On hydrodynamics with Lifshitz symmetry

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BS Kim, Y Oz 1304.7481, 1309.6794, 1402.2981,
S Chapman, Y Oz 1312.6380, A. Meyer, Y. Oz to appear

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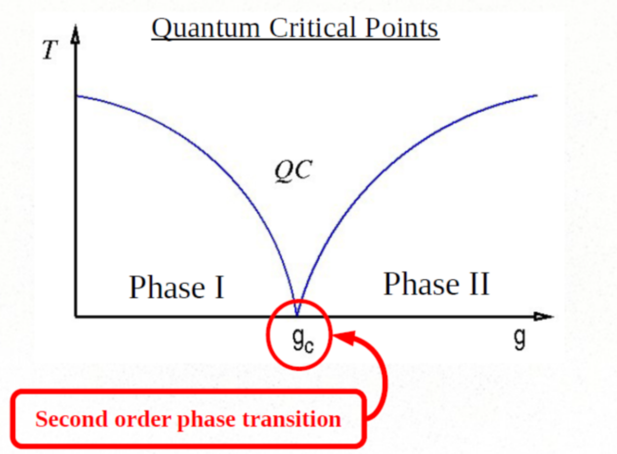
Outline

- Preliminaries
- Lifshitz hydrodynamics
- Non-relativistic limit
- Application to strange metals

Quantum Critical Points



In materials with quantum critical behavior:

- Interesting scaling behavior (maybe anomalous?) of transport coefficients

$$\rho_{xx} \sim T$$

- Planck dissipation: the only apparent scale is the temperature

$$\frac{1}{\tau} \sim \frac{k_B T}{\hbar}$$

Close to "minimal value"

- Interaction rate among electrons $\gg$ Interaction rate with lattice and impurities

- Hydrodynamic description expected to be valid [Damle, Sachdev; Andreev, Kivelson,Spivak; Davison, Schalm, Zaanen]

- Scaling symmetry

Symmetries

We will assume that the symmetries of the critical point are:

- Time and space translations

- Spatial rotations

- Anisotropic scaling

$$t \to \lambda^z t, \qquad x^i \to \lambda x^i$$

Boost invariance?

The associated Ward identities are

- Translations and rotations:

$$\partial_\mu T^{\mu\nu} = 0,\ T^{ij} = T^{ji}$$

- Scaling

$$zT^{00} - \sum_i T^{ii} = 0.$$

- If there is no boost invariance

$$T^{0i} \neq T^{i0} \quad \text{or} \quad J^i \neq T^{0i}$$

How should we implement this in hydrodynamics?

The $z=2$ Lifshitz scalar

$$S=\frac{1}{2}\int dtd^dx\left((\partial_t\phi)^2-\frac{\kappa}{2}(\nabla^2\phi)^2\right)$$

Finite temperature T = Lifshitz fluid at rest

- Rotational symmetry is unbroken

$$\langle T^{0i}\rangle=\langle T^{i0}\rangle=0,\quad\langle T^{ij}\rangle\propto\delta^{ij}$$

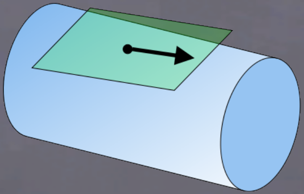
- The Ward identity and scaling are satisfied

$$z\varepsilon=z\langle T^{00}\rangle=d\langle T^{ii}\rangle=dp\propto T^{(d+z)/z}$$

How do we describe the moving fluid?

First let's write the action in a more useful form

$$\boxed{S = \frac{1}{2}\int dtd^dxe\left(t^at^be_a^\mu e_b^\nu\partial_\mu\phi\partial_\nu\phi - \frac{\kappa}{2}(P_t^{ab}e_a^\mu e_b^\nu\partial_\mu\partial_\nu\phi)^2\right)}$$



- We define a rest frame of the Lifshitz fluid at each point on the tangent space $t^a = (1, 0, \cdots, 0)$ ($P_t^{ab} = \eta^{ab} + t^at^b$)

- The geometry is flat $e_a^\mu = \delta_a^\mu$

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$$x^\mu(\tau) = \tau v^\mu$$

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$$\Lambda^\mu_\alpha e^\alpha_b (\Lambda^{-1})^b_a = e^\mu_a$$

- This will change the vector that defines the rest frame of the Lifshitz fluid $t^a \longrightarrow \Lambda^a_b t^b \equiv u^a$

- For an observer moving at constant velocity relative to the Lifshitz fluid the right action is

$$\boxed{S = \frac{1}{2}\int dt d^d x e \left(u^a u^b e_a^\mu e_b^\nu \partial_\mu \phi \partial_\nu \phi - \frac{\kappa}{2}(P^{ab} e_a^\mu e_b^\nu \partial_\mu \partial_\nu \phi)^2 \right)}$$

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- At non-zero T The expectation value of the energy-momentum tensor is

$$\langle T^{\mu\nu} \rangle = \varepsilon u^\mu u^\nu + p P^{\mu\nu}$$

Where u^μ is the velocity of the fluid relative to the observer and $P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$

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- The Ward identity for the scaling symmetry becomes

$$zT^{\mu\nu}u_\mu u_\nu - T^{\mu\nu}P_{\mu\nu} = 0$$

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- In general we expect deviations proportional to derivatives of the velocity

Hydrodynamics

The effective low energy description at nonzero temperature

- Variables: $u^\mu(x)$, $T(x)$, $\mu(x)$

- Constitutive relations

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + pP^{\mu\nu} + \pi^{\mu\nu}$$

$$J^\mu = qu^\mu + \nu^\mu$$

- Conservation equations $\Rightarrow$ dynamical equations

$$\partial_\mu T^{\mu\nu} = F^{\nu\alpha} J_\alpha, \quad \partial_\mu J^\mu = 0.$$

- Scaling Ward identity (at least at the ideal order)

$$zT^{\mu\nu}u_\mu u_\nu - T^{\mu\nu}P_{\mu\nu} = 0$$

It turns out this is true also at first order for theories with a holographic dual... But maybe the symmetry is larger? [Hartong, Kiritsis, Obers]

- Rotational invariance

$$(T^{\mu\nu} - T^{\nu\mu})P_{\mu\alpha}P_{\nu\beta} = 0$$

- Breaking of boost invariance

$$(T^{\mu\nu} - T^{\nu\mu})P_{\mu\alpha}u_\nu \neq 0$$

- Generic form of derivative terms if there is no boost invariance

$$\pi^{\mu\nu} = u^{[\mu}V_A^{\nu]} + \pi_S^{\mu\nu}$$

Where $V_A^\mu u_\mu = 0$, $\pi_S^{\mu\nu} = \pi_S^{\nu\mu}$

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Where $V_A^\mu u_\mu = 0$, $\pi_S^{\mu\nu} = \pi_S^{\nu\mu}$

- Landau frame $T^{\mu\nu}u_\nu = -\varepsilon u^\mu$

$$\pi^{\mu\nu} = u^{\mu}V_A^{\nu} + \pi_S^{\mu\nu}$$

Where $\pi_S^{\mu\nu} u_\nu = 0$

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- The equation for the entropy current can be derived from the hydrodynamic equations

$$\partial_\mu T^{\mu\nu} u_\nu + \mu \partial_\mu J^\mu = F^{\mu\nu} u_\mu J_\nu = -E^\nu J_\nu$$

Parity preserving fluid

- The entropy current takes the form

$$j_s^\mu = s u^\mu - \frac{\mu}{T}\nu^\mu$$

- The divergence is

$$\partial_\mu j_s^\mu = -\frac{1}{T}\pi_S^{\mu\nu}\partial_\mu u_\nu + \nu^\mu\left[\frac{E_\mu}{T} - \partial_\mu\left(\frac{\mu}{T}\right)\right] - \frac{1}{T}V_A^\mu a_\mu$$

Where $a^\mu = u^\alpha \partial_\alpha u^\mu$ is the acceleration of the fluid

- Since $\partial_\mu j_s^\mu \geq 0$ any explicit derivative term should be squared, and no other terms are allowed

$$V_A^\mu = -T\alpha_1 a^\mu - T\alpha_2 \left[\frac{E^\mu}{T} - P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right]$$

$$\nu^\mu = -\alpha_3 a^\mu + \sigma T \left[\frac{E^\mu}{T} - P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right]$$

$$\pi_S^{\mu\nu} = -\eta\sigma^{\mu\nu} - \zeta P^{\mu\nu}(\partial_\alpha u^\alpha)$$

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- The positivity constraints are

$$\sigma \geq 0,\ \eta \geq 0,\ \zeta \geq 0,\ \sigma T\alpha_1 \geq \alpha_-^2$$

Parity breaking fluid

- We will work in $3+1$ dimensions

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\alpha\beta\gamma}u_\alpha\partial_\beta u_\gamma, \quad B^\mu = \frac{1}{2}\epsilon^{\mu\alpha\beta\gamma}u_\alpha F_{\beta\gamma}$$

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- The entropy current can be modified by additional parity-breaking terms

$$j_s^\mu = su^\mu - \frac{\mu}{T}\nu^\mu + D\omega^\mu + D_B B^\mu$$

- The divergence becomes

$$\partial_\mu j_s^\mu = -\frac{1}{T}\pi_S^{\mu\nu}\partial_\mu u_\nu + \nu^\mu\left[\frac{E_\mu}{T} - \partial_\mu\left(\frac{\mu}{T}\right)\right] - \frac{1}{T}V_A^\mu a_\mu$$

$$+\omega^\mu\left(\partial_\mu D - \frac{2D}{\varepsilon+p}\partial_\mu p + 2\left(\frac{qD}{\varepsilon+p} - D_B\right)E_\mu\right)$$

$$+B^\mu\left(\partial_\mu D_B - \frac{D_B}{\varepsilon+p}\partial_\mu p + \frac{qD_B}{\varepsilon+p}E_\mu\right)$$

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$$+B^\mu\left(\partial_\mu D_B - \frac{D_B}{\varepsilon+p}\partial_\mu p + \frac{qD_B}{\varepsilon+p}E_\mu - C\frac{\mu}{T}E_\mu\right)$$

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- Anomalous current: $\partial_\mu J^\mu = C E_\mu B^\mu$

- To the derivative terms we had before we should add the parity breaking terms

$$V^{\mu}_{A,\not{P}} = -T\beta\omega^{\mu} - T\beta_B B^{\mu}$$

$$\nu^{\mu}_{\not{P}} = \xi\omega^{\mu} + \xi_B B^{\mu}$$

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- Even if $C = 0$ the coefficients ξ and ξ_B are non-zero if β, β_B are non-zero

- Chiral Magnetic and Vortical Effects without anomalies!

Summary so far

- Hydrodynamic analysis reveals that Lifshitz fluids have exotic transport properties:
  - New terms depending on the acceleration of the fluid a^μ
  - Chiral Effects without anomalies

- We can extend the analysis to superfluids, essentially the new terms depend on the acceleration a^μ

What is the phenomenology of the new terms?

Non-relativistic limit

- Condensed matter systems are not relativistic: take $c \to \infty$ limit

$$u^\mu = \left(1 + \frac{v^2}{2c}\right)\left(1, \frac{v^i}{c}\right), \quad \varepsilon = \rho c^2 + U - \rho \frac{v^2}{2}$$

$$q = \rho c - \rho \frac{v^2}{2c}, \quad \mu = c + \frac{\mu_{NR}}{c}$$

- This has to be done in the Eckart frame $u^\mu \longrightarrow u^\mu - \frac{1}{q}\nu^\mu \Rightarrow J^\mu = q u^\mu$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + p P^{\mu\nu} + u^\mu V_A^\nu + \pi_S^{\mu\nu} - \frac{\varepsilon + p}{q}(u^\mu \nu^\nu + u^\nu \nu^\mu)$$

Galilean invariant: [Kaminski,Moroz; Jensen,Karch]

Hydrodynamic equations

- Continuity equation (mass conservation)

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

- Momentum conservation equation (Navier-Stokes)

$$\partial_t \left(\rho v^i - \alpha a^i + \beta \omega^i\right) + \partial_k \left((\rho v^i - \alpha a^i + \beta \omega^i) v^k\right) + \partial^i p$$

$$= \rho \left(E^i + \epsilon^{ijk} v_j B_k\right) - \partial_k \left(\eta \sigma^{ki}\right) - \partial^i \left(\zeta \partial_k v^k\right)$$

Where $a^i = D_t v^i = (\partial_t + v^k \partial_k) v^i$ and $\omega^i = \frac{1}{2}\epsilon^{ijk}\partial_j v_k$

- Energy conservation

- Galilean boost invariance is broken

$$J^i = \rho v^i \neq T^{0i} = \rho v^i - \alpha a^i + \beta \omega^i$$

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- Lifshitz scaling $[\partial_t] = z$, $[\partial_i] = 1$

$$[\rho] = d + 2 - z, \;\; [p] = [U] = z + d$$

$$[T] = z, \;\; [\mu_{NR}] = 2(z-1), \;\; [v^i] = z - 1$$

$$[E_i] = 2z - 1, \quad [B_i] = z$$

Note that ρ is the mass density and the electromagnetic fields include the factor e/m

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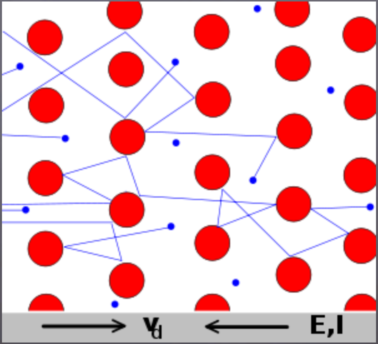
- Symmetry algebra: Lifshitz $+$ conserved charge

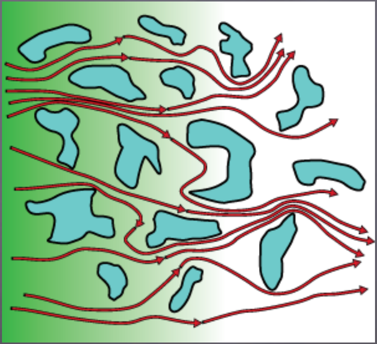
$$[D, H] = -zH,\ \ [D, P_i] = -P_i,\ \ [D, N] = (z-2)N$$

Drude model

The electronic transport of normal metals can be understood as a gas of free electrons (actually quasiparticles) scattering with heavy objects (lattice ions, impurities)



The electrons in a strange metal are strongly coupled, they cannot be approximated by a free gas. We expect them to behave as an almost perfect liquid moving through a porous medium



- We parametrize the effect of the medium by a drag term in the equations of motion

$$\partial_t \left(\rho v^i - \alpha a^i + \beta\omega^i\right) + \partial_k \left((\rho v^i - \alpha a^i + \beta\omega^i)v^k\right) + \partial^i p$$

$$= -\lambda\rho v^i + \rho\left(E^i + \epsilon^{ijk} v_j B_k\right) - \partial_k\left(\eta\sigma^{ki}\right) - \partial^i\left(\zeta\partial_k v^k\right)$$

- λ is the inverse of the electron mobility $[\lambda] = z$. For $B_i = 0$ and $E_i =$ constant

$$J^i = \rho v^i = \sigma_{xx} E^i = \frac{\rho}{\lambda} E^i$$

- This predicts naturally $\rho_{xx} \sim T$ at high temperatures for any dimension
Although the right effective theory may have anomalous scalings [Hartnoll, Karch; Khveshchenko]

- Possible experiment: linearly polarized light source

$$E_x = E(y)e^{i\omega t} + E^*(y)e^{-i\omega t}, \quad B_y(y) = B(y)e^{i\omega t} + B^*(y)e^{-i\omega t}$$

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- The AC velocity produced by the electric field has the form

$$v_{AC} = V_i(y)e^{i\omega t} + V_i^*(y)e^{-i\omega t}$$

$$(-\eta\partial_y^2 + \lambda\rho + \alpha\omega^2 + i\rho\omega)V_x + \frac{i\omega\beta}{2}\partial_y V_z = \rho E$$

$$(-\eta\partial_y^2 + \lambda\rho + \alpha\omega^2 + i\rho\omega)V_z - \frac{i\omega\beta}{2}\partial_y V_x = 0$$

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- Non-linear effects in the fluid produce a DC current

$$(-\eta\partial_y^2 + \lambda\rho)v_{DC}^i = \rho\epsilon^{ijk}(V_j B_k^* + V_j^* B_k)$$

- For $\beta = 0$, $v^x_{DC} = 0$ and $v^z_{DC} \neq 0$. In fact,

$$\vec{J}_{DC} \propto \vec{E} \times \vec{B}$$

$\Rightarrow$ Photon drag effect

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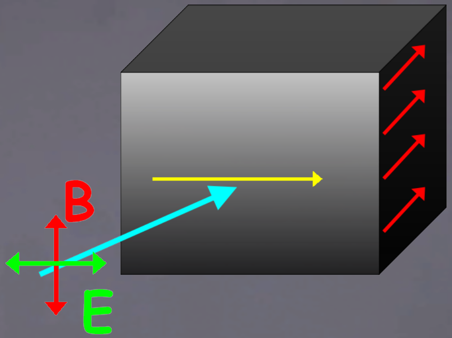
- For $\beta \neq 0$, $v^x_{DC} \neq 0$

$$\vec{J}_{\chi,DC} \propto \beta(\vec{\nabla} \times \vec{E}) \times \vec{B}$$

$\Rightarrow$ Chiral photon drag effect



Assuming electrons are the only mobile particles, the chiral effect is allowed only if

- Parity is broken
- There is no Galilean boost invariance

Smoking gun of parity breaking Lifshitz fluid

If parity is not broken the distinction between Lifshitz and others becomes more difficult to measure. Some new effects are

- Non-linear effects: dependence of the conductivity on electric field

- Frequency dependence of penetration depth in the metal

- Anisotropic contributions to the heat current in superfluids

To do: parity breaking Lifshitz fluids in $2+1$ dimensions

$\epsilon\upsilon\chi\alpha\rho\iota\sigma\tau\acute{\omega}$!