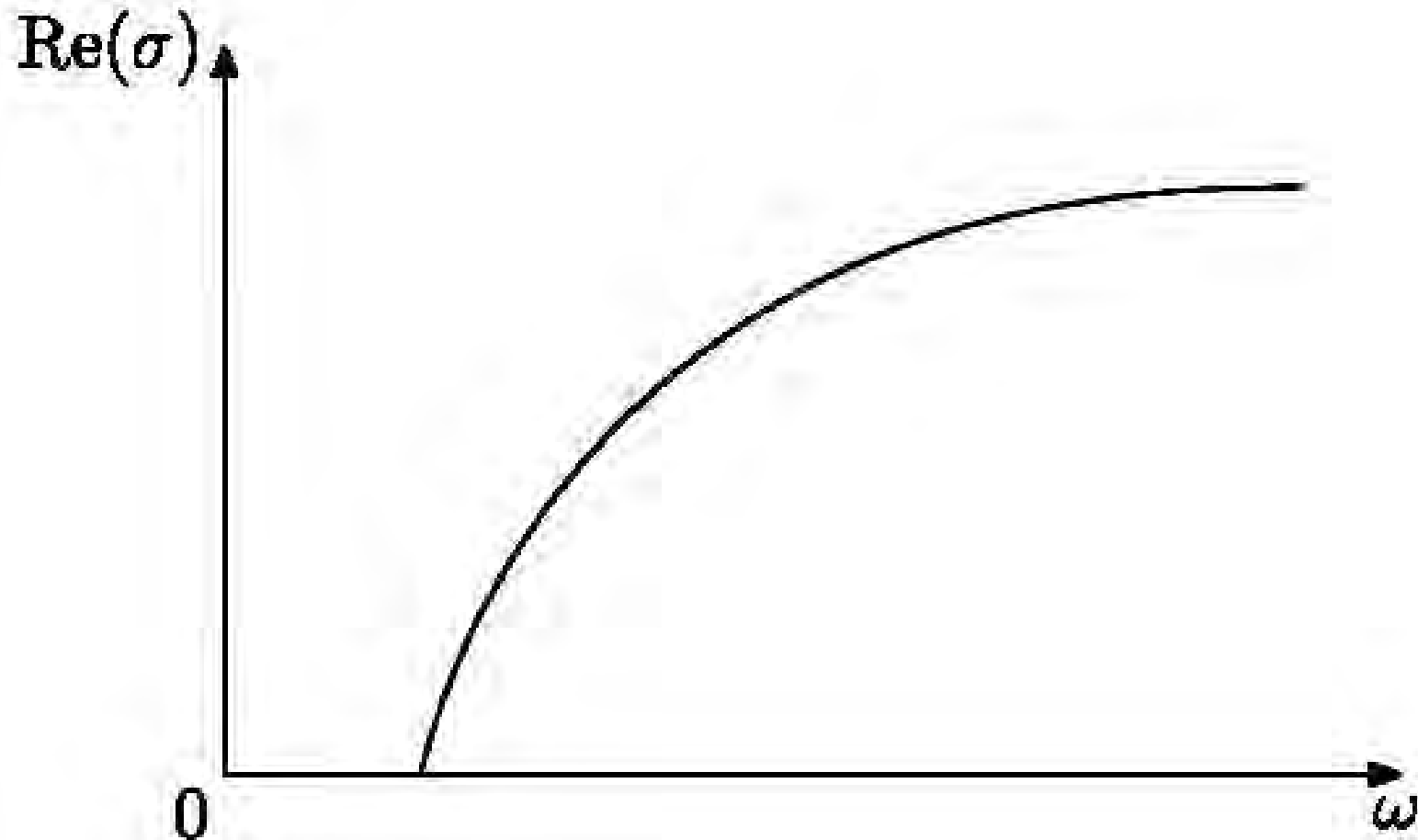
- 3) **Mott localization**, where strong onsite interactions localize electrons, [?]. This has been argued to be at work in insulating anti-ferromagnets, [?].

- 4) A new mechanism at strong coupling: **Momentum-dissipating interactions become relevant (strong)** in the IR, and they inhibit conduction

Donos+Hartnoll

- This mechanism was generalized to Einstein-Maxwell-Dilaton (EMD) theories with many saddle points found corresponding to **insulators, bad metals and conventional metals**.

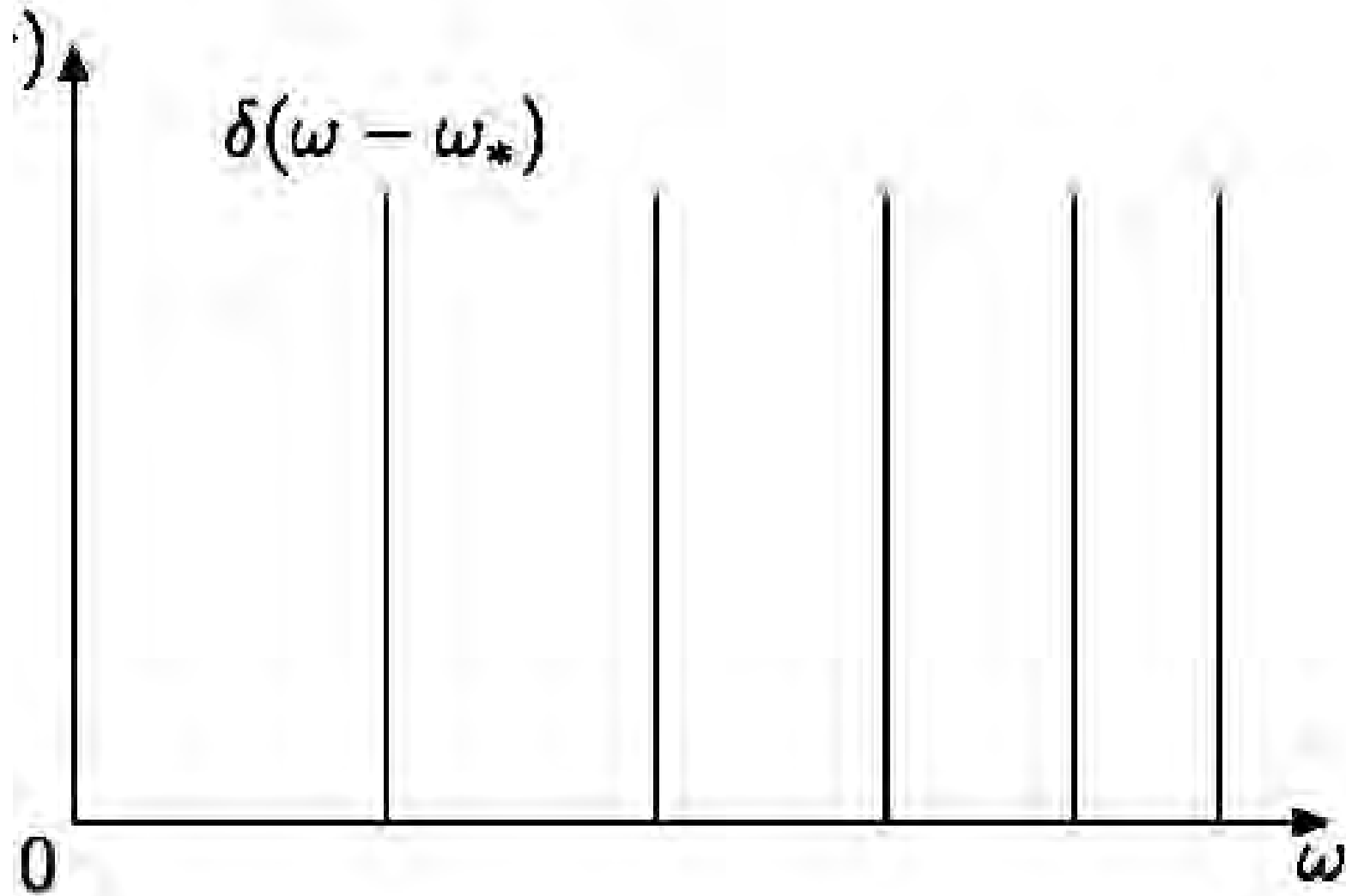
Donos+Gouteraux+Kiritsis



Supersolids

- A **supersolid** is a generalization of a superfluid state. It is characterized by a spontaneously broken $U(1)$ symmetry which guarantees a superfluid component (a zero frequency δ -function).
- It has (spontaneously) **broken translational invariance**.
- Therefore the appropriate two-point function has a discretely localized spectral density.
- If probed at a generic non-zero frequency it is non-responsive. If it is probed at zero frequency it behaves as a superfluid.
- They have been theoretically anticipated and studied, especially in the last 2-3 years.
Legget, Fisher+Nelson, Anderson, Nicolis+Penco+Rosen
- There are **proposed realizations with cold atoms**.
Keilmann+Cirac+Roskilde
- **There have been claims for presence in solid Helium⁴ as well as a recent refutation.**

Kim+Chan



The Plan of the rest

- Conductivity in Holographic Theories
- Gapped ground states in EMD theories
- Adding momentum dissipation.
- Scaling of the AC conductivity

The Wilsonian setup

- Holographic theories are generically RG flows between fixed points.
- The first step: **Classification of Scale Invariant/Fixed-point theories (The Wilsonian approach in holography)**.
- The strategy is to use **Effective Holographic Theories** (in order to explore **all possible QC holographic scale invariant theories** with given symmetries.
Charmousis+Gouteraux+Kim+Kiritsis+Meyer (2010)
- Once fixed points are classified for a given bulk action, the conductivity can be studied first “locally” (in the IR fixed point) and then globally (along the whole flow).

Generic Scaling Geometries: A classification of all QC points in holography

*Charmousis+Gouteraux+Kim+Kiritsis+Meyer (2010), Gouteraux+Kiritsis (2011),
Huisje+Sachdev+Swingle, (2011)*

- Assume translation and rotational invariance in space and time.
- The QC points generically **break hyperscaling invariance** and are characterized by several exponents. Two appear in the metric (z, θ) .
- $z \rightarrow$ dynamical exponent
- $\theta \rightarrow$ hyperscaling violation exponent

$$ds^2 = r^{\frac{2\theta}{d}} \left[\frac{dt^2}{r^{2z}} + \frac{dr^2 + dx_1^2 + dx_2^2 + \dots + dx_d^2}{r^2} \right]$$

- There is invariance under:

$$x_i \rightarrow \lambda x_i \quad , \quad t \rightarrow \lambda^z t \quad , \quad r \rightarrow \lambda r \quad , \quad ds \rightarrow \lambda^{\frac{\theta}{d}} ds$$

- The entropy scales as

$$S \sim T^{\frac{d-\theta}{z}}$$

which gives an interpretation to the hyperscaling violation exponent.

- There is a third exponent, associated with the charge density, the **conduction exponent** ζ :

Gouteraux+Kiritsis, Gouteraux

$$A_t = Q r^{\zeta-z}$$

Conductivity:basics

- It can be calculated in the linear regime from the correlators of the currents

$$\sigma_{ij}(\omega, \vec{k}) = \frac{1}{i\omega} \int d^p x dt e^{-i\omega t - i\vec{k}\cdot\vec{x}} \langle J_i(t, \vec{x}) J_j(0, 0) \rangle$$

or more generally as the (non-linear) response to an external electric field

$$J_i = \sigma_{ij} E_j$$

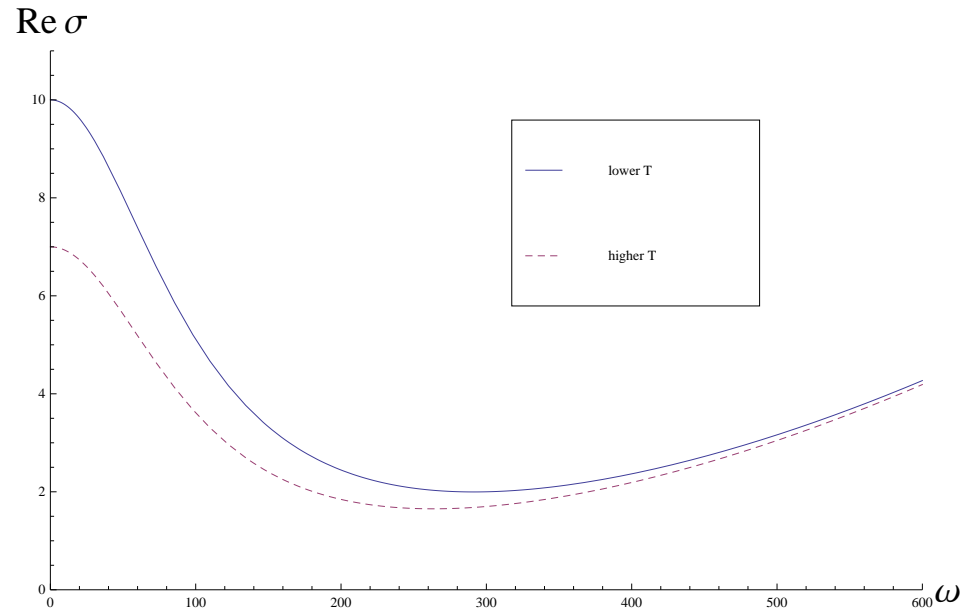
- **Translational invariance** and **finite density** imply a pole at zero frequency for the conductivity.

$$\sigma(\omega) \simeq K \left(\delta(\omega) + \frac{i}{\omega} \right) + \dots, \quad K = \frac{4\rho^2}{\varepsilon + p}$$

- K is the **Drude weight**.
- Weak scattering over ions or impurities, resolves the zero frequency pole

$$\sigma(\omega) \simeq \frac{K\tau}{1 - i\omega\tau} + \dots, \quad \frac{1}{\tau} = \text{scattering rate}$$

- This is the so-called **Drude peak** and defines the response of a **metal** at low frequencies.



- As $T \rightarrow 0$, $\tau \rightarrow \infty$. In this limit $\tau \rightarrow \infty$ we obtain back the zero frequency pole and δ -function.
- **Breaking translation invariance in holography renders computations harder.**
- When the U(1) symmetry is **spontaneously broken** there is a δ -function at zero frequency marking the **superfluid**. The Drude weight is then the **superfluid density**.

General features of holographic conductivities

- When the bulk action of the U(1) gauge field is linear in F^2

$$S_F \sim \int d^4x \sqrt{g} Z(\phi) F^2$$

the DC conductivity has the schematic form

$$\sigma_{DC} \sim \sigma_{pair} + \sigma_{drag}$$

Blake+Tong, Donos+Gauntlett

- The first term $\sigma_{DC} \sim Z$ was interpreted as a term coming from the pair-production of charge.

Karch+O'Bannon

- It is non-zero even when $Q = 0$.
- It does not contribute to thermal transport.

Donos+Gauntlett

- It is **finite**, even when there is no momentum dissipation (**does not contribute to the Drude weight**).

- $\sigma_{drag} \sim \tau$ is due to momentum-dissipating interactions.

- If more than one mechanisms of momentum dissipation are at work,

$$\sigma_{drag} = \sum_I \sigma_{drag}^I \quad , \quad \tau = \sum_I \tau_I$$

(inverse Mathiessen law)

- In the limit of vanishing dissipation, $\tau \rightarrow \infty$, it generates the Drude δ -function.

- When charge dynamics is described by the DBI action then

$$\sigma_{DC} = \sqrt{\sigma_{pp}^2 + \sigma_{drag}^2}$$

Holographic gapped systems at finite density

- There are two ways that a system can be insulating at $T=0$:
 - ♠ **Charged excitations are gapped.** In that case the conductivity is non-zero only above the gap.
 - ♠ There is **no gap** but the limit $\omega \rightarrow 0$ gives a **vanishing conductivity**. In both cases the operator/mechanism that breaks translation invariance, **is relevant in the IR.**
- **Gapped holographic systems** are known at zero charge density, and they are in use as models of YM.
Witten, '98, Gursoy+Kiritsis+Nitti, '07, Nishioka+Ryu+Takatanagi, '10
- At **finite density**, many saddle-points **with discrete spectrum** were found, in the classification of QC points in EMD Theories.
Charmousis+Gouteraux+Kiritsis+Kim+Meyer, '10, McGreevy+Balasubramanian, '10

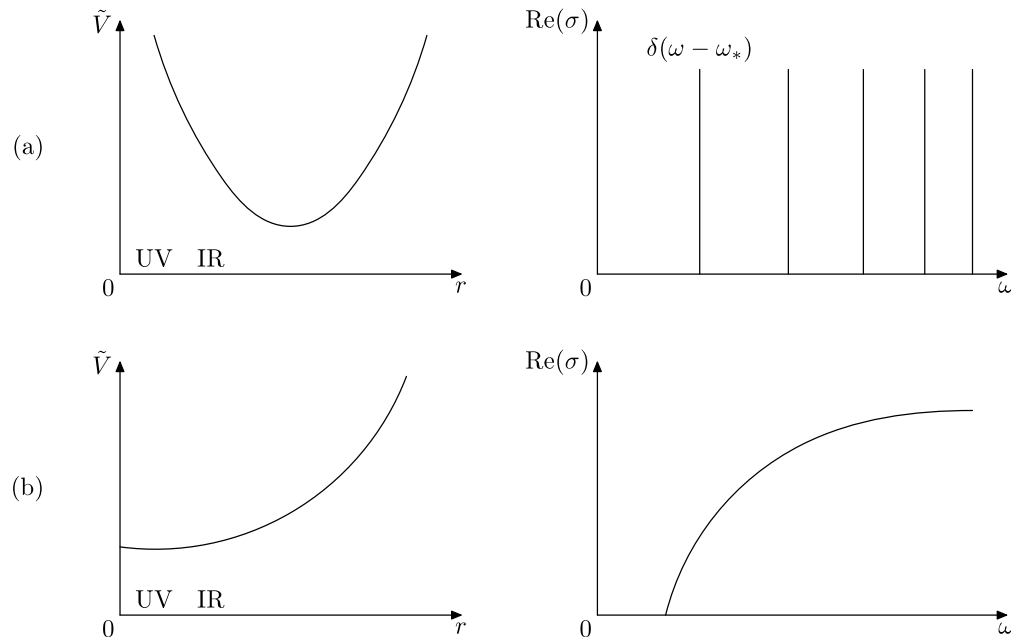
$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4} F^2 \right] , \quad V \sim e^{-\delta\phi} , \quad Z \sim e^{\gamma\phi}$$

- The conductivity is obtained by solving for the fluctuations of the gauge field, $\delta A_i = a_i(r) e^{i\omega t}$.

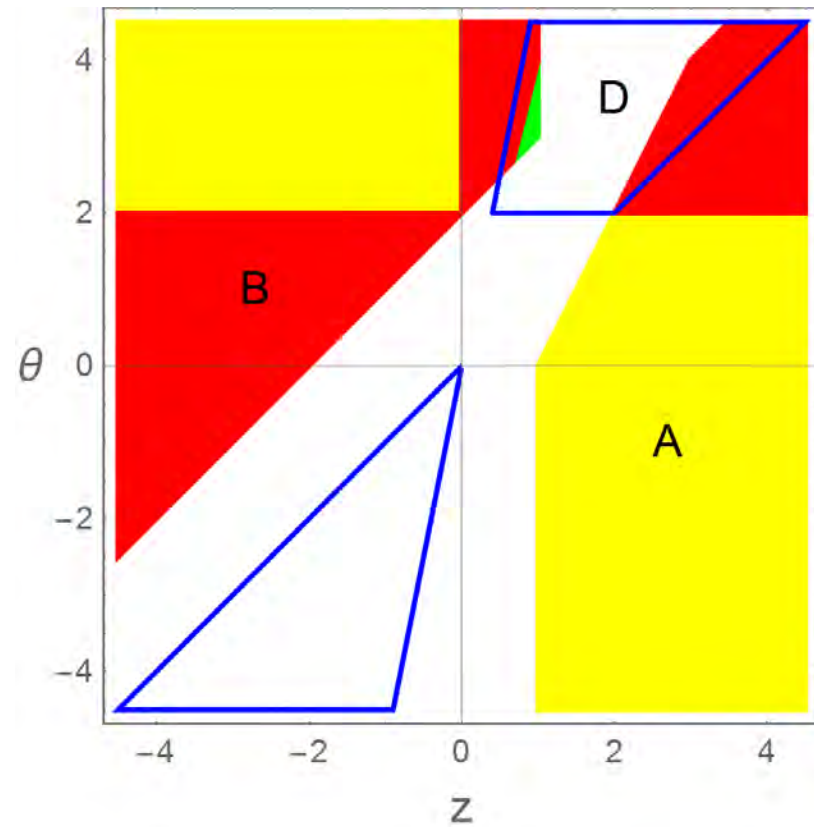
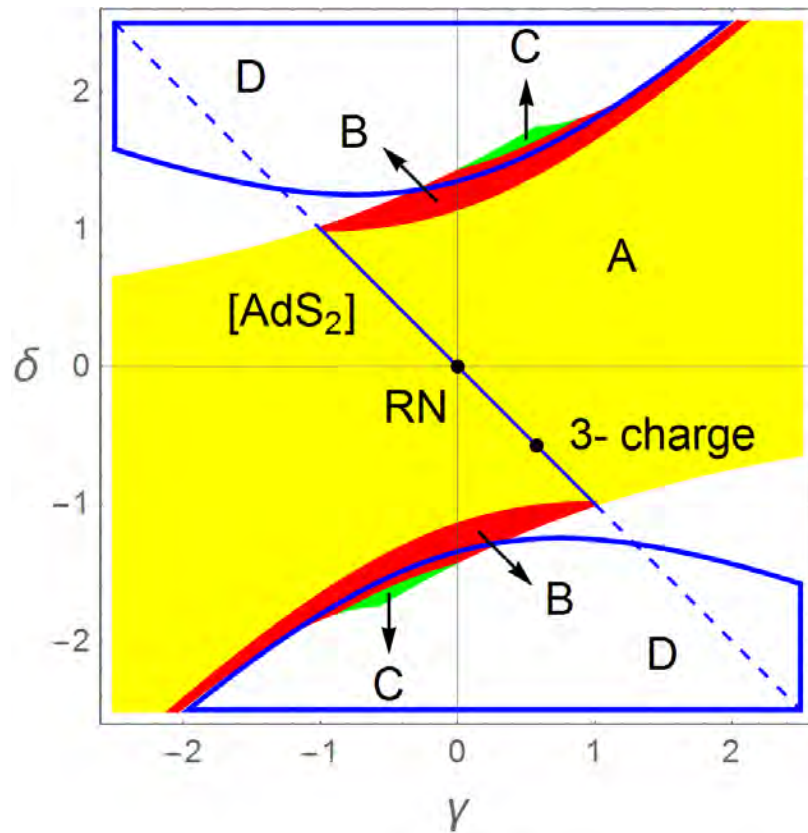
$$\frac{1}{Z} \sqrt{\frac{g_{rr}}{g_{tt}}} \partial_r \left(Z \sqrt{\frac{g_{tt}}{g_{rr}}} a_i' \right) + \left[\frac{g_{rr}}{g_{tt}} \omega^2 - \frac{Q^2 g_{rr}}{Z g_{xx}^2} \right] a_i = 0$$

- By a field and coordinate redefinitions it can be mapped into a Schrödinger problem

$$-\psi'' + V_{eff} \psi = \omega^2 \psi$$



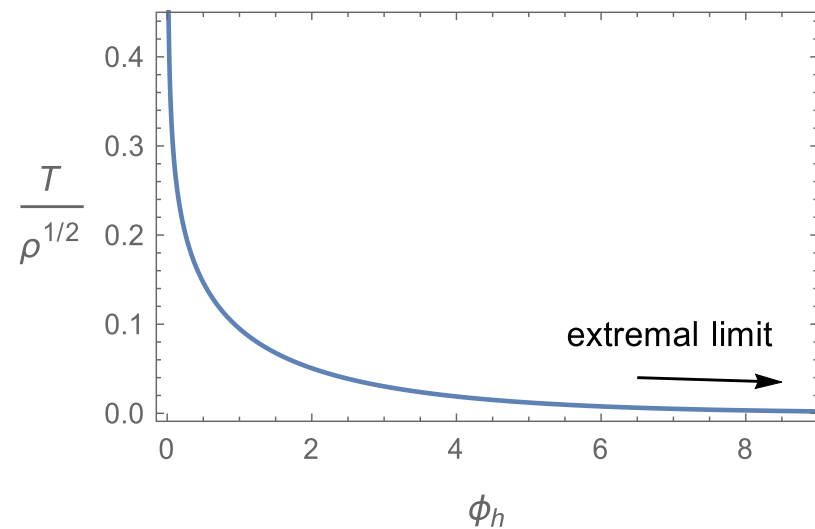
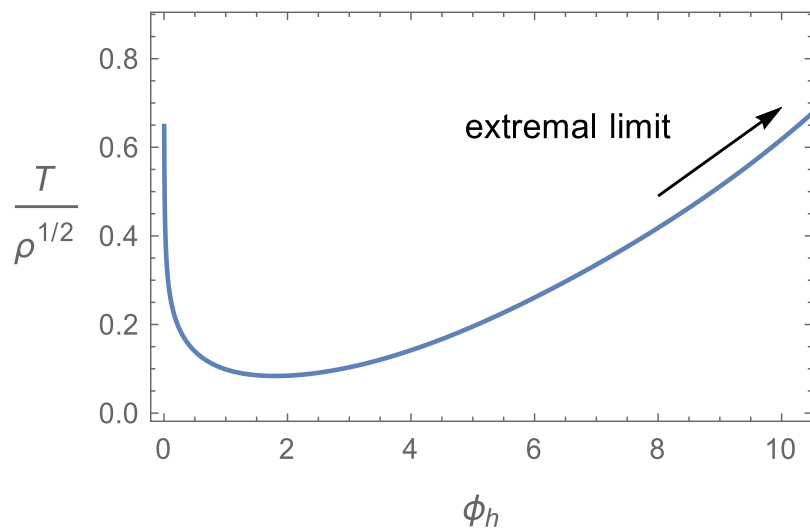
The parameter space



The regions A, B, and C are the parameter space constrained by the Gubser criterion. In region A (yellow), the extremal limit is at $T \rightarrow 0$, and the current-current correlator is gapless. In region B (red), the extremal limit is at $T \rightarrow \infty$, and the current-current correlator is gapped. In region C (green), the extremal limit is at $T \rightarrow \infty$, and the current-current correlator is gapless. Region D (enclosed by blue boundaries) is holographically unreliable.

The finite temperature picture

- The gapped systems above are at finite density, and have therefore a zero-frequency δ -function: **They are perfect conductors.**
- **Note also that we are at $T = 0$.** In this respect as far as the δ -function is concerned they resemble real metals at $T = 0$.
- Up to $T = T_{min}$ this is **the only saddle-point for the system.** For $T > T_{min}$ there are also two black holes that are competing at the same temperature.



- At $T = T_c > T_{min}$ there is a **first order phase transition to the large black hole phase** that is a gapless plasma phase. This is very similar to the **confinement-deconfinement phase transition** in gauge theories.

Adding Momentum dissipation

- We use axions as a source of momentum dissipation.

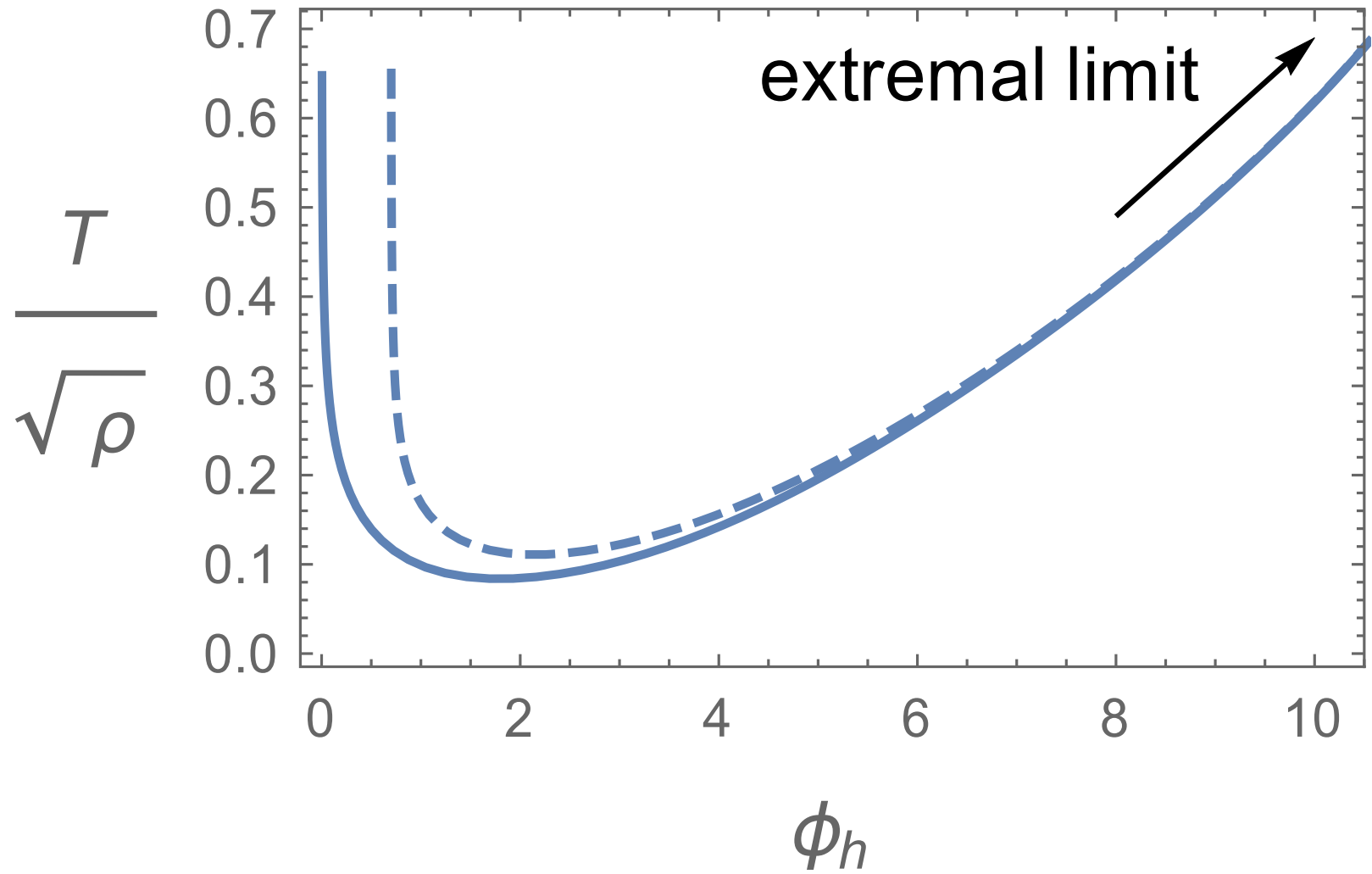
$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{Y(\phi)}{2} \sum_{i=1}^2 (\partial\psi_i)^2 \right],$$

$$V(\phi) \sim e^{-\delta\phi}, \quad Z(\phi) \sim e^{\gamma\phi}, \quad Y(\phi) \sim e^{\lambda\phi}.$$

$$\psi_1 = kx \quad , \quad \psi_2 = ky$$

- We also choose λ in the region where the axions are “irrelevant” in the IR.

$$\gamma=0.4, \delta=-1.2$$



- Momentum dissipation is expected to remove the zero frequency δ -function but:

(a) We are in the $T=0$ geometry as long as $T < T_c$

(b) The **axions are irrelevant in the deep IR.**

- A detailed analysis of the Drude weight is needed.

- The fluctuation equations for the conductivity involve **not only δA_i but also the axions.**

- There is a direct formula for the DC conductivity in the presence of a regular (non-extremal) horizon.

$$\sigma_{\text{DC}} = Z_h + \frac{q^2}{k^2 (g_{xx})_h Y_h} \equiv Z_1 + \frac{q^2}{k^2} Z_2$$

Gouteraux, Donos+Gauntlett

- There is also a direct formula for the **Drude weight**:

$$\Pi(\omega) = fH\lambda'_1 - \frac{q}{k^2}fZ_2 \left(\frac{Z_1}{Z_2}\right)' \lambda_2, \quad , \quad \partial_r \Pi = \mathcal{O}(\omega^2)$$

$$\lambda_1 = \frac{Z_1}{H} \left(a_x - \frac{q}{k^2} \frac{Z_2}{Z_1} b_x \right), \quad \lambda_2 = \frac{Z_2}{H} (q a_x + b_x).$$

- It can be shown that $\Pi(0)$ is the Drude weight. We can then establish that:

(a) when $\sigma_{DC} \rightarrow \infty$ at extremality then there is a zero-frequency δ -function, and this happens in the gapless geometries.

(b) when $\sigma_{DC} \rightarrow 0$ at extremality, then there is no zero-frequency δ -functions and this happens for the gapped geometries.

- We have therefore found holographic system with **a charged spectrum that is gapped and discrete.**
- These are insulators that share properties of both **band-gap insulators and Mott insulators.**

The AC conductivity in a holographic strange metal

Kim+Kiritsis+Panagopoulos

- The bulk is the AdS Schwarzschild black hole.
- The charge is described by a standard DBI action coupled to the metric

$$S_{DBI} = \mathcal{N} \int d^5x \sqrt{\det(g + F)}$$

- The ansatz for the ground state is

$$A = (Ey + h_+(u))dx^+ + (b^2Ey + h_-(u))dx^- + (b^2Ex^- + h_y(u))dy,$$

- It is a stationary solution in lightcone coordinates with a nontrivial **charge density** and a **light-cone electric field** $F_{+y} = E$
- The DBI equations can be solved exactly and the conductivity computed à la Karch-O'Bannon.

The DC conductivity

- The parameters are E, J^+, T . E is similar to the **doping parameter** of cuprates.
- They can be combined in one **scaling variable** t and parameter J .

$$t = \frac{\pi \ell T}{2\sqrt{E}} \quad , \quad J^2 = \frac{J_+^2}{(2N)^2 \sqrt{2} (E)^3} ,$$

and the DC conductivity becomes

$$\sigma_{DC} = \sigma_0 \sqrt{\sigma_{DR}^2 + \sigma_{QC}^2} ,$$

$$\sigma_{DR}^2 = \frac{J^2}{t^2 A(t)} \quad , \quad A(t) = t^2 + \sqrt{1 + t^4} \quad , \quad \sigma_{QC}^2 = \frac{t^3}{\sqrt{A(t)}} .$$

- We will also define the ratio

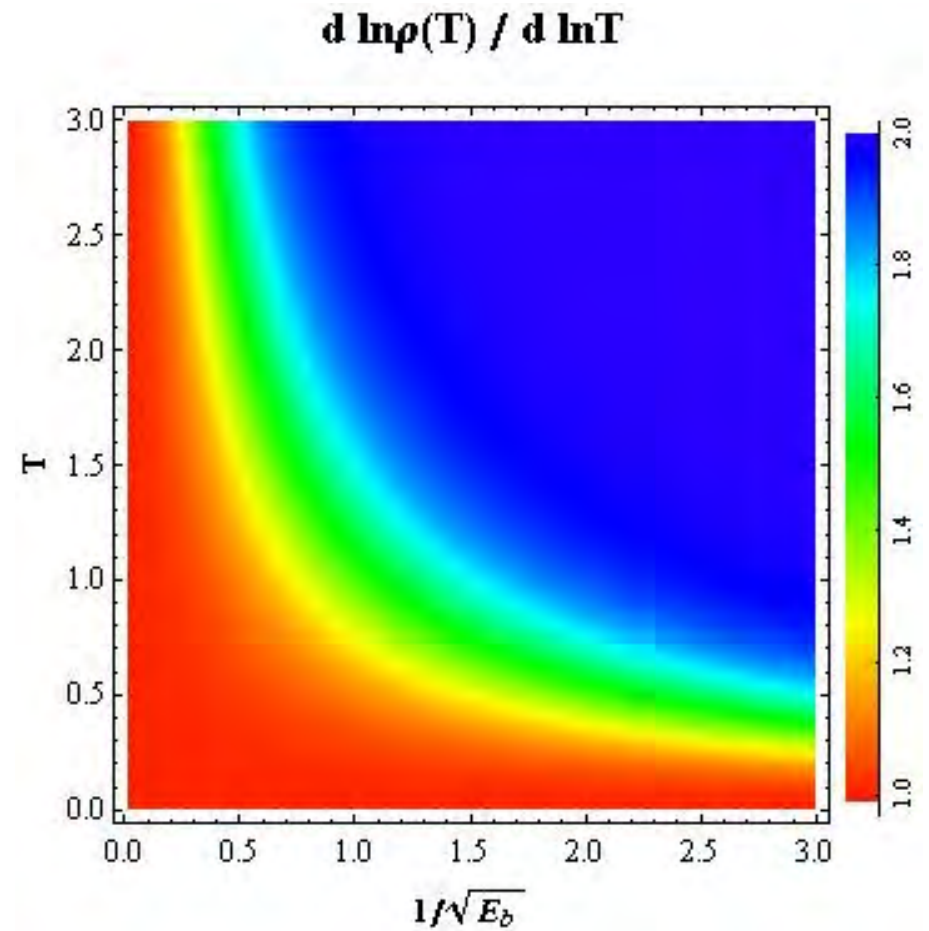
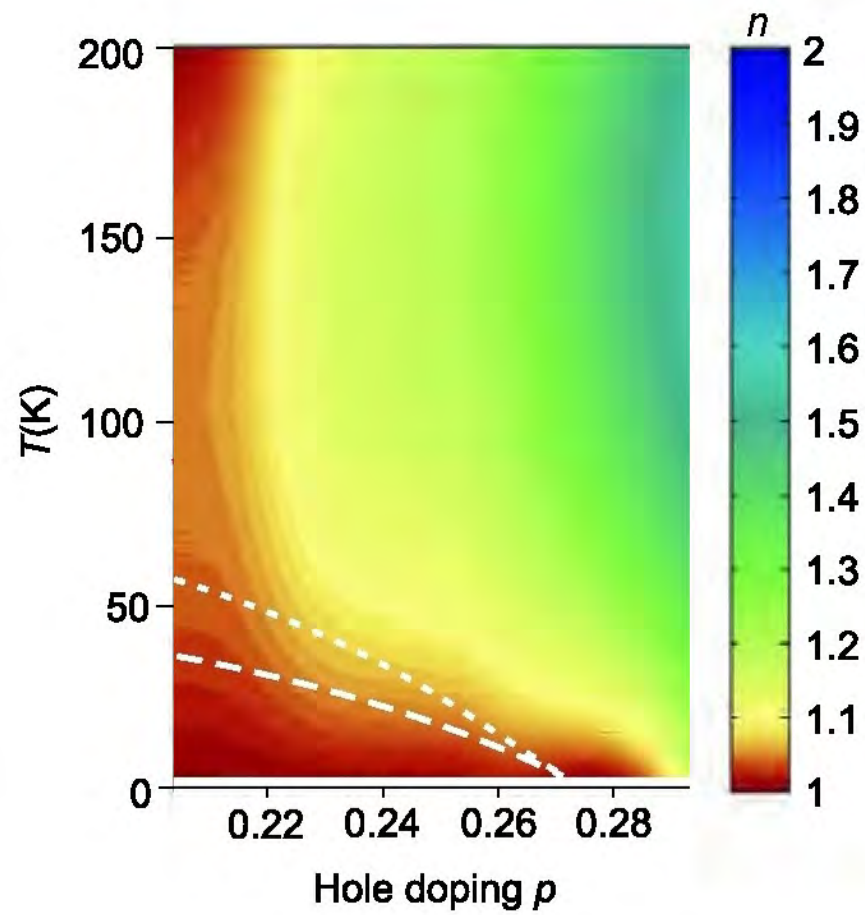
$$q \equiv \frac{\sigma_{DR}^2}{\sigma_{QC}^2}$$

that determines in which regime we are.

- The conductivity is as follows:

$$\rho = \left\{ \begin{array}{ll}
 (\sigma_0 J)^{-1} t & q \gg 1, t \ll 1 \\
 \sqrt{2} \sigma_0^{-1} J t^2 & q \gg 1, t \gg 1 \\
 \sigma_0^{-1} t^{-3/2} & q \ll 1, t \ll 1 \\
 \sqrt{2} \sigma_0^{-1} t^{-1/2} & q \ll 1, t \gg 1
 \end{array} \right. \begin{array}{l}
 \text{linear} \\
 \text{quadratic} \\
 \text{regime I} \\
 \text{regime II}
 \end{array} \left| \begin{array}{l}
 \text{Drude regime (DR)} \\
 \text{Quantum critical (QC)}
 \end{array} \right.$$

(1)



Kim+Kiritsis+Panagopoulos

The AC conductivity

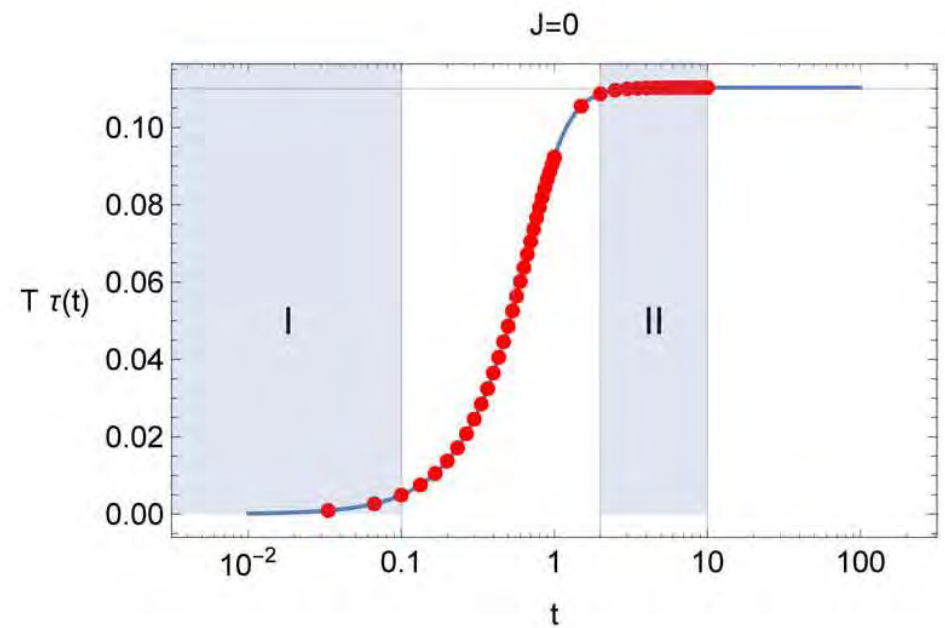
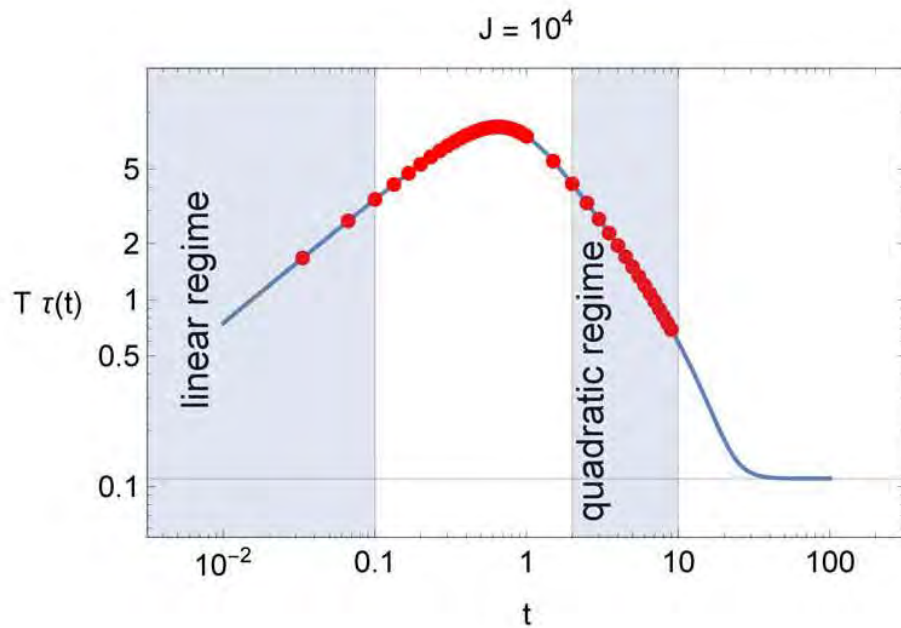
- From the AC conductivity equations one can compute the asymptotics
- The large ω behavior

$$(b^2 \ell \mathcal{N} T)^{-1} \sigma(\omega) = i \left(\frac{2\pi}{3} \right)^{7/3} \frac{2}{\Gamma(1/3) \Gamma(7/3)} \left(\frac{\omega}{T} \right)^{-1/3} e^{i\pi/6}$$

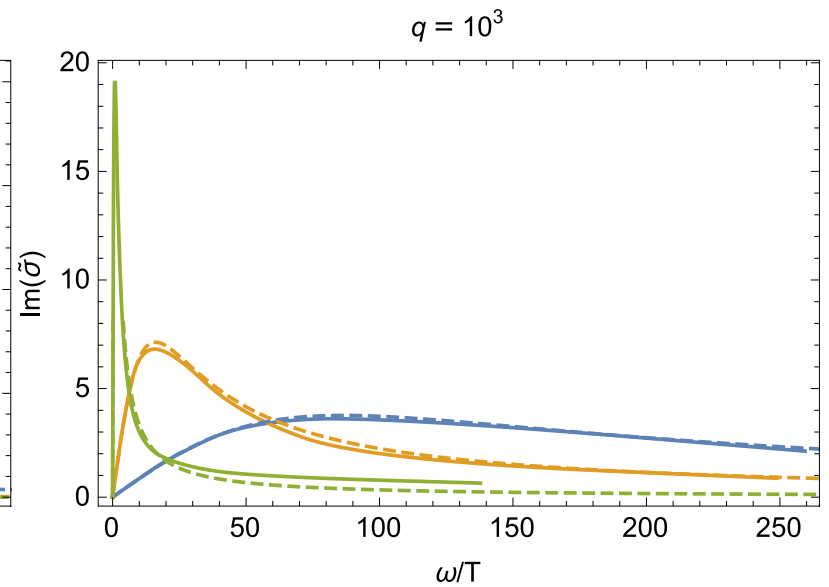
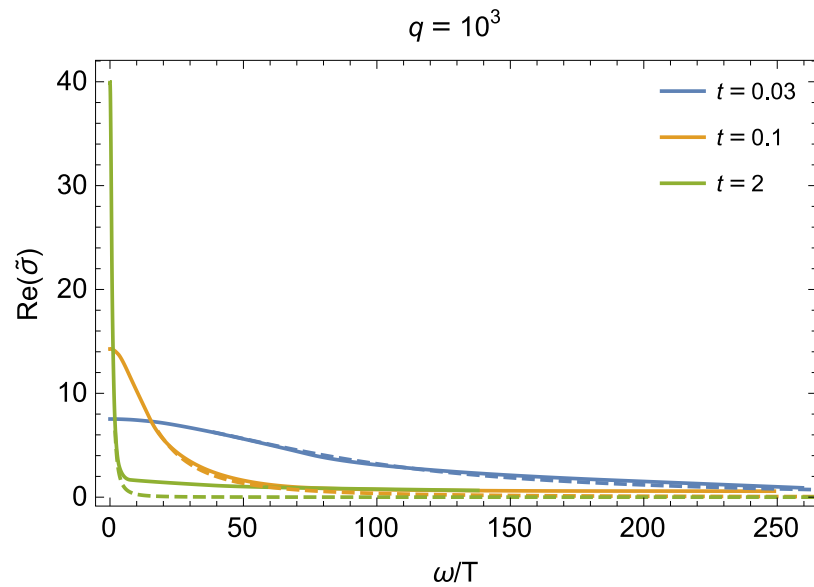
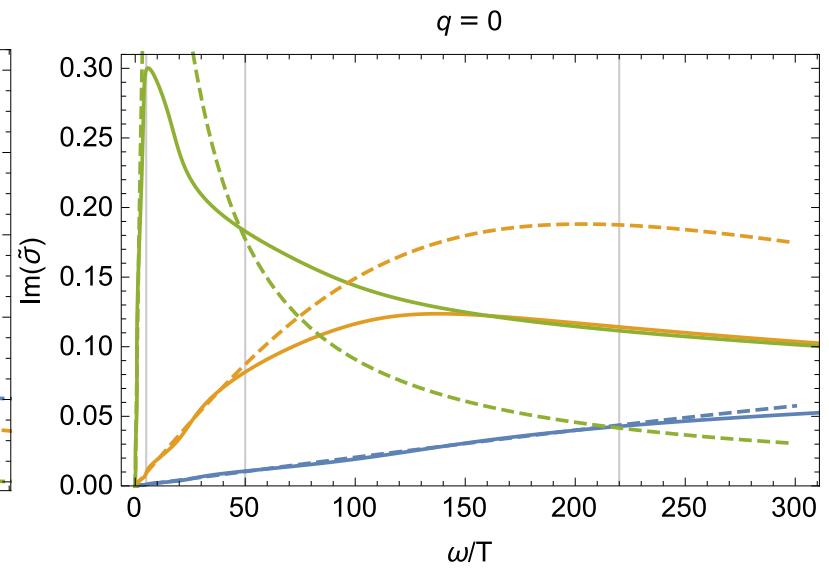
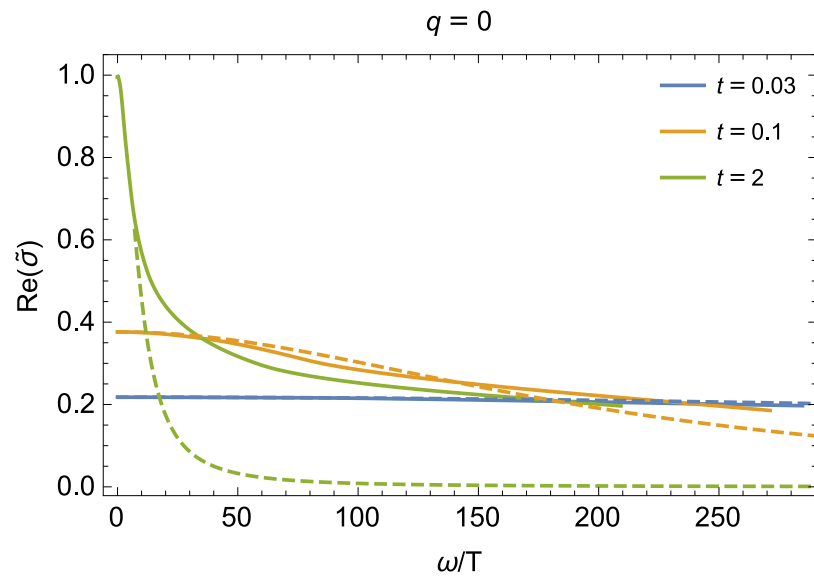
- The generalized relaxation time defined as

$$\sigma(\omega) \approx \sigma_{DC} \left(1 + i\tau\omega + \mathcal{O}(\omega^2) \right),$$

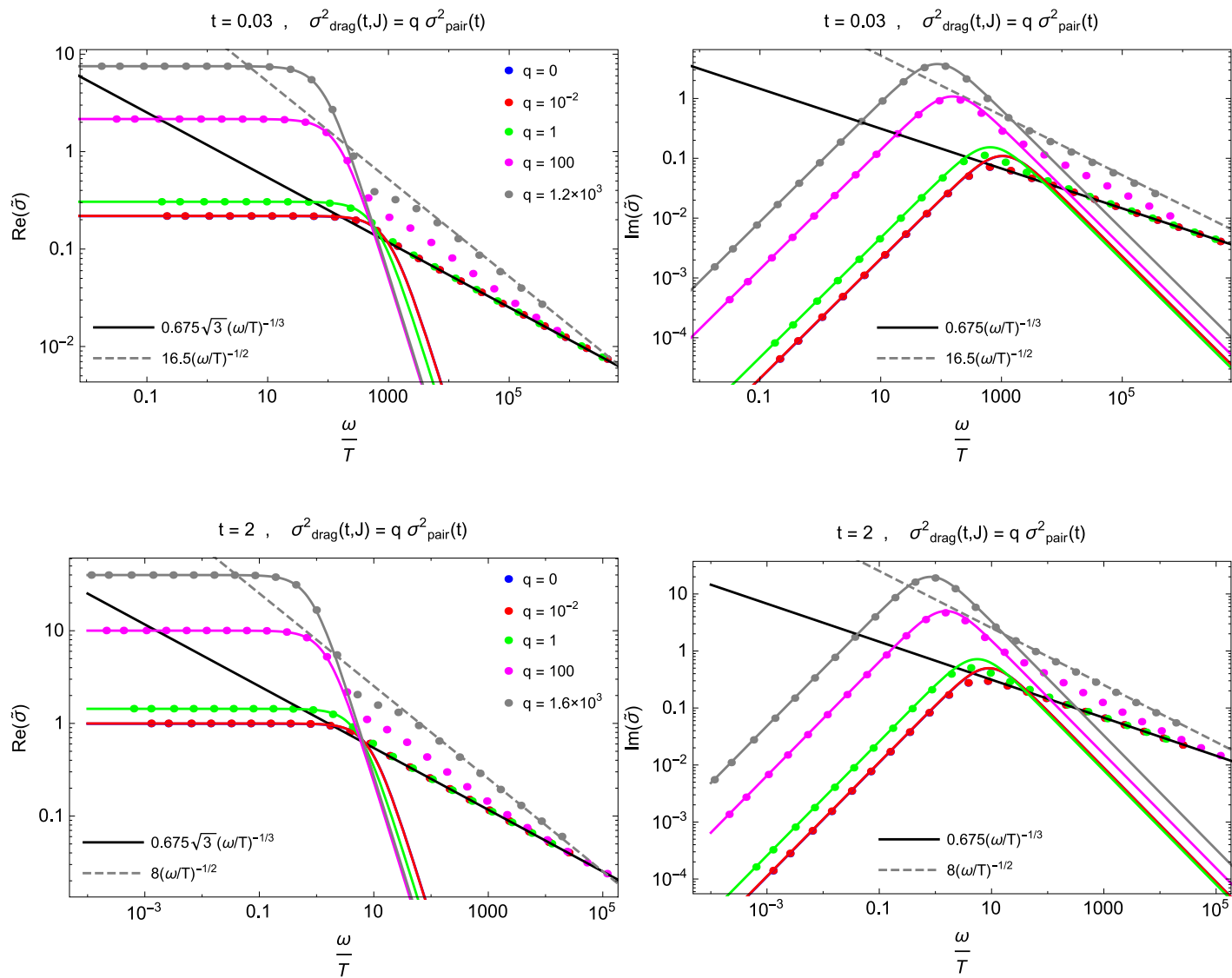
can be computed analytically but has a complicated formula.



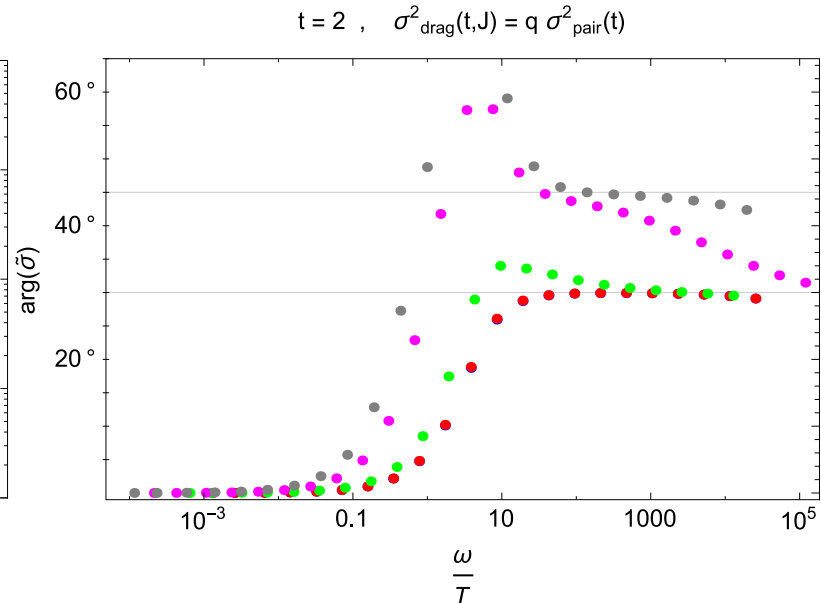
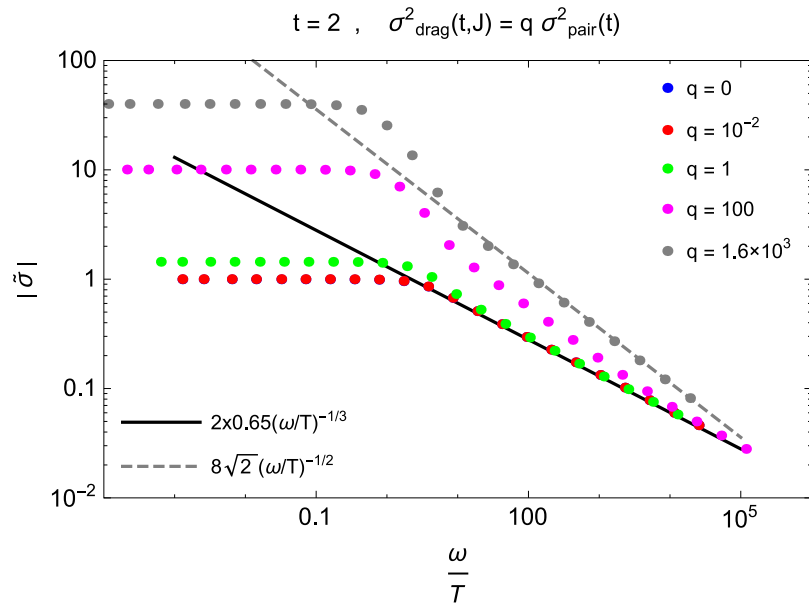
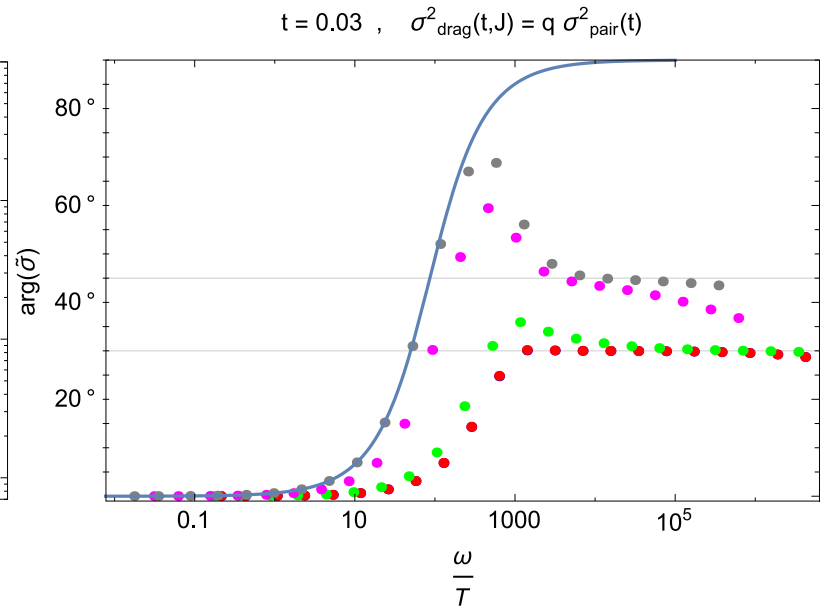
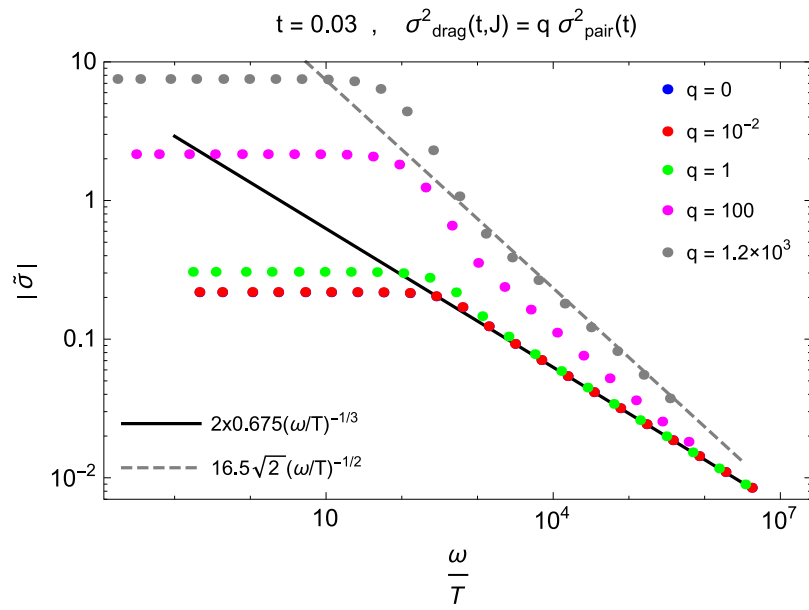
The generalized relaxation time τ as a function of the scaling temperature variable t . The left plot shows the behaviour of the dimensionless quantity $T\tau$ (T is the temperature) deep in the Drude regime (DR). The right plot shows the behavior in the Quantum Critical regime (QCR) at zero density. Dots show numerical data and the continuous line is the analytic formula obtained in the text using perturbation theory .



Conductivity for three different temperatures in the QC (up) and in the DR (down) regimes. The left figures show the real part of the conductivity while the right figures the imaginary part of the conductivity. Continues lines represent the numerical data, dashed lines correspond to a fit to the Drude peak formula.



The colored continuous lines show the Drude fitting using the analytic computation of the generalized relaxation time. Straight black and dashed gray lines show the UV and the intermediate power law behavior of the AC conductivity.



Absolute value (left) and argument (right) part of the conductivity. The blue continuous line shows the Drude fit using the analytic computation of the generalized relaxation time. Straight black and dashed gray lines show the UV and the intermediate power law behavior of the AC conductivity.

Lessons learned

- When the conductivity is dominated by the **drag mechanism (momentum dissipation)** there is a **clear Drude peak**.
- When the conductivity is dominated by the **pair-production mechanism** there is **no Drude peak**.
- There is an associated **scaling tail in the conductivity** (here $\sigma \sim \omega^{-\frac{1}{3}}$). This **always survives beyond the Drude peak** as it falls off slower than ω^{-1} .
- It has the features seen in experiment by Van der Marel et al.:
 - a) **Constant phase matching the power falloff.**
 - b) **Temperature independence.**
- These properties seem to hold more generally and beyond the system under study.

Outlook

- Holography provides a rich **landscape of possibilities for conductivity**.
- It realizes novel types of **insulators** and **supersolids**.
- More work is needed to formulate **the general theory of conductivity** in holography. We have just scratched the surface.

THANK YOU

Overview of the solutions

- The gravitational action:

$$S = M^3 \int d^5x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z_1(\phi)}{4} F_1^2 - \frac{Z_2(\phi)}{4} F_2^2 \right]$$

$$\text{with } F_{\mu\nu}^i \equiv \partial_\mu A_\nu^i - \partial_\nu A_\mu^i$$

and

$$V(\phi) \rightarrow_{IR} \sim V_0 e^{-\delta\phi}, \quad Z_1(\phi) \underset{IR}{\sim} Z_{10} e^{\gamma_1\phi}, \quad Z_2(\phi) \underset{IR}{\sim} Z_{20} e^{\gamma_2\phi}.$$

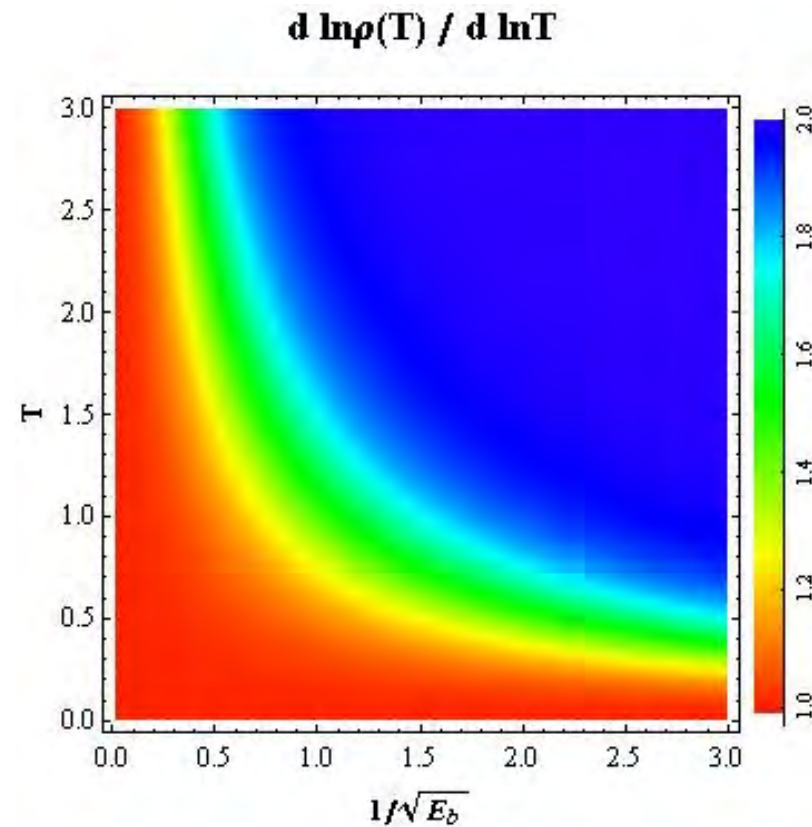
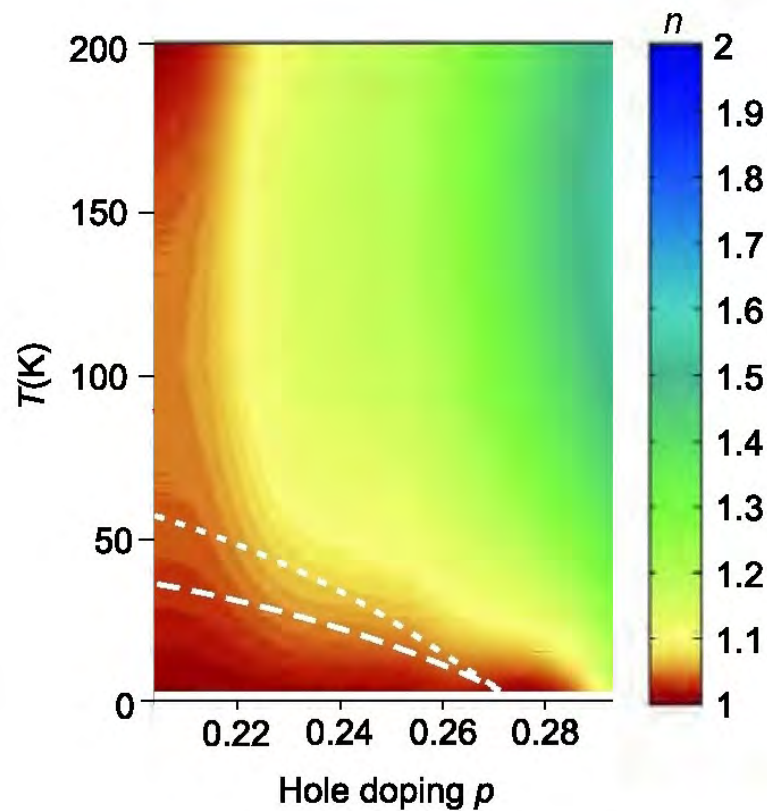
- The metric ansatz has helical symmetry and we parametrize the scaling solutions as :

$$ds^2 = r^{\frac{2\theta}{3}} \left[-\frac{dt^2}{z^{2z}} + \frac{L^2 dr^2 + \omega_1^2}{r^2} + \frac{1}{r^2 z^2} \left(\omega_2^2 + \frac{\lambda}{r^2} \omega_3^2 \right) \right], \quad A_1 \sim r^{\zeta-z}$$

where

$$\omega_1 = dx_1, \quad \omega_2 = \cos(kx_1)dx_2 + \sin(kx_1)dx_3, \quad \omega_3 = \sin(kx_1)dx_2 - \cos(kx_1)dx_3$$

Critical lines vs critical points in Cuprates



Kim+Kiritsis+Panagopoulos

Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$.

R.A. Copper et. al. 2009

Holographic conductivity,

Elias Kiritsis

Breaking Translation Invariance

- **Charge lattices** lead to non-linear (gravitational) PDEs that are in general very difficult to solve
Horowitz+Santos+Tong (2012), Lin+Liu+Wu+Xiang (2013), Donos+Kiritsis (2013)
- Some other string mechanisms of translation invariance breaking (momentum dissipation) are simpler to analyze:
 - ♠ An effective “phenomenological” treatment of momentum dissipation using massive gravity
Vegh (2013), Davison (2013), Blake+Tong (2013), Davison+Schalm+Zannen (2013)
 - ♠ Interactions of charge with a **bulk sector** that carries most of the energy (DBI probe approximation)
Karch+O’Bannon (2007), Charmousis+Gouteraux+Kiritsis+Kim+Meyer (2010)
 - ♠ Interactions with **string axions** that model a kind of **homogeneous disorder**.
Andrade+Withers (2013), Gouteraux (2013), Donos+Gauntlett (2013)
 - ♠ The use of **random field disorder** in holography.
Hartnoll+Herzog (2008), Davison+Schalm+Zaenen (2013), Lucas+Sachdev+Schalm (2014)
 - ♠ Study of **saddle points with helical symmetry**.
Kachru et al. (2012), Donos+Gauntlett (2012), Donos+Hartnoll (2012)

Holographic conductivity,

Elias Kiritsis

DC conductivity

- The general structure of the holographic DC conductivity at strong coupling:

$$\sigma_{\text{DC}} = \sqrt{(\sigma_{\text{DC}}^{pp})^2 + (\sigma_{\text{DC}}^{\text{drag}})^2} \quad , \quad \sigma_{\text{DC}} = \sigma_{\text{DC}}^{pp} + \sigma_{\text{DC}}^{\text{drag}}$$

- σ^{pp} is the **pair-creation contribution**: it exists also when $Q = 0$.
- σ^{drag} is the **“drag” contribution**. It originates from the force and dissipation a fractionalized charge (a string) feels as it is moving in the strongly coupled medium.

Karch+O'Bannon, (2007)

Ground-states with helical symmetry

Work with A. Donos and B. Gouteraux, to appear

- The gravitational action:

$$S = M^3 \int d^5x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z_1(\phi)}{4} F_1^2 - \frac{Z_2(\phi)}{4} F_2^2 \right]$$

$$\text{with } F_{\mu\nu}^i \equiv \partial_\mu A_\nu^i - \partial_\nu A_\mu^i$$

and

$$V(\phi) \rightarrow_{IR} \sim V_0 e^{-\delta\phi}, \quad Z_1(\phi) \underset{IR}{\sim} Z_{10} e^{\gamma_1\phi}, \quad Z_2(\phi) \underset{IR}{\sim} Z_{20} e^{\gamma_2\phi}.$$

- The metric ansatz has helical symmetry and we parametrize the scaling solutions as :

$$ds^2 = r^{\frac{2\theta}{3}} \left[-\frac{dt^2}{z^{2z}} + \frac{L^2 dr^2 + \omega_1^2}{r^2} + \frac{1}{r^{2z_2}} \left(\omega_2^2 + \frac{\lambda}{r^2} \omega_3^2 \right) \right], \quad A_1 \sim r^{\zeta-z}$$

where

$$\omega_1 = dx_1, \quad \omega_2 = \cos(kx_1)dx_2 + \sin(kx_1)dx_3, \quad \omega_3 = \sin(kx_1)dx_2 - \cos(kx_1)dx_3$$

Anisotropic saddle points

(a) IR marginal current ($\zeta = \theta - 2 - 2z_2$)

$$S_a \sim T^{\frac{2z_2+2-\theta}{z}}$$

vanishes at $T \rightarrow 0$ • For the DC conductivity

$$\sigma_{DC}^a \sim T^{\frac{\zeta-2}{z}}$$

- σ_{DC} along the helical axis vanishes always at $T = 0$ (power insulators)
- For the IR limit of AC conductivity we find

$$\sigma_{AC}^a(T = 0) \sim \omega^{\frac{\theta-2z_2-4}{z}}$$

- This is the same power as the DC conductivity and $\sigma_{AC}^a(T = 0) \rightarrow 0$ as $\omega \rightarrow 0$.

- (b) IR Irrelevant current ($z = \frac{3}{2}z_2$).

$$S_b \sim T^{\frac{(2z_2+2-\theta)}{z}} ,$$

- For the DC conductivity

$$\sigma_{DC}^b \sim T^{\frac{\zeta-2}{z}}$$

- It is sometimes diverging (metal) and sometimes vanishing (insulator).
- σ_{DC} comes always for the pair creation term. Therefore in the metallic case we expect no Drude peak (incoherent metals) .
- For the AC conductivity at zero T:

$$\sigma_{AC}^a(T=0) \sim \omega^{\min\{n_1, n_2\}}$$

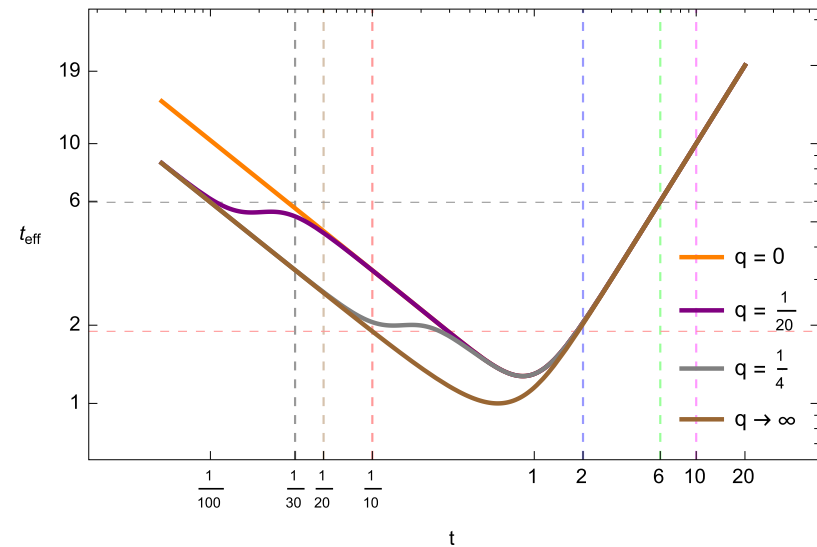
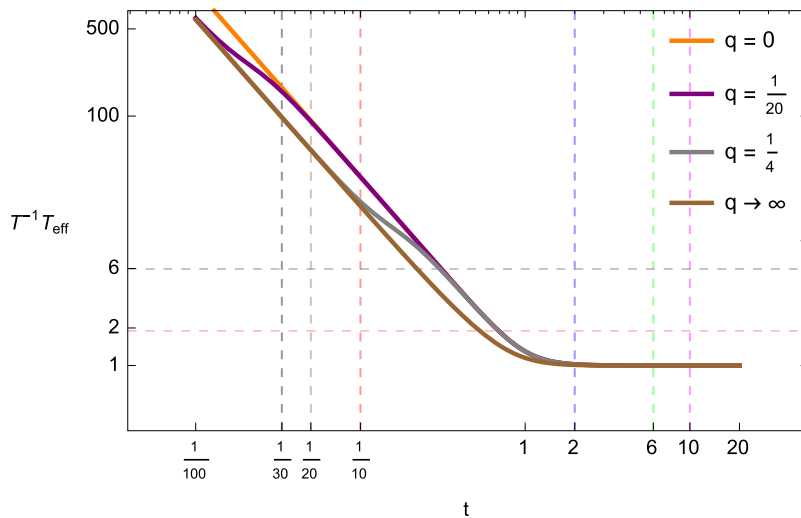
$$n_1 = -1 + \left| \frac{-4 + 2\zeta + 3z_2}{3z_2} \right| , \quad n_2 = -1 + \frac{1}{3} \sqrt{\frac{4\theta^2 + 96z_2 - 76\theta z_2 + 217z_2^2}{z_2^2}}$$

- Whenever **the system is an insulator**, n_1 dominates and the the AC and DC exponents are equal.
- When n_2 dominates, **the system is metallic** with a decaying low-frequency AC tail. A $\delta(\omega)$ may bridge this behavior.
- When the system is metallic and n_1 dominates, it can have:
 1. Diverging AC frequency power tail and match the DC scaling
 2. Diverging AC frequency power tail without matching the DC scaling
 3. Decaying AC frequency power tail.
- Whenever $\sigma_{AC} \sim \omega^0$ then $\sigma_{DC} \sim T^0$ or $\sigma_{DC} \sim T^{-2}$

The effective temperature

The effective temperature on the brane is not t for non-trivial E :

$$t_{eff} = t \sqrt{\frac{J^2 A(t)^2 \sqrt{A(t) - 2t^2} + t^5 (t^2 A(t) + 2t^4 + 3)}{2\sqrt{2}t^3 (t^5 \sqrt{A(t)} + J^2)}}$$



Dependence of the world-volume temperature T_{eff} with the parameters J and t , the effective temperature is always bigger than the background temperature T .

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 2 minutes
- Strongly Correlated Electrons 7 minutes
- Insulators 9 minutes
- Supersolids 11 minutes
- The plan for the rest 12 minutes
- The Wilsonian Program 13 minutes
- Generic Scaling Geometries: A classification of all QC points in holography 15 minutes

- Conductivity [19 minutes](#)
- General features of holographic conductivity [21 minutes](#)
- Holographic gapped systems at finite density [26 minutes](#)
- The finite temperature picture [28 minutes](#)
- Adding Momentum dissipation [35 minutes](#)
- The AC conductivity in a holographic strange metal [37 minutes](#)
- The DC conductivity [41 minutes](#)
- The AC conductivity [49 minutes](#)
- Outlook [49 minutes](#)

- Critical lines vs critical points 51 minutes
- Breaking translation invariance 52 minutes
- DC conductivity 53 minutes
- Ground-states with helical symmetry 54 minutes
- Anisotropic saddle-points 58 minutes
- The effective temperature 60 minutes