BPS Black Holes in AdS_4-NUT

Nick Halmagyi

CNRS
and
Laboratoire de Physique Théorique et Haute Energies,
Université Pierre et Marie Curie

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based on work done since 2013 with
Harold Erbin, Alessandra Gnecchi, Michela Petrini, Thomas Vanel, Alberto Zaffaroni

Motivation

- Asymptotically flat BPS black holes have triggered great advances in our understanding of BH entropy and string theory in general
- Black holes in AdS_{d+1} should in principle admit a study from the dual CFT_d but BPS black holes have proved to be rare finds.
- Gutowski-Reall BPS rotating black hole in 5d and various limits of the Plebanski-Demianski solution in 4d
- Motivated in part by search for ground states for AdS/CMT
- Can we construct supersymmetric black holes in AdS space much like their flat space cousins? Multiple centers requires running scalar fields
- Could they be related to a topological string on flux backgrounds

Four Dimensional Black Holes in Einstein-Maxwell Theory

Demianski-Plebanski (1976) produced a solution of Einstein-Maxwell theory which unified all previous known black holes with the following parameters:

- Λ: cosmological constant
- M: mass
- N: NUT charge
- P: magnetic charge
- Q: electric charge
- J: angular momentum
- a: acceleration

In addition there is a parameter $\kappa = \{-1, 0, 1\}$ for the curvature of the horizon $\{\mathbb{H}^2/\Gamma, \mathbb{R}^2/\Gamma', S^2\}$

TABLE I

Known exact solutions of the Einstein and Einstein–Maxwell Equations of type D

		$m + in, a + ib, e + ig, \lambda$			
$m + in, a, e + ig, \lambda$ Plebanski [3]		$m + in, a + ib, e + ig$ Kinnersley [2]		$m + in, b, e + ig, \lambda$	
$m + in, a, e\lambda$ Carter [11]		$m + in, a, e + ig$ Demianski, Newman [12]	$m + in, a + ib, \lambda$ Carter [11]	$m + in, b, e, \lambda$ Carter [11]	
$m + in, a, \lambda$ Frolov [22]			$m + in, a + ib$ Kinnersley [13]	$m + in, b, e$ Levi-Civita [4] Newman, Tamburino [5] Robinson, Trautman [6] Ehlers, Kundt [5]	
m, a, λ Demianski [14]	m, a, e Newman *et. al.* [10] Perjes [25] Ernst [26]	$m + in, a$ Demianski [15] Kramer, Neugebauer [24] Robinson, J. Robinson Zund [27]	$m + in, e$ Brill [23]		$m + in, \lambda$ Demianski [16] Frolov [22]
m, λ Kottler [17]	m, e Reisner, Nordstrom [18]	m, a Kerr [9]	$m + in$ Newman, Tamburino, Unti [19] Taub [21]	e, b Bertotti [28] Robinson [29]	
λ de Sitter [30]			m Schwarzschild [20]		

DP Solution in Minimal Gauged Sugra: $\{\Lambda, M, Q, P\}$

- Minimal gauged sugra: $(g_{\mu\nu}, A_\nu, \Psi_\mu)$
  The bosonic sector is Einstein-Maxwell theory
- Romans (1992) studied the parameters (Λ, M, Q, P) in minimal gauged sugra. In other words, the DP solution with

$$N = J = a = 0$$

- The metric is

$$\begin{aligned} ds_4^2 &= -e^{2U}dt^2 + e^{-2U}dr^2 + r^2 d\Omega_2^2 \\ e^{2U} &= \frac{1}{r^2}\Big(-\frac{\Lambda}{3}r^4 + r^2 - 2Mr + Z^2\Big) \end{aligned}$$

- Three important issues: BPS, supersymmetry and extremality
- SUSY $\Rightarrow$ BPS but BPS $\not\Rightarrow$ SUSY
- SUSY $\Rightarrow \begin{cases} \text{P=0} & \text{naked singularity} \\ \text{M=0} & \text{naked singularity} \end{cases}$

DP Solution in Minimal Gauged Sugra: $\{\Lambda, M, Q, P, J\}$

- Kostelecky-Perry (1995) include rotation into the analysis of Romans, so they have DP solution with

$$(M, N = 0, P, Q, J, a = 0)$$

- They find a BPS bound

$$M \geq |Z| + g|J|$$

- Caldarelli-Klemm (1998) checked integrability and found that mag charge must vanish $P = 0$
- Nonetheless there exist 4d BPS rotating black holes, regularity of the solutions puts a lower bound on J, no static limit.
- BPS bounds in 4d examined by Hristov-Toldo-Van Doren (2012)

$$P = 0 : \;\; M \geq Q + g|J|$$
$$P \neq 0 : \;\; M \geq 0$$

DP Solution in Minimal Gauged Sugra $\{\Lambda, M, N, Q, P\}$

- The integrability conditions were computed for the general DP solution Ortin et al. (2000)
- (M, N, Q, P) preserves $SU(2) \times U(1)$ (or $SL(2,\mathbb{R}) \times U(1)$)
- There are two branches of supersymmetric solutions
- quarter-BPS solutions: susy parameters are singlets under the symmetry of the horizon $\mathfrak{sl}_2(\mathbb{R})$, which has no analogue in asymptotically flat space
- half-BPS solutions: susy parameters are doublets, generalizing the asymptotically flat BPS solutions.

DP Solution in Minimal Gauged Sugra: $\{\Lambda, M, N, Q, P, J, a\}$

- (J, a) break $SU(2) \times U(1) \to U(1) \times U(1)$
- Killing spinor computed with $M = N$ Martelli, Passias (2013)
- sufficiency of integrability shown Klemm-Ozawa (2013)
- no half-BPS conditions due to breaking of $SU(2)$

Holography for Demianski-Plebanski

Martelli, Passias, Sparks et al (2011-2013)

In various limits, they

- obtain the Euclidean continuation of the supersymmetric Demianski-Plebanski solution
- compute the four dimensional on-shell action
- compare to the strong coupling limit of the supersymmetric free-energy of ABJM on M_3, where SUSY is preserved in an $SU(4)$ invariant manner
- M_3 is a fibration of S^1 over S^2 where the fiber is stretched and the S^2 is squashed

Nishioka (2014)

- computes super-Renyi entropy on $S^1 \times \Sigma_g$ and compares to euclidean on-shell action for DP with (Λ, M, P) and $\kappa = -1$

PD as Deformation of AdS_4

It is instructive to consider the PD solution as a deformation of the asymptotic ground state, AdS_4:

Metric Deformations

(ℓ, m)	Non-Normalizable	Normalizable
$(0,0)$	Temp T	Mass M
$(1,0)$	NUT N	x
$(2,0)$	x	Rotation J
$(3,0)$	Acceleration a	x

Gauge Field Deformations

$$A \sim \frac{Q}{r} dt + P \cos\theta d\phi$$

(ℓ, m)	Non-Normalizable	Normalizable
$(0,0)$	Constant gauge transformation	Electric Charge Q
$(1,0)$	Magnetic Charge P	x

Black Holes with Scalar Fields

- Duff-Liu (1998): superstars in 4d $\mathcal{N} = 8$ gauged sugra. These are singular half-BPS asympt' AdS

$$\{M, q_\Lambda | N = 0, q_\Lambda = 0, J = 0, a = 0\}$$

- Generalized to non-BPS regular black holes in STU-model Chong-Cvetic-Lu-Pope (2005). They have the solution with

$$\{M, N, q_0 = q_1, q_2 = q_3, J | p^\Lambda = 0, a = 0\}$$

found by "inspired guesswork" modelled on black holes in ungauged sugra

- Rotating case in Chow-Compere (2013) and Gnecchi,Klemm,Hristov,Toldo,Vaughn (2013)

$$\{M, p^0 = p^1, p^2 = p^3, q_0 = q_1, q_2 = q_3, J | a = 0\}$$

- Static case developed further in Chow-Compere (2013) with

$$\{M, p^\Lambda, q_\Lambda | N = 0, J = 0, a = 0\}$$

Regular SUSY Black Holes with Scalar Fields

- A substantial development was Cacciatori-Klemm (2009) who considered the STU model of $\mathcal{N} = 2$ gauged supergravity with the metric ansatz

$$
\begin{aligned}
ds_4^2 &= -e^{2U} dt^2 + e^{-2U} dr^2 + e^{2(V-U)} d\Sigma_g^2 \\
e^V &= r(\frac{r}{R} + v_0)\,, \qquad v_0 > 0\,.
\end{aligned}
$$

- Recall that in the DP solution

$$g_{tt} g_{\theta\theta} = P_4(r)$$

a fourth order polynomial in r. The CK solution has

$$P_4(r) = P_2(r)^2$$

- The CK solution has four magnetic charges and one constraint from Dirac quantization
- For the one-modulus example $F = -X^0 X^1$ and $q_\Lambda = 0$, Colleoni-Klemm (2013) have included NUT charge

Wrapped Branes

- To give perspective to these solutions of Cacciatori-Klemm one looks to the work of Maldacena-Nunez (2000)
- Conformal M2 brane theory has $SO(8)$ (BLG) or $SU(4) \times U(1)$ R-symmetry (ABJM)
- We can study this field theory on $\mathbb{R} \times \Sigma_g$ and supersymmetry is preserved by twisting
- The Cartan subgroup of the R-symmetry group is $U(1)^4$ and we twist by $U(1)$, structure group of Σ_g
- equivalent to twisting four line bundles over Σ_g, the four Chern numbers must satisfy

$$\sum_{\Lambda=0}^{3} p^{\Lambda} = 2 - 2g$$

to preserve the CY_5 condition and thus supersymmetry

- M5-branes wrapped on Σ_g give AdS_7 black 3-branes which are known numerically. The dual SCFT has $\mathcal{N} = 2$ Gaiotto (2009) or $\mathcal{N} = 1$ Bah-Beem-Bobev-Wecht (2012) in four dimensions
- D3-branes wrapped on Σ_g give AdS_5 black strings. The solutions are known numerically, recently the dual $(0, 2)$ SCFT was studied and its central charge computed Benini-Bobev (2013). Constant scalar black string is known analytically Chamseddine-Sabra (2001)
- D4-D8 system wrapped on Σ_g gives AdS_6 black 2-branes, Nunez et al (2001)
- Numerous more complicated AdS black objects are known with more elaborate embeddings into string/M-theory.
- Gauged sugra with hypermultiplets Halmagyi, Petrini, Zaffaroni (2013) lifts to $M2 - M5$ bound state wrapped

M-theory lift

In one particular duality frame the STU model has prepotential

$$F = -2i\sqrt{X^0 X^1 X^2 X^3}$$

and is a consistent truncation of the N=8, de-Wit Nicolai theory Cvetic et al (1999). This in turn is a consistent truncation of M-theory on S^7

- M: tension of M2 branes
- N: D6 branes in IIA, squashing of bdy M_3 and or orbifolding
- p^Λ: quantized twists of S^7 over Σ_g
- q_Λ: rotation along $U(1)^4$ of S^7
- J: rotation in 4d spacetime
- a: squashing of boundary M_3

The CK solutions have $q_\Lambda = 0$, we would like the supersymmetric solution for general $(p^\Lambda, q_\Lambda, N, M)$

Symplectic Covariance

- Dall'agata and Gnecchi (2010) present a duality covariant formulation of static black holes in $\mathcal{N} = 2$ gauged sugra

$$\mathcal{G} = \begin{pmatrix} g^\Lambda \\ g_\Lambda \end{pmatrix} , \qquad \mathcal{Q} = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}$$

- The formalism transforms covariantly (not invariantly) under $Sp(2n_v + 2, \mathbb{R})$

$$\Lambda \in Sp(2n_v + 2, \mathbb{R}) , \qquad \mathcal{G} \to \Lambda\mathcal{G} , \qquad \mathcal{Q} \to \Lambda\mathcal{Q}$$

- if there is a prepotential it transforms non-linearly
- $\mathcal{G}$ specifies the theory, $\mathcal{Q}$ specifies a selection sector of the vacuum. Acting with Λ generates a different but equivalent theory.

- The STU which embeds into M-theory has

$$\mathcal{G} = \begin{pmatrix} g^\Lambda \\ g_\Lambda \end{pmatrix} , \quad g^\Lambda = - \begin{pmatrix} 0 \\ g \\ g \\ g \end{pmatrix} , \quad g_\Lambda = \begin{pmatrix} g \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

There exists a one parameter deformation of the $SO(8)$ de-Wit Nicolai theory. On the truncation to the STU-model ω corresponds to a symplectic rotation of $\mathcal{G}$ by Lu-Pang-Pope (2014)

$$\mathcal{G} \to \mathcal{O}_\omega \cdot \mathcal{G} \, , \qquad \mathcal{O} = \begin{pmatrix} \mathbb{1} \cos\omega & \mathbb{1} \sin\omega \\ -\mathbb{1} \sin\omega & \mathbb{1} \cos\omega \end{pmatrix}$$

Very Special Geometry

- We consider theories where $\mathcal{M}_v$ is a very special geometry

$$F = -d_{ijk}\frac{X^i X^j X^k}{X^0}$$

- $\mathcal{M}_v$ is a symmetric space
- sporadic cases

$$\Big[\frac{SU(1,1)}{U(1)}\Big]^2, \quad \Big[\frac{SU(1,1)}{U(1)}\Big]^3, \quad \frac{Sp(6)}{U(3)}, \quad \frac{E_7}{E_6 \otimes U(1)}$$

$$\frac{SU(3,3)}{SU(3)\otimes SU(3)\otimes U(1)}, \quad \frac{SO(12)}{SU(6)\otimes U(1)}$$

- infinite series

$$\frac{SU(1,1)}{U(1)} \otimes \frac{SO(P+2,2)}{SO(P)\otimes SO(2)}$$

- Several more infinite families if one allows for homogeneous spaces Van Proeyen/de Wit (1992)

Static Black Holes and Special Geometry

The metric ansatz is

$$ds_4^2 = -e^{2U}(dt + 2N\cos\theta d\phi)^2 + e^{-2U}dr^2 + e^{2(V-U)}d\Sigma_g^2$$

The BPS Equations are

$$\begin{array}{rcl} 2e^V\partial_r\big(\mathrm{Im}\,\widetilde{\mathcal{V}}\big) &=& I_4'(\mathcal{G},\mathrm{Im}\,\widetilde{\mathcal{V}},\mathrm{Im}\,\widetilde{\mathcal{V}}) + 2N\kappa\mathcal{G}r - \mathcal{Q} \\ \partial_r\big(e^V\big) &=& 2\langle\mathcal{G},\mathrm{Im}\,\widetilde{\mathcal{V}}\rangle \\ 3N\kappa e^V + 2\langle\mathcal{Q},\mathrm{Im}\,\widetilde{\mathcal{V}}\rangle &=& -2e^V\langle\mathrm{Im}\,\widetilde{\mathcal{V}},\partial_r(\mathrm{Im}\,\widetilde{\mathcal{V}})\rangle + 4N\kappa r\langle\mathcal{G},\mathrm{Im}\,\widetilde{\mathcal{V}}\rangle \\ \langle\mathcal{G},\mathcal{Q}\rangle &=& -\kappa \qquad (\in\mathbb{Z}\ \text{Dirac Qu.}) \end{array}$$

where $\widetilde{\mathcal{V}} = e^{V-U}e^{-i\psi}\mathcal{V}$

- the scalar fields only appear through $\mathrm{Im}\,\widetilde{\mathcal{V}}$
- The $\mathcal{G}\to 0$ limit reproduces the flat space attractor equations
- from $\mathrm{Im}\,\widetilde{\mathcal{V}}$ and e^V we can obtain e^U and $\mathrm{Re}\,\widetilde{\mathcal{V}}$ (and thus z^i)

$$e^{4(V-U)} = \frac{3}{2}I_4(\mathrm{Im}\,\widetilde{\mathcal{V}})\,, \qquad \mathrm{Re}\,\widetilde{\mathcal{V}} = 2e^{2(U-V)}I_4'(\mathrm{Im}\,\widetilde{\mathcal{V}})$$

Quartic Invariant

- The quartic invariant is known from flat space BPS black holes

$$I_4(\mathcal{Q}) = -(p^\Lambda q_\Lambda)^2 - \frac{1}{16} p^0 \widehat{d}^{ijk} q_i q_j q_k + 4 q_0 d_{ijk} p^i p^j p^k + \frac{9}{16} d_{ijk} \widehat{d}^{ilm} p^j p^k q_l q_m$$

$$\widehat{d}^{ijk} = \frac{g^{im} g^{jn} g^{kp} d_{mnp}}{(d_{rst} y^r y^s y^t)^2}$$

- this gives a four index symmetric tensor

$$t^{MNRS} = \partial^M \partial^N \partial^R \partial^S I_4(\mathcal{Q}) \,, \qquad \partial^M = \frac{\partial}{\partial \mathcal{Q}_M}$$

- then we define the derivative of this tensor

$$I_4'(A,B,C)_M = \Omega_{MN} t^{NQRS} A_Q B_R C_S$$

and

$$I_4(A,B,C,D) = t^{MNQR} A_M B_N C_Q D_R$$

- The AdS_4 entropy is given by

$$R_{\mathrm{AdS}_4}^{-4} = I_4(\mathcal{G})$$

Horizon Equations N.H. 1308.1439

- Metric

$$ds_4^2 = R_1^2 ds^2_{AdS_2} + R_2^2 d\Sigma_g^2$$

- Define the complexified charges

$$\mathfrak{p}^\Lambda = p^\Lambda + iR_2^2 g^\Lambda\,, \quad \mathfrak{q}_\Lambda = q_\Lambda + iR_2^2 g_\Lambda$$

and the quantities

$$\Pi_j = d_{ijk}\mathfrak{p}^k\mathfrak{p}^k - \frac{1}{3}\mathfrak{p}^0\mathfrak{q}_i$$

- the scalar fields are given by

$$\begin{aligned} z^i &= \frac{\mathfrak{p}^i + \overline{\widehat{\mathcal{Z}}}^i}{\mathfrak{p}^0} \\ \overline{\widehat{\mathcal{Z}}}^i &= \frac{1}{2}\frac{\widehat{d}^{ijk}\Pi_j\Pi_k}{\sqrt{\widehat{d}^{lmn}\Pi_l\Pi_m\Pi_n}} \end{aligned}$$

- The radii can also be solved for

$$e^{i\psi}\frac{R_2^2}{R_1} = \frac{\mathfrak{p}^0}{2ie^{K/2}} - 6e^{K/2} d_{ijk} \overline{\widehat{\mathcal{Z}}}^i y^j y^k$$

- Finally the radius R_2 from which we get the entropy, is found from the solution to

$$I_4(\widehat{\mathcal{Q}}) = 0\,, \qquad \widehat{\mathcal{Q}} = \begin{pmatrix} \mathfrak{p}^\Lambda \\ \mathfrak{q}_\Lambda \end{pmatrix}$$

which gives

$$R_2^4 = \frac{I_4(\mathcal{G},\mathcal{G},\mathcal{Q},\mathcal{Q}) \pm \sqrt{I_4(\mathcal{G},\mathcal{G},\mathcal{Q},\mathcal{Q})^2 - I_4(\mathcal{G})I_4(\mathcal{Q})}}{I_4(\mathcal{G})}$$

- there is one constraint

$$\begin{aligned} 0 &= I_4(\mathcal{Q})I_4(\mathcal{Q},\mathcal{G},\mathcal{G},\mathcal{G})^2 + I_4(\mathcal{G})I_4(\mathcal{Q},\mathcal{Q},\mathcal{Q},\mathcal{G})^2 \\ &\quad - I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q})I_4(\mathcal{G},\mathcal{G},\mathcal{Q},\mathcal{Q})I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q}) \end{aligned}$$

Comments

- The charges are large in Planck units but not in AdS units.
- The large entropy comes from the large AdS_4 radius
- For the STU-model this of course comes from the large number of M2-branes
- We can do better and solve the whole black hole...

Static $\frac{1}{4}$-BPS Black Holes with Dyonic Charges

N.H. and Alessandra Gnecchi 1312.2766

N.H. 1408.2831

N.H. and Harold Erbin 1502.XXXX

Pair of real double roots

$$e^{2V} = r^2\Big(\frac{r}{R_{AdS_4}} + \sqrt{v_2}\Big)^2 , \qquad \mathrm{Im}\,\widetilde{\mathcal{V}} = \frac{1}{\langle \mathcal{G}, A_1\rangle} A_1 + r A_3$$

- The whole solution is given by the UV and IR boundary conditions
- The NUT charge is non-normalizable but still subleading in the BPS equations
- BPS equations vastly overdetermined but nonetheless one can find analytic solutions

The UV solution is AdS_4

$$A_3 = \frac{I_4'(\mathcal{G})}{4 I_4(\mathcal{G})^{1/4}} , \qquad R_{\mathrm{AdS}_4} = I_4(\mathcal{G})^{-3/4}$$

The IR solution is given by

$$A_1 = a_1 I_4'(\mathcal{G},\mathcal{G},\mathcal{G}) + a_2 I_4'(\mathcal{G},\mathcal{G},\mathcal{Q}) + a_3 I_4'(\mathcal{G},\mathcal{Q},\mathcal{Q}) + a_4 I_4'(\mathcal{Q},\mathcal{Q},\mathcal{Q})$$

with

$$a_3 = -\frac{a_3 I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q})}{3 I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q})}$$

$$a_3 = \frac{a_3}{6} \frac{I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q}) I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q})^2}{I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q})^2 I_4(\mathcal{Q}) - I_4(\mathcal{G}) I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q})^2}$$

$$a_3 = \frac{9\big(I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q}) I_4(\mathcal{G}) - I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q}) I_4(\mathcal{Q})\big)}{I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q}) I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q}) \big[\langle I_4'(\mathcal{G},\mathcal{G},\mathcal{G}), I_4'(\mathcal{Q},\mathcal{Q},\mathcal{Q})\rangle + \kappa I_4(\mathcal{G},\mathcal{G},\mathcal{Q},\mathcal{Q})\big]}$$

$$a_3 = -\frac{a_2 I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q})}{3 I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q})}$$

Since we enforced that the IR is of the form $AdS_2 \times \Sigma_g$ we have one constraint from the horizon solution

$$\begin{aligned} 0 &= I_4(\mathcal{Q}) I_4(\mathcal{Q},\mathcal{G},\mathcal{G},\mathcal{G})^2 + I_4(\mathcal{G}) I_4(\mathcal{Q},\mathcal{Q},\mathcal{Q},\mathcal{G})^2 \\ &\quad - I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q}) I_4(\mathcal{G},\mathcal{G},\mathcal{Q},\mathcal{Q}) I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q}) \end{aligned}$$

The metric is given by

$$v_2 = \sqrt{2\langle \mathcal{G}, A_1 \rangle}$$

The NUT charge is fixed by the equations between UV and IR

$$N\kappa = -\frac{I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q})^2 I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q})}{144\sqrt{2}\, I_4(\mathcal{G})^{1/4}} \times$$

$$\Big[18\langle \mathcal{G}, \mathcal{Q}\rangle I_4(\mathcal{G},\mathcal{G},\mathcal{Q},\mathcal{Q}) - \langle I_4'(\mathcal{Q},\mathcal{Q},\mathcal{Q}), I_4'(\mathcal{G},\mathcal{G},\mathcal{G})\rangle\Big]^{1/2} \times$$

$$\big[\big(I_4(\mathcal{G}) I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q})^2 - I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q})^2 I_4(\mathcal{Q})\big)^2 + 16\big[I_4(\mathcal{G},\mathcal{G},\mathcal{G},\mathcal{Q}) I_4(\mathcal{G},\mathcal{Q},\mathcal{Q},\mathcal{Q})\big]^3\big]^{1/2}$$

Single Double Root

- More generally, there are solutions with only a single double root in e^{2V}
- This double root ensures an $\mathrm{AdS}_2 \times \Sigma_g$ IR horizon region
- The ansatz is then

$$e^{2V} = r^2\left(\frac{r^2}{R^2_{AdS_4}} + v_3 r + v_2\right)$$

The sections are

$$\mathrm{Im}\,\widetilde{\mathcal{V}} = e^{-V}\left[A_1 r + A_2 r^2 + A_3 r^3\right]$$

where

$$\begin{aligned} A_1 &= a_1 I_4'(\mathcal{G},\mathcal{G},\mathcal{G}) + a_2 I_4'(\mathcal{G},\mathcal{G},\mathcal{Q}) + a_3 I_4'(\mathcal{G},\mathcal{Q},\mathcal{Q}) + a_4 I_4'(\mathcal{Q},\mathcal{Q},\mathcal{Q}) \\ A_2 &= b_1 I_4'(\mathcal{G},\mathcal{G},\mathcal{G}) + b_2 I_4'(\mathcal{G},\mathcal{G},\mathcal{Q}) + b_3 I_4'(\mathcal{G},\mathcal{Q},\mathcal{Q}) + b_4 I_4'(\mathcal{Q},\mathcal{Q},\mathcal{Q}) \end{aligned}$$

Comments

- With a single double root the solution for A_2 is somewhat complicated but totally explicit
- The NUT charge is then arbitrary, the b_i but not the a_i, depend non-trivially on N
- The IR region is still $\mathrm{AdS}_2 \times \Sigma_g$
- When the NUT charge satisfies the previous constraint, the solution will degenerate to a pair of double roots
- In principle we could have four complex roots (two conjugate paris). Indeed, this is the root structure of the constant scalar solution

Conclusions and Future Work

- Half BPS branch of solutions for static black holes
- NUT charge should resolve the singular superstar geometry while preserving half-BPS
- Fully BPS horizon geometries?
- Scalar hair, our ansatz has picked out certain solutions
- Euclidean continuation of general solution with running scalars, computation of on-shell action
- Partition function for general twists of ABJM
- multiple centers for (M, N); multipole moments and/or separated M2-branes in eleven dimensions?
- BPS gauged sugra solutions with (J, a) and running scalars
- Generating rotation from 3d reduction in gauged sugra