Reconstructing holography (fluid/gravity) as a RG Flow

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Based on

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N. Behr and A. Mukhopadhyay, “Holography as a highly efficient RG flow: Part 2”, arXiv:1503.xxxx

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S. Kuperstein and A. Mukhopadhyay “The unconditional RG flow of the relativistic holographic fluid”, arXiv:1105.4530 (published in JHEP)

Inspired by

I. I. Heemskerk and J. Polchinski, “Holographic and Wilsonian Renormalisation Groups,” arXiv:1010.1264

II. T. Faulkner, H. Liu and M. Rangamani, “Integrating out geometry: Holographic Wilsonian RG and the membrane paradigm,” arXiv: 1010.4036

III. S. S. Lee, “Quantum Renormalization Group and Holography,” arXiv:1305.3908

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Part 0: Motivation and Plan

Motivation

Can the holographic duality lead to a broader effective framework for quantum many-body systems with strongly interacting degrees of freedom?

We need to reconstruct the holographic duality as a RG Flow.

Plan

- Part 1: Three principles for reconstructing holography
- Part 2: The invertible mapping between RG flow and gravity
- Part 3: Reconstructing fluid/gravity
- Part 4: Outlook

Part 1: Three principles

Definitions

Single-trace operators: Those which generate the full algebra of local operators

Strongly interacting limit: The limit where only a few single-trace operators have parametrically small anomalous dimensions.

Large N limit: The limit where expectation values of multi-trace operators factorise.

Pure gravity sector: The sector of states where only the em-tensor expectation value is independent.

First Principle

Consider the most general coarse-graining consistent with large N expansion - the cut-off in momentum space is a functional of elementary fields and external sources.

We expect that in the strongly interacting large N limit in pure gravity sector:

$$\frac{\partial t^{\mu}{}_{\nu}(\Lambda)}{\partial \Lambda} = \text{a non-linear functional of } t^{\mu}{}_{\nu}(\Lambda) \text{ and its spacetime derivatives.}$$

In a CFT, the R.H.S can be expanded systematically for $\Lambda \gg \Lambda_{\mathrm{IR}}$ (a state-dependent IR scale).

The above RG flow equation is a “Legendre transform” of the beta-function flow in the usual local RG.

Consider this special RG flow:

$$\frac{\partial t^{\mu}{}_{\nu}(\Lambda)}{\partial \Lambda} = \frac{1}{\Lambda^3}\cdot\frac{1}{2}\Box t^{\mu}{}_{\nu}(\Lambda) - \frac{1}{\Lambda^5}\cdot\left(\frac{1}{4}\,\delta^{\mu}{}_{\nu} t^{\alpha}{}_{\beta}(\Lambda) t^{\beta}{}_{\alpha}(\Lambda) - \frac{9}{128}\,\Box^2 t^{\mu}{}_{\nu}(\Lambda)\right) - \\ -\frac{1}{\Lambda^5}\,\log\,\Lambda\cdot\frac{1}{32}\cdot\Box^2 t^{\mu}{}_{\nu}(\Lambda) + \mathcal{O}\left(\frac{1}{\Lambda^7}\,\log\,\Lambda\right).$$

Generally speaking: $$\partial_{\mu} t^{\mu}{}_{\nu}{}^{\infty} = 0, \quad \text{but} \quad \partial_{\mu} t^{\mu}{}_{\nu}(\Lambda) \neq 0.$$

In our RG flow:

$$\partial_{\mu} t^{\mu}{}_{\nu}(\Lambda) = \frac{1}{\Lambda^4}\cdot\left(\frac{1}{16}\partial_{\nu}\left(t^{\alpha}{}_{\beta}(\Lambda) t^{\beta}{}_{\alpha}(\Lambda)\right) - \frac{1}{8} t^{\alpha}{}_{\nu}(\Lambda)\partial_{\alpha}\mathrm{Tr}\, t(\Lambda)\right) + \\ +\frac{1}{\Lambda^6}\cdot\left(\frac{1}{48} t^{\alpha}{}_{\beta}(\Lambda)\partial_{\nu}\Box t^{\beta}{}_{\alpha}(\Lambda) - \frac{1}{48} t^{\alpha}{}_{\nu}(\Lambda)\partial_{\alpha}\Box\mathrm{Tr}\, t(\Lambda)\right) + \\ +\mathcal{O}\left(\frac{1}{\Lambda^8}\right).$$

What’s so special?

This effective scale-dependent Ward identity related to local energy momentum conservation can be re-written in the standard form:

$$\nabla_{(\Lambda)\mu} t^{\mu}{}_{\nu}(\Lambda) = 0$$

in a “fictitious” scale-dependent background metric:

$$\begin{aligned} g_{\mu\nu}(\Lambda) = \quad & \eta_{\mu\nu} + \frac{1}{\Lambda^4} \cdot \frac{1}{4} \eta_{\mu\alpha} t^{\alpha}{}_{\nu}(\Lambda) + \frac{1}{\Lambda^6} \cdot \frac{1}{24} \eta_{\mu\alpha} \Box t^{\alpha}{}_{\nu}(\Lambda) + \\ & + \frac{1}{\Lambda^8} \cdot \left(\frac{1}{32}\, \eta_{\mu\alpha} t^{\alpha}{}_{\rho}(\Lambda) t^{\rho}{}_{\nu}(\Lambda) - \frac{7}{384}\, \eta_{\mu\nu} t^{\alpha}{}_{\beta}(\Lambda) t^{\beta}{}_{\alpha}(\Lambda) + \frac{11}{1536}\, \eta_{\mu\alpha} \Box^2 t^{\alpha}{}_{\nu}(\Lambda) \right) - \\ & - \frac{1}{\Lambda^8}\, \log\Lambda \cdot \frac{1}{516} \cdot \eta_{\mu\alpha} \Box^2 t^{\alpha}{}_{\nu}(\Lambda) + \mathcal{O}\left(\frac{1}{\Lambda^{10}}\, \log\Lambda \right) \end{aligned}$$

Reproduces Einstein’s equations in Fefferman-Graham gauge with $r = \Lambda^{-1}$!

We will call such RG flow as **highly efficient RG flow**.

Highly Efficient RG Flow: An RG flow where the effective Ward identities for conservation of energy, momentum and global charges take the same form at each scale, but in redefined backgrounds which absorb the contributions of the multi-trace operators.

Corresponding to any gauge-fixing of classical gravity equations, there is a highly efficient RG flow.

Other than FG gauge, variables parametrising the state (gravity solution) explicitly appear in RG flow equation — however these disappear by redefining the scale, coordinates and the em-tensor.

Note not any highly efficient RG flow will NOT reproduce classical gravity equations. We need more conditions.

Second Principle

The RG flow should have a special **lifted Weyl symmetry** which reduces to Weyl symmetry in the UV. This also carries *complete information of the gauge fixing* in the dual gravity equations.

Example (Penrose-Brown-Henneaux) for the FG gauge!

$$\Lambda = \tilde{\Lambda} \cdot (1 - \delta\sigma(\tilde{x}))\,,$$

$$x^\mu = \tilde{x}^\mu + \chi^\mu(\tilde{x}, \tilde{\Lambda}), \quad \text{with} \quad \chi^\mu(\tilde{x}, \tilde{\Lambda}) = \int_{\tilde{\Lambda}}^{\infty} \frac{\mathrm{d}\hat{\Lambda}}{\hat{\Lambda}^3} \cdot g^{\mu\nu}(\hat{\Lambda}, \tilde{x}) \cdot \frac{\partial}{\partial \tilde{x}^\nu} \delta\sigma(\tilde{x})$$

transformation of g is a shift in scale + Weyl + a d-diffeomorphism

$$\tilde{g}_{\mu\nu}(\tilde{\Lambda}, \tilde{x}) = g_{\mu\nu}(\tilde{\Lambda}, \tilde{x}) - \tilde{\Lambda} \cdot \delta\sigma(\tilde{x}) \cdot \frac{\partial g_{\mu\nu}(\tilde{\Lambda}, \tilde{x})}{\partial \tilde{\Lambda}} + 2 \cdot \delta\sigma(\tilde{x}) \cdot g_{\mu\nu}(\tilde{\Lambda}, \tilde{x}) + \mathcal{L}_\chi g_{\mu\nu}(\tilde{\Lambda}, \tilde{x})$$

Above should be the symmetry of the RG flow corresponding to *any* classical gravity in FG gauge. It determines the transformation of $t^\mu{}_\nu(\Lambda)$ but this depends on dual classical gravity.

Consider the highly efficient RG Flow in the conformally flat background metric

$\eta_{\mu\nu} e^{2\sigma(x)}$:

$$
\begin{aligned}
\frac{\partial}{\partial\Lambda} t^{\mu}{}_{\nu}(\Lambda) = \qquad & -\frac{1}{\Lambda^3}\Big(2 t^{\mu}_{(2)\nu}{}^{*} + \\
& + \frac{1}{2}\Big(- e^{-2\sigma} \eta^{\alpha\beta}\, \nabla^{(\sigma)}_{\alpha} \nabla^{(\sigma)}_{\beta} \, \Big(t^{\mu}{}_{\nu}(\Lambda) - t^{\mu}_{(0)\nu}{}^{*} \Big) + \\
& + 4 \Big(t^{\mu}{}_{\beta}(\Lambda) - t^{\mu}_{(0)\beta}{}^{*} \Big)\, e^{-2\sigma} \eta^{\beta\gamma}\, g_{(2)\gamma\nu} + \\
& + 6\, \delta^{\mu}{}_{\nu} \, \Big(t^{\gamma}{}_{\beta}(\Lambda) - t^{\gamma}_{(0)\beta}{}^{*} \Big)\, e^{-2\sigma} \eta^{\beta\delta}\, g_{(2)\gamma\delta} \Big) + \\
& + t^{\mu}_{(2a)\nu} \Big) + 2 \frac{1}{\Lambda^3} \, \log\,\Lambda \, t^{\mu}_{(2a)\nu} + \mathcal{O}\left(\frac{1}{\Lambda^5} \log\,\Lambda \right)
\end{aligned}
$$

where:

$$
g_{(2)\mu\nu} = -(\partial_\mu \sigma)(\partial_\nu \sigma) + \partial_\mu \partial_\nu \sigma + \frac{1}{2} \eta_{\mu\nu} \eta^{\alpha\beta} (\partial_\alpha \sigma)(\partial_\beta \sigma), \quad \text{etc.}
$$

This is invariant under the stated transformations.

When the highly efficient RG flow maps to classical gravity equations, there is a unique “g”.

Transformation of “g” gives the transformation of “t” which is:

$$
\begin{aligned}
\tilde{t}^{\mu}{}_{\nu} = \quad & t^{\mu}{}_{\nu} + \Bigg(\frac{1}{\tilde{\Lambda}^2} e^{-2\sigma}\,\delta\sigma\,\frac{1}{2}\Big(-\eta^{\alpha\beta}\,\nabla^{(\sigma)}_{\alpha}\nabla^{(\sigma)}_{\beta} t^{\mu}{}_{\nu} + 4t^{\mu}{}_{\beta}\,\eta^{\beta\gamma}\,g_{(2)\gamma\nu} + 6\delta^{\mu}{}_{\nu}\,t^{\gamma}{}_{\beta}\,\eta^{\beta\delta}\,g_{(2)\gamma\delta}\Big) \\
& + 4\,\delta\sigma\,t^{\mu}{}_{\nu} - \frac{1}{2}\frac{1}{\tilde{\Lambda}^2}\mathcal{L}_{e^{-2\sigma}\eta^{\alpha\beta}\partial_{\beta}\delta\sigma}\; t^{\mu}{}_{\nu} + \\
& + \frac{e^{-2\sigma}}{2\tilde{\Lambda}^2}\Bigg(\eta^{\mu\beta}\partial_{\beta} t^{\alpha}{}_{\nu}\,\partial_{\alpha}\delta\sigma + \partial_{\nu} t^{\mu}{}_{\alpha}\,\partial_{\beta}\delta\sigma\,\eta^{\alpha\beta} + t^{\mu}{}_{\alpha}\,\partial_{\nu}\partial_{\beta}\delta\sigma\,\eta^{\alpha\beta} + t^{\alpha}{}_{\nu}\,\partial_{\beta}\partial_{\alpha}\delta\sigma\,\eta^{\beta\mu} - \\
& - 2\partial_{\alpha} t^{\mu}{}_{\nu}\,\partial_{\beta}\delta\sigma\,\eta^{\alpha\beta} - t^{\mu}{}_{\nu}\,\partial_{\alpha}\partial_{\beta}\delta\sigma\,\eta^{\alpha\beta} - \delta^{\mu}{}_{\nu} t^{\alpha}{}_{\beta}\,\partial_{\alpha}\partial_{\gamma}\delta\sigma\,\eta^{\beta\gamma}\Bigg) + \\
& + \mathcal{O}\left(\frac{1}{\tilde{\Lambda}^4}\right)\Bigg) + \mathcal{O}(\partial^3)
\end{aligned}
$$

What is the **lifted Weyl symmetry** for the RG flow corresponding to other gauges?

Consider an arbitrary gauge infinitesimally away from FG gauge. This is related to FG gauge by unique transformations $\mathcal{G}$

$$\Lambda = \tilde{\Lambda} - \rho(\tilde{\Lambda}, \tilde{x}) \cdot \tilde{\Lambda}^2, \quad x^\mu = \tilde{x}^\mu + \chi^\mu(\tilde{\Lambda}, \tilde{x}).$$

The transformation of “g” which determines transformation of “t” is again shift in scale + Weyl + d-diffeomorphism

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \rho\,\tilde{\Lambda}^2\,\frac{\partial g_{\mu\nu}}{\partial\tilde{\Lambda}} - 2\,\rho\,\tilde{\Lambda}\,g_{\mu\nu} + \mathcal{L}_\chi g_{\mu\nu}$$

The lifted Weyl symmetry corresponding to the new gauge is: $\mathcal{G}\mathcal{P}^{\delta\sigma}_{\mathrm{PBH}}\mathcal{G}^{-1}$

In this form it is independent of the dual classical gravity. Indeed it carries complete information about the gauge fixing in dual classical gravity equations.

Third Principle

What dual classical gravity theory should the RG flow in the specific field theory correspond to?

It is determined by our 3rd principle: **the RG flow should end at a sensible infrared fixed point (after necessary re-scalings)**.

In the hydrodynamic limit, this is the **incompressible non-relativistic Navier Stokes fluid at the scale of the temperature (horizon)** [S. Kuperstein, AM 2013] after the rescalings:

$$T^{-1} - \Lambda^{-1} = \xi \cdot \lambda^{-1}, \quad t = \frac{\tau}{\xi}, \quad \xi \to 0.$$

The third principle is **equivalent to horizon regularity** [S. Kuperstein, AM 2013].

It also follows from [S. Kuperstein, AM 2013] that to **fix the relation between "g" and "t" uniquely** the third principle of IR regularity of RG flow is required.

Summary

Holographic duality is equivalent to a highly efficient RG flow in a strongly interacting large N field theory which satisfies the three principles:

(i) it **preserves the form of effective Ward identity** for conservation of energy, momentum and global charges in same form but in scale dependent backgrounds,
(ii) it has a **special kind of symmetry** which reduces to Weyl symmetry in the UV and is a simple combination of a shift in scale +Weyl transformation+d-diffeomorphism for the effective background metric “g” and sources at any scale, and
(iii) it has a **sensible IR limit**.

Claim : The above principles are not not only necessary but also sufficient to imply the holographic duality

Part 2: The invertible map to gravity

ADM variables

In any gauge, the (d+1)-spacetime geometry dual to the QFT state is:

$$\mathrm{d}s^2 = \alpha(r,x)\mathrm{d}r^2 + \gamma_{\mu\nu}(r,x)\left(\mathrm{d}x^\mu + \beta^\mu(x,r)\mathrm{d}r\right)\left(\mathrm{d}x^\nu + \beta^\nu(r,x)\mathrm{d}r\right).$$

Momentum constraints of (d+1)-gravity $\quad \nabla_{(\gamma)\mu} T^\mu{}_\nu{}^{\mathrm{ql}} = 0$

As for example in Einstein's gravity:

$$T^\mu{}_\nu{}^{\mathrm{ql}} = -\frac{1}{8\pi G_N}\,\gamma^{\mu\rho}\left(K_{\rho\nu} - K\gamma_{\rho\nu}\right)$$

So we expect: $\quad \gamma_{\mu\nu} \approx g_{\mu\nu}, \quad T^\mu{}_\nu{}^{\mathrm{ql}} \approx t^\mu{}_\nu$

Some simple requirements fix this uniquely.

Disappearance of “l”

The AdS radius “l” has no direct meaning in the dual QFT.

It can only appear in the relations between ADM variables and “g” via

$$N^2 \approx \frac{l^{d-1}}{16\pi G_N}$$ and

$$\frac{\alpha'_{(i)}}{l^2} \approx f\left(g_{(A)}\right), \quad \text{such that } f \to 0 \text{ as } g_{(A)} \to \infty.$$

$\alpha'^{2n}_{(i)}$ gives higher derivative corrections to Einstein's gravity

$g_{(A)}$s are couplings in the CFT

Dimensional analysis gives and gauge independence gives: $r = \Lambda^{-1}$

Obtaining “g”

Mapping to RG flow requires that the map between ADM variables and “g” is independent of the state (gravity solution) and also takes the same form in each gauge.

The first principle requires

$$\nabla_{(\gamma)} = \nabla_{(g)}$$

which implies $$g_{\mu\nu} = f\left(\frac{r}{l}\right) \cdot \gamma_{\mu\nu}$$

Furthermore, $$g_{\mu\nu}(\Lambda = \infty) = \eta_{\mu\nu}$$

and “l” does not appear n “g” explicitly. Finally:

$$\boxed{g_{\mu\nu} = \frac{r^2}{l^2} \cdot \gamma_{\mu\nu}}$$

Obtaining “t”

Mapping to RG flow requires that the map between ADM variables and “t” is independent of the state (gravity solution) and also takes the same form in each gauge.

The first principle requires

$$t^{\mu}{}_{\nu} = h\left(\frac{r}{l}\right) \cdot \left(T^{\mu}{}_{\nu}{}^{\mathrm{ql}} + T^{\mu}{}_{\nu}{}^{\mathrm{ct}}\right)$$

$$T^{\mu}{}_{\nu}{}^{\mathrm{ct}} = \; -\frac{1}{8\pi G_N}\left[C_{(0)}\left(\frac{r}{l},\frac{\alpha'_{(i)}}{l^2}\right)\cdot\frac{1}{l}\cdot\delta^{\mu}{}_{\nu} + C_{(2)}\left(\frac{r}{l},\frac{\alpha'_{(i)}}{l^2}\right)\cdot l\cdot\left(R^{\mu}{}_{\nu}[\gamma] - \frac{1}{2}R[\gamma]\delta^{\mu}{}_{\nu}\right) + \right. \\ \left. + \cdots\right]$$

Requiring that “l” does not appear explicitly implies

$$\boxed{h\left(\frac{r}{l}\right) = \left(\frac{l}{r}\right)^{d}, \quad C_{(i)}\left(\frac{r}{l},\frac{\alpha'_{(i)}}{l^2}\right) = C_{(i)}\left(\frac{\alpha'_{(i)}}{l^2}\right) \text{ and thus independent of } \frac{r}{l}.}$$

The counterterm coefficients which are numerical constants are determined uniquely by the third principle, namely requirement of good IR fixed point for RG flow [S Kuperstein, AM 2013].

Note we do not need to impose UV finiteness as in traditional holographic renormalisation — rather this follows from demanding a sensible IR fixed point.

This is crucial because the RG flow is fundamentally first order, so if it can reconstruct holography then IR condition should be sufficient!

How to invert?

Let us see this in FG gauge first.

Define $$z^{\mu}{}_{\nu} = g^{\mu\rho} \cdot \frac{\partial g_{\rho\nu}}{\partial r}$$

Then:

$$t^{\mu}{}_{\nu} = \quad \frac{l^{d-1}}{16\pi G_N}\left[\frac{1}{r^{d-1}} \cdot \left(z^{\mu}{}_{\nu} - (\mathrm{Tr}\, z)\, \delta^{\mu}{}_{\nu}\right) + \frac{2}{d-2} \cdot \frac{1}{r^{d-2}} \cdot C_{(2)} \cdot \left(R^{\mu}{}_{\nu}[g] - \frac{1}{2}R[g]\delta^{\mu}{}_{\nu}\right) + \cdots\right]$$

Inverting we get

$$z^{\mu}{}_{\nu} = r^{d-1} \cdot \frac{16\pi G_N}{l^{d-1}} \cdot \left(t^{\mu}{}_{\nu} - \frac{\mathrm{Tr}\, t}{d-1}\delta^{\mu}{}_{\nu}\right) - \frac{2r}{d-2}\left(R^{\mu}{}_{\nu} - \frac{R}{2(d-1)}\delta^{\mu}{}_{\nu}\right) + ... + \cdots\Big]$$

The general UV expansion for “t” is

$$
\begin{aligned}
t^{\mu}{}_{\nu} = \quad & t^{\mu}{}_{\nu}{}^{\infty} + b_2\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot\frac{1}{\Lambda^2}\cdot\Box t^{\mu}{}_{\nu}{}^{\infty} + \\
& + \frac{1}{\Lambda^4}\cdot\Bigg(b_{4a}\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot t^{\mu}{}_{\rho}{}^{\infty} t^{\rho}{}_{\nu}{}^{\infty} + b_{4b}\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot\delta^{\mu}{}_{\nu}\cdot t^{\alpha}{}_{\beta}{}^{\infty} t^{\beta}{}_{\alpha}{}^{\infty} + \\
& + b_{4c}\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot\Box^2 t^{\mu}{}_{\nu}{}^{\infty}\Bigg) + \\
& + \frac{1}{\Lambda^4}\,\log\Lambda\cdot\tilde{b}_{4c}\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot\Box^2 t^{\mu}{}_{\nu}{}^{\infty} + \mathcal{O}\left(\frac{1}{\Lambda^6}\log\Lambda\right)
\end{aligned}
$$

Solve all coefficients by substituting this in the highly efficient RG flow equation.

The general UV expansion for “g” is

$$
\begin{aligned}
g_{\mu\nu} = \quad & \eta_{\mu\nu} + c_4\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot\frac{1}{\Lambda^4}\cdot\eta_{\mu\alpha}{t^{\alpha}}_{\nu}{}^{\infty} + c_6\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot\frac{1}{\Lambda^6}\cdot\eta_{\mu\alpha}\Box{t^{\alpha}}_{\nu}{}^{\infty} + \\
& + \frac{1}{\Lambda^8}\cdot\left(c_{8a}\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot\eta_{\mu\alpha}{t^{\alpha}}_{\rho}{}^{\infty}{t^{\rho}}_{\nu}{}^{\infty} + c_{8b}\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot\eta_{\mu\nu}\cdot{t^{\alpha}}_{\beta}{}^{\infty}{t^{\beta}}_{\alpha}{}^{\infty} + \right. \\
& \left. + c_{8c}\left(\frac{\alpha_{(i)}}{l^2}\right)\cdot\eta_{\mu\alpha}\Box^2{t^{\alpha}}_{\nu}{}^{\infty}\right) + \\
& + \mathcal{O}\left(\frac{1}{\Lambda^{10}}\right)
\end{aligned}
$$

Substitute it in inverted z-t relation to solve for the coefficients. Has a unique solution as in the UV, “g” should be Minkowki space.

This is the FG expansion of “g” as obtained from gravity equations!

Part 3: Reconstructing Fluid-Gravity

RG flow of fluid (?)

The fluid/gravity correspondence gives the **exact asymptotic hydrodynamic series expansion** of the field-theory dynamics near thermal equilibrium.

This has a Borel pole [Heller, Janik, Witaszczyk 2013].

It is exactly like the case of “normal solutions” of Boltzmann equation at weak coupling.

As with any asymptotic perturbative series, we can try to consider an improved series with different coefficients, but reproduces the same result at the appropriate scale with fewer coefficients.

This is the RG flow of the fluid that is captured by gravity.

In the infrared we get incompressible non-relativistic Navier-Stokes as expected.

Recapitulation

Landau-Lifshitz definitions of hydrodynamic variables:

$$\boxed{\begin{array}{c} t^{\mu}{}_{\nu} u^{\nu} = -\epsilon(T) u^{\mu}, \quad \text{with } u^{\mu} g_{\mu\nu} u^{\nu} = -1 \text{ and} \\ \epsilon(T) \text{ determines } T \text{ locally via equation of state.} \end{array}}$$

"t" can be parametrised in terms of these hydro variables:

$$\boxed{\begin{aligned} \langle t_{\mu\nu} \rangle = \quad & \epsilon(\mathrm{T}) u_{\mu} u_{\nu} + P(\mathrm{T}) \Delta_{\mu\nu} - \zeta(\mathrm{T}) (\nabla \cdot u) \Delta_{\mu\nu} - \eta(\mathrm{T}) \sigma_{\mu\nu} \\ & + \sum_{n=2}^{\infty} \sum_{m=1}^{n_s} \gamma_s^{(n,m)}(\mathrm{T})\, S^{(n,m)} \Delta_{\mu\nu} + \sum_{n=2}^{\infty} \sum_{m=1}^{n_t} \gamma_t^{(n,m)}(\mathrm{T})\, T_{\mu\nu}^{(n,m)} \end{aligned}}$$

Hydrodynamic equations follow from:

$$\boxed{\nabla_{\mu} \langle t^{\mu}{}_{\nu} \rangle = 0.}$$

Generalised coarse-graining

The most general coarse-graining compatible with hydrodynamic limit (up to overall cut-off factor) is:

$$\boxed{\begin{aligned} u^\mu(\Lambda,x) &= \int \mathrm{d}^d k\ e^{ik\cdot x}\ \Theta\left(1-\frac{k^2}{\Lambda^2}\right)\Bigg[v^{(0)}\left(\frac{T^\infty(x)}{\Lambda}\right)\delta^\mu_{\ \nu} + i v^{(1)}_{\mathrm{v}}\left(\frac{T^\infty(x)}{\Lambda}\right)u^\alpha(\Lambda,x)\frac{k_\alpha}{T^\infty(x)}\delta^\mu_{\ \nu} \\ &\quad + i v^{(1)}_{\mathrm{s}}\left(\frac{T^\infty(x)}{\Lambda}\right)u^\mu(\Lambda,x)\frac{k_\nu}{T^\infty(x)} + \mathcal{O}\left(\frac{k^2}{T^\infty(x)^2}\right)\Bigg]\tilde{u}^{\nu\infty}(k)\,, \\ T(\Lambda,x) &= \int \mathrm{d}^d k\ e^{ik\cdot x}\ \Theta\left(1-\frac{k^2}{\Lambda^2}\right)\Bigg[w^{(0)}\left(\frac{T^\infty(x)}{\Lambda}\right) + i w^{(1)}_{\mathrm{s}}\left(\frac{T^\infty(x)}{\Lambda}\right)u^\alpha(\Lambda,x)\frac{k_\alpha}{T^\infty(x)} \\ &\quad + \mathcal{O}\left(\frac{k^2}{T^\infty(x)^2}\right)\Bigg]\tilde{T}^\infty(k) \end{aligned}}$$

Highly efficient RG flow

We require:

(i) $u^\mu(\Lambda)$ and $T(\Lambda)$ follow standard hydro equations in some background $g_{\mu\nu}(\Lambda)$ with Λ dependent transport coefficients.

(ii) $t^\mu{}_\nu(\Lambda)$ follows an RG flow equation that depends on only $t^\mu{}_\nu(\Lambda)$ and Λ explicitly (up to possible redefinitions).

(iii) the lifted Weyl symmetry exists.

(iv) the RG flow ends at incompressible NR Navier-Stokes fixed point.

The flow of hydro variables and transport coefficients has complete information about “g” and hence the gravity equations [S. Kuperstein AM 2013].

We have found the solution for “coarse-graining parameters” and the transport coefficients which reproduce Einstein’s equations.

Question: Is our solution the unique one that satisfies all the four principles? If yes, we have completed the reconstruction of holography as a RG flow when it reduces to the the fluid/gravity correspondence.

Outlook

Prove sufficiency: Requires study of Osborn consistency conditions and showing that it reproduces Dirac's surface deformation algebra.

Beyond pure gravity: There is a simple generalisation of the first two principles with a bosonic operator. Sketchily,

$$\nabla_{(\Lambda)\mu} t^{\mu}{}_{\nu}(\Lambda) = O(\Lambda)\partial_\mu J(\Lambda)$$

$$\tilde{J} = J - \delta\sigma\,\tilde{\Lambda}\,\frac{\partial}{\partial\tilde{\Lambda}} J + (d-\Delta)\,\delta\sigma\,J + \mathcal{L}_\chi J$$

Does the quantum effective action [Kiritsis, Li, Nitti '14] reproduce the J?

Quantum gravity and holography: Does extension of our approach to finite N solve some recent paradoxes of the holographic duality [Almheiri, Dong, Harlow '14, AMPS, …]?

Entanglement renormalization: Does our three principles follow naturally from efficient ways of coarse-graining quantum information?

Semi-holographic framework: Can we derive a semi-holographic framework which generalises holographic duality and gives a broad general framework for many body quantum systems with strongly interacting degrees of freedom?

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