# Hydrodynamic properties of charged black branes in (super)gravity

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Based on work with Andreas Vigand Pedersen and Adriana Di Dato.

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## Outline

## Setting: Fluid/gravity correspondence

[Bhattacharyya, Hubeny, Minwalla, Rangamani '07]

Applied to asymptotically flat *dilatonic* black $p$-branes sourcing either:

- a $(p+1)$-form gauge potential or
- a Maxwell gauge potential.

## Original motivation

- What is the effect when you add charge to the brane?

  Schwarzschild black $p$-brane [Camps, Emparan, Haddad '10]
  Reissner-Nordström black brane [JG, Pedersen '13]

## Stepping stone to

- Supergravity multi-charged bound state solutions.
- Viscous Lifshitz hydrodynamics (hyperscaling-violating).

## Input for the relationship between flat/AdS

- Generalizations of the "AdS/Ricci flat correspondence".
  Neutral branes [Caldarelli, Camps, Gouteraux, Skenderis '12]
- Effective hydrodynamics of D$p$-branes.
  Black D3-brane [Emparan, Hubeny, Rangamani '13]

## Highlights

- New examples of fluid transport - input to universal bounds.
- Access to stability properties and GL instability.
- Modified version of the “AdS/Ricci flat correspondence”.

# Outline

## Action

We consider $D$ dimensional $p$-brane solutions

$$I = \int_D \left( R * 1 - 2\, \mathrm{d}\phi \wedge * \mathrm{d}\phi - \frac{1}{2} \sum_{q \in \mathcal{I}} \mathcal{F}_{(q+2)} \wedge * \mathcal{F}_{(q+2)} \right) \,,$$

with $\mathcal{F}_{(q+2)} = e^{a_q \phi}\, \mathrm{d}C_{(q+1)}$ and $\mathcal{I}$ the collective set of gauge potentials.

## Special cases D/NS/M-branes

- $D = 10$, type II supergravity with $\mathcal{I}_{\mathsf{NS}} = \{1\}$ and either
  - A with $\mathcal{I}_{\mathsf{RR}} = \{0, 2\}$ or
  - B with $\mathcal{I}_{\mathsf{RR}} = \{1, 3\}$.
- $D = 11$, supergravity with $\mathcal{I}_{\mathsf{M}} = \{2\}$.

## Charged branes

We consider singly charged $p$-branes,
[Caldarelli, Emparan, Van Pol '10]

$$\mathrm{d}s^2 = \mathrm{d}s^2_{p+2}(x^a, r) + h(r)\mathrm{d}\Omega^2_{n+1} \ , \quad C_{(q+1)}(x^a, r) \ , \quad \phi(r) \ ,$$

with either fundamental charge $q = p$ or Maxwell charge $q = 0$.

- Solutions are spanned by two parameters $r_0$ and $\gamma_0$.
- In contrast to AdS (co-dimension 1) branes, we consider branes of co-dimension $n + 2$.
- Dilaton coupling is related to the (intersection) number $N$

$$a_q^2 = \frac{4}{N} - \frac{2(q+1)(D-q-3)}{D-2} \ .$$

## Outline

## Fluid/gravity lore

There is a one-to-one correspondence between the solutions to Einstein's equations and the relativistic Navier-Stokes equations,

$$\mathsf{div}\, T = 0 \ , \quad \mathsf{d} \star j = 0 \ .$$

- The effective $T$ and $j$ encompass the asymptotic data of a (perturbed) solution and the correspondence allows one to reconstruct the full gravitational solution to any given order in the derivatives.

Indeed, to leading order the asymptotic stress tensor and current are

$$T_{ab} = \varrho\, u_a u_b + P \Delta_{ab} \ , \quad j = \mathcal{Q}_q \mathsf{Vol}_{q+1} \ ,$$

with fluid velocity $u^a$ and projector $\Delta_{ab} = \eta_{ab} + u_a u_b$.

## Remarks

At lowest order (no derivatives) and flat intrinsic geometry $\eta_{ab}$,

- The correspondence between fluid dynamics and gravity is a convenient repackaging of black hole thermodynamics in terms of a relativistic (perfect) fluid.

- If one abandons the requirement of flat intrinsic geometry $\eta_{ab} \to \gamma_{ab}$, the statement becomes an equivalence between gravity and perfect fluid dynamics on a curved $p$-submanifold.
  - This is *the blackfold approach* and is a non-trivial statement.

[Emparan, Harmark, Niarchos, Obers '09]

## Outline

## Ingredients:

- In order to ensure that the perturbative problem is well-posed, we need to cast the solution into Eddington-Finkelstein.
  - Solution behave nicely at the horizon.
  - Gravitational perturbations move along null lines, i.e., $|\mathrm{d}r| = 0$.

- Derivative expansion
  - The solution is part of a larger class of solutions $\mathrm{d}s_f^2$, for which the parameters $\xi = (u^a, r_0, \gamma_0)$ are worldvolume fluctuating functions.

$$\mathrm{d}s_f^2 = \mathrm{d}s^2 + \mathrm{d}s_\partial^2 + \mathcal{O}(\partial^2) \ , \quad \text{etc.}$$

## Solving:

- Gauge choice and ansatz for perturbations $\psi_\partial = (g_\partial, A_\partial, \phi_\partial)$
- Equations
  - *Constraint equations:*

$$\mathbb{C}_\partial \ \partial\xi + \mathcal{O}(\partial^2) = 0 \ ,$$

  - *Dynamical equations:*

$$\mathbb{L}_r^{(1)}\mathbb{L}_r^{(2)} \ \psi_\partial = s_\partial(r) + \mathcal{O}(\partial^2) \ .$$

- Solving and fixing freedom and boundary conditions
  - 1. Horizon regularity.
  - 2. Homogenous solution.
  - 3. Asymptotically flatness.

## First-order corrected solution ($q = p$):

- Corrected effective stress tensor

$$T_{ab} = \varrho\, u_a u_b + P\Delta_{ab} - 2\eta\sigma_{ab} - \zeta\vartheta\Delta_{ab} + \mathcal{O}(\partial^2)\ .$$

- No correction to the charge current $j = Q \star 1$.

- Transport coefficients

$$\eta = \frac{s}{4\pi}\ , \quad \frac{\zeta}{\eta} = 2\left(\frac{1}{p} + \frac{(C-2n)\gamma_0}{n+1+C\gamma_0} + \frac{(n+1)\big(1+(C-2n)\gamma_0\big)}{(n+1+C\gamma_0)^2}\right)\ ,$$

with $C \equiv 2 - n(N-2)$.

  - The neutral limit can be obtained independently by taking either $\gamma_0$ or $N$ to zero.

## First-order corrected solution ($q = 0$):

- Shear and bulk viscosity

$$\eta = \frac{s}{4\pi} \ , \quad \frac{\zeta}{\eta} = \frac{2}{p} + \frac{2}{C}\left(2 - N + \frac{(n+1)N}{(n+1+C\gamma_0)^2}\right) \ .$$

- Corrected effective current

$$j^a = \mathcal{Q}u^a - \tilde{\mathfrak{D}}\Delta^{ab}\partial_a\left(\frac{\Phi}{\mathcal{T}}\right) + \mathcal{O}(\partial^2) \ ,$$

with diffusion coefficient

$$\frac{\mathfrak{D}}{\eta} = \frac{4\pi r_0(1+\gamma_0)}{nN\gamma_0\sqrt{(1+\gamma_0)^N}} \ .$$

## Outline

## First-order fluid conservation

- Introduce small long-wavelength perturbations in the effective fluid

$$\mathcal{T} \to \mathcal{T} + \delta\mathcal{T} e^{i(\omega t + k_i x^i)} \ , \quad \Phi \to \Phi + \delta\Phi e^{i(\omega t + k_i x^i)} \quad \text{etc.}$$

## Dispersion relations

- Sound mode(s)

$$\omega(k) = \pm c_s k + i\mathfrak{a}_s k^2 \ .$$

- Shear mode

$$\omega(k) = \frac{i\eta}{w} k^2 \ .$$

## Leading order

- Speed of sound

$$c_s^2 = \left(\frac{\partial P}{\partial \varrho}\right)_{Q_p} = -\frac{1+(2-nN)\gamma_0}{n+1+C\gamma_0} \quad .$$

- Stability threshold

$$\bar{\gamma}_0 = \frac{1}{nN-2} \quad .$$

[Emparan, Harmark, Niarchos, Obers '11]

  - It is a transition point between an unstable branch and a stable branch of configurations.
  - The point of maximal charge.
  - The stable regime is where the electrostatic energy is dominant, i.e., large $\gamma_0$.

## First order

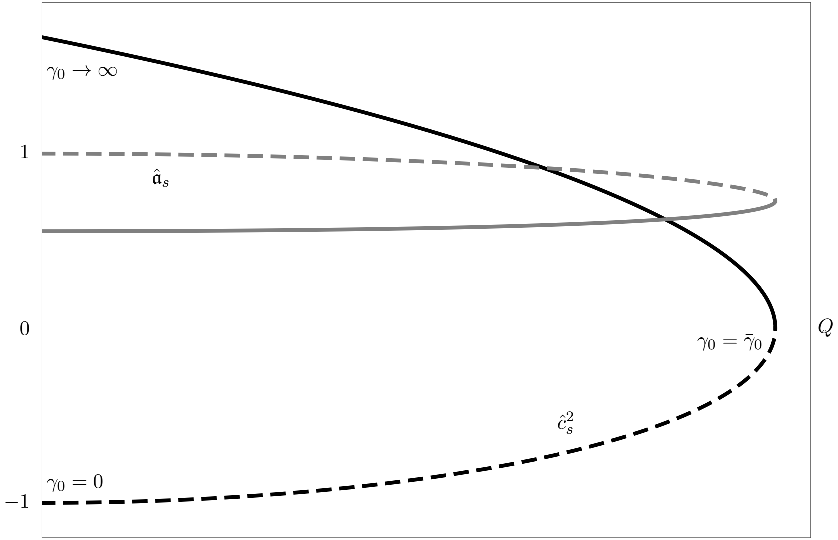
- In the absence of charge diffusion, the attenuation of the sound mode takes the exact same form as a neutral fluid

$$\mathfrak{a}_s = \frac{1}{w}\left(\left(1 - \frac{1}{p}\right)\eta + \frac{\zeta}{2}\right) \ .$$

## Features

- The charge only plays a role in the equation of state.
- The attenuation coefficient is always positive.
  - The stability is therefore fully determined by the linear order, i.e., by the speed of sound.
  - This is in contrast to configurations with smeared charges where the attenuation coefficient plays an important role for the stability properties of the effective fluid.

# *Sound mode coefficients ($q = p$) for fixed $\mathcal{T}$*



## Correlated stability conjecture

[Gubser, Mitra '00, Reall '01]

- An occurrence of a dynamical GL-like instability is intercorrelated with a thermodynamical stability.
- The condition for thermodynamic stability is positivity of the specific heat $c_{Q_p}$.
- In fact one finds perfect agreement, since there is a direct relation between the speed of sound and the specific heat

$$c_s^2 = \left(\frac{\partial P}{\partial \varrho}\right)_{Q_p} = s\left(\frac{\partial \mathcal{T}}{\partial \varrho}\right)_{Q_p} = \frac{s}{c_{Q_p}} \quad .$$

## Diffusion

- Additional longitudinal mode

$$\omega(k) = i\mathfrak{a}_{\mathfrak{D}} k^2 \quad .$$

  - Attenuation of the diffusion of charge.
  - This is a first-order derivative effect.
  - Stability to first-order requires both $\mathfrak{a}_s$ and $\mathfrak{a}_{\mathfrak{D}}$ to be positive.

## Leading order

- Speed of sound

$$c_s^2 = \left(\frac{\partial P}{\partial \varrho}\right)_{\frac{s}{\mathcal{Q}}} = -\frac{1 + (2 - N)\gamma_0}{(1 + \gamma_0 N)(n + 1 + C\gamma_0)} \quad .$$

- Stability threshold

$$\bar{\gamma}_0 = \frac{1}{N - 2} \quad .$$

  - It is a transition point between an unstable branch and a stable branch of configurations.
  - The point *is not* where the configuration has maximal charge.
- At leading order the analysis is similar to $q = p$.

## First order

- With charge diffusion, one finds the modification to the dispersion relation for the sound mode

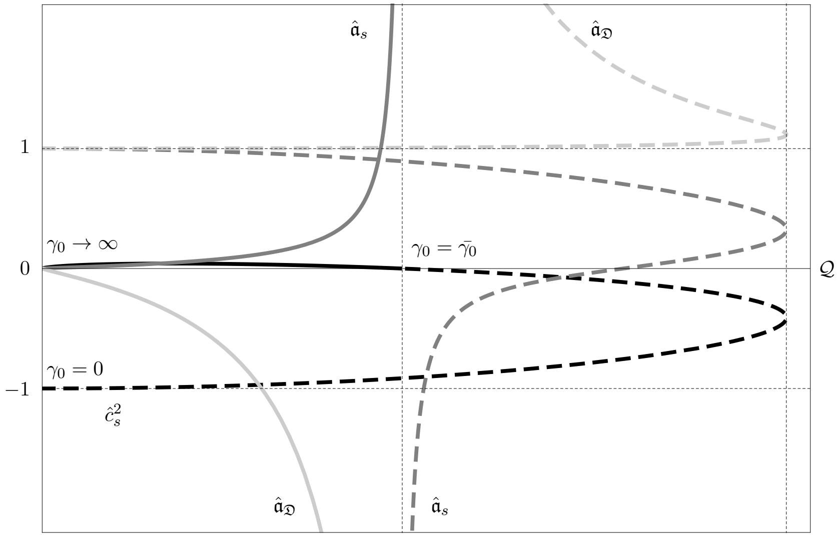
$$\mathfrak{a}_s = \frac{1}{w}\left(\left(1-\frac{1}{p}\right)\eta + \frac{\zeta}{2} + \frac{2}{\mathcal{T}}\frac{\mathcal{Q}^2}{c_s^2}\left(\frac{\mathcal{Q}^2}{w\Phi}\,\frac{c}{C_{\mathcal{Q}}}\right)^2\mathfrak{D}\right) \quad .$$

Here, $C_{\mathcal{Q}}$ is the specific heat capacity and $c$ is the (inverse) isothermal permittivity.

- For the diffusion mode,

$$\mathfrak{a}_{\mathfrak{D}} = \frac{\mathcal{Q}^2}{c_s^2 w}\,\frac{c}{C_{\mathcal{Q}}}\mathfrak{D} \quad .$$

# *Sound and diffusion ($q = 0$) for fixed $\mathcal{T}$*



## Correlated stability conjecture

- The condition for thermodynamic stability is positivity of the specific heat $c_{Q_p}$ and isothermal permittivity $c$,

$$C_{\mathcal{Q}} = \left(\frac{\partial\varrho}{\partial\mathcal{T}}\right)_{\mathcal{Q}} = \left(\frac{n+1+C\gamma_0}{(nN-2)\gamma_0-1}\right)s \quad ,$$
$$c = \left(\frac{\partial\Phi}{\partial\mathcal{Q}}\right)_{\mathcal{T}} = \left(\frac{1}{(\gamma_0+1)(1-(nN-2)\gamma_0)}\right)\frac{1}{s\mathcal{T}} \quad .$$

## Observations

- Complementary behaviour - exactly as in dynamical stability analysis for $\mathfrak{a}_s$ and $\mathfrak{a}_{\mathfrak{D}}$.
- Agreement with the correlated stability conjecture.
- However, the exchange threshold is $\gamma_0 = {}^{1}/_{(nN-2)}$ (maximal charge configuration).

# Outline

## Non-trivial check

- Modification to the "AdS/Ricci flat correspondence"
  - Originally a mapping asymptotically locally AdS spacetimes and Ricci-flat spacetimes. [Caldarelli, Camps, Gouteraux, Skenderis '12]
  - We refer to the two sides of the map as the Flat and the AdS side.

- We need to introduce a gauge potential.
  One minimalistic modification is to perform a Kaluza-Klein reduction, thus connecting us to EMD theory

$$\mathcal{L} = R \quad \longrightarrow \quad \mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\mathcal{F}_2^2 \ .$$

- This fixes the coupling constant $a = a_{\mathsf{KK}}$.

## Theories, Branes and Hydro

| | Flat side | AdS side |
|---|---|---|
| Einstein-Hilbert | $\mathcal{L} = R$ [Camps et al. '10] | $\mathcal{L}_\Lambda = R - 2\Lambda$ [Minwalla et al. '08] |
| | Kaluza-Klein $\downarrow$ | |
| EMD theory | $I(R, \phi, A_1)$ [JG et al. '15] | $I_{(d+1),\Lambda}(R, \phi, A_1, \Lambda)$ [Goutéraux et al. '11] |
| | $\Omega_{n+1} \downarrow$ | $\mathbb{T}^{d-p-1} \downarrow$ |
| $(p+2)$-theory | $\mathcal{I}(\mathcal{R}, \chi, \phi, A_1)$ | $\mathcal{I}_\Lambda(\mathcal{R}, \chi, \phi, A_1, \Lambda)$ |

## Inspection

- Say we have closed analytic expressions of solutions to the reduced action for any positive integer $d$.
- Extend the domain via an analytic continuation of the parameter $d$ to any real value.
- It makes sense to consider solutions for negative values of $d$.
- Apply same reasoning for integer $n$ and inspect that

$$\mathrm{Vol}(S^{n+1})\,\mathcal{I}_\Lambda \;\leftrightarrow\; \mathrm{Vol}(\mathbb{T}^\beta)\,\mathcal{I} \quad ,$$

under the identification $d \leftrightarrow -n$.

# Outline

## The broader perspective

- 2nd order transport coefficients
  - Simply by starting from the known second-order results in AdS. [Bhattacharyya, Loganayagam, Mandal, Minwalla, Sharma '08]
- Smeared D0-brane solutions are related to charged D$p$-branes by T-duality.
  - This requires considering the Buscher rules in a derivative expansion.

$$I(\mathcal{R}, \phi, \mathcal{F}_2) \quad \longrightarrow \quad I(\mathcal{R}, \phi, \mathcal{F}_{p+2}) \; .$$

  - Two approaches to transport coefficients.
- Hydrodynamic limit for branes with general smeared charge ($q < p$) and multi-charged bound state solutions.
  - Requires a extended fluid formalism.

## Lifshitz hydrodynamics

- It is possible to generalize the mapping between actions. However, if one wants to uplift to higher dimensions this leads to pathological theories.
- At the reduced level one has explicit access to the first-order corrected solutions with (hyperscaling-violating) Lifshitz symmetry.