Comments on AdS Compactification

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Aspects of Conformal Frontiers at Thessaloniki (CFT)

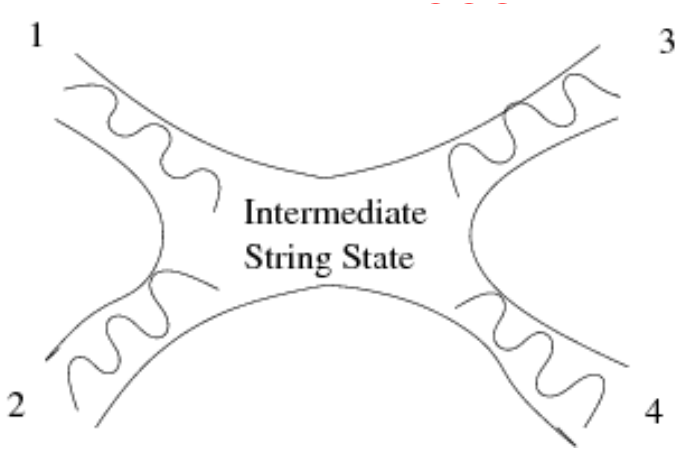
Issue

In AdS Spacetime,
How to make Kaluza-Klein compactification?
What to do with boundary conditions?

Seungho Gwak, Jaewon Kim, SJR, 1510.nnnn

Motivation (1)

- exchange of massive string states to spin ≤ 2 processes $\rightarrow$ Veneziano, Virasoro-Shapiro amplitudes
- exchange of massive higher spins to spin ≤ 2 processes $\rightarrow$ Maldacena et.al



- (massive HS)-(massless)-(massless) cubic interaction
- causality of spin≤ 2 processes in flat and AdS space
- a consistent theory of massive + massless HS in AdS and its flat space limit

Motivation (2)

- non-unitary AdS/CFT: non-Hermitian action

$$I_{\rm CFT} = \int d^d x \phi \cdot \Box^\ell \phi, \qquad [\sigma] := [\phi^2] = d - 2\ell$$

  or non-local action

$$I_{\rm CFT} = \int d^d x \int d^d y \frac{\phi(x) \cdot \phi(y)}{|x-y|^{2\ell}}, \qquad [\sigma] := [\phi^2] = 2d - 2\ell$$

- can non-unitary AdS/CFT embedded inside unitary?
- Recall: intersecting conformal branes, defect branes within conformal branes, twisted sector of quiver gauge theories, anomaly inflows, etc.
- Recall: logarithmic CFTs as a limit of unitary CFTs, open systems, large-N limit of coupled CFTs

Ideas

- consider a unitary, massless HS theory in $\mathrm{AdS}_{\mathbf{D}}$
- compactify on $\mathbb{M}^{D-d}$; keep all KK + light modes
- get massive+massless (non)unitary HS theory in $\mathrm{AdS}_{\mathbf{d}}$

$$\boxed{AdS_D \quad \rightarrow \quad AdS_d \times \mathbb{M}^{D-d}, \qquad \partial M^{D-d} \neq 0}$$

- ex1. AdS_{D-1} hypersurface foliation of AdS_D
- ex2. Janus geometry $\mathbb{S}^1/\mathbb{Z}_2$
- ex3. charged BH in AdS_4
- previous attempt: spin-0

How many distinct boundary conditions?

Are they all self-adjoint?

If not, what are their roles?

Main Results

- AdS compactification on $\mathbb{S}_+/\Gamma$
- Classification of admissible boundary conditions
  $2s+1$ boundary condition; $s-1$ are AdS intrinsic
- Boundary conditions fix gauge invariance and spectra
- HS Spectra:
  KK massive via Stueckelberg $\oplus$
  AdS massive via Stueckelberg, massless, partially massless
- spin-$s \to$ massless spin-s plus massive spin-$(s-2)$ towers of $M^2 = 3[2(s-2)+d-2]$
- Quadratic action of massive+massless HS completely fixed
- Casimir energy: agreement with dual CFT
- Interactions in progress

important previous works: Metsaev, Zinoviev, Buchbinder

Massive HS fields in AdS_3

- massive HS fields in AdS_3 has 2 propagating DOF

$$\phi_s = \begin{pmatrix} u_s \\ d_s \end{pmatrix}$$

- equation of motion for massive HS state

$$\boxed{\partial_z^2 + \Box_{d-1} - \frac{1}{z^2}\mathfrak{M}_s^2 = 0}$$

- $\mathfrak{M}_s^2$ = mass-squared operator

$$\mathfrak{M}_s^2 = (E_0 - 1 - s\sigma_3)^2 - \frac{1}{4}$$

- unitary representations: $E_0 - 1 \geq s$
- massive $s = 0$ scalar field:

$$\boxed{\mathfrak{M}_0^2 = (mR)^2 + \frac{d(d-2)}{4}}$$

AdS Compactification

What's Wrong with translational compactification?

- translational compactification

$$AdS_4 \to AdS_3 \times \mathbb{R}^1/\mathbb{Z}$$

$$ds^2 = \frac{1}{z^2}(dz^2 - dx_0^2 + dx_1^2 + dy^2), \qquad y \simeq y + 2\pi\mathbb{Z}$$

- AdS_4 massless scalar:

$$[(\Box_2 + \partial_z^2) + \partial_y^2]\Phi(x, z, y)$$

- Fourier expand $\Phi = \sum_n e^{iny}\phi_n(x, z)$

$$[(\Box_2 + \partial_z^2) - n^2]\phi_n(x, z) = 0$$

- WRONG – nothing like massive AdS scalar
- non-tensorial: $\nabla_m h_{ny} \to \nabla_m A_n + \delta_{mz}\frac{1}{2}A_n$

Well-behaved rotational compactification

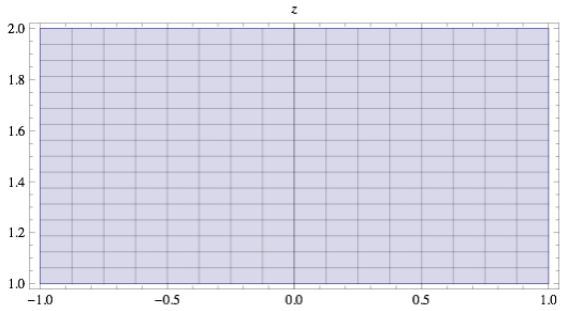
- rotational compactification

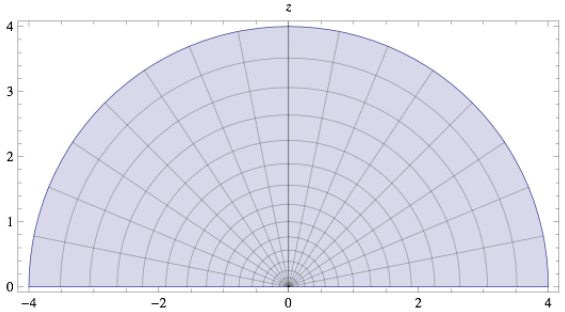
$$\boxed{AdS_4 \to AdS_3 \times_{\text{warped}} \mathbb{S}^1/\Gamma}$$

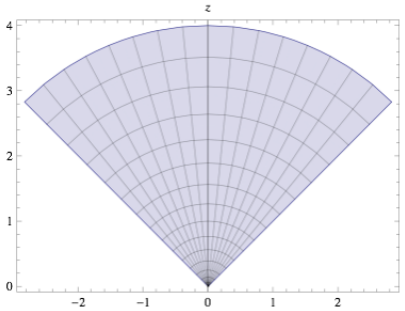
- semi-circle: $z = \rho\cos\theta, y = \rho\sin\theta$

$$\begin{aligned} ds_4^2 &= \frac{1}{z^2}[-dt^2 + dz^2 + d\mathbf{x}_3^2] \\ &= \frac{1}{\rho^2\cos^2\theta}[-dt^2 + d\mathbf{x}_1^2 + d\rho^2 + \rho^2 d\theta^2] \\ &= \frac{1}{\cos^2\theta}[ds_3^2 + d\theta^2] \end{aligned}$$

- tensorial: $\nabla_m h_{n\theta} \to \nabla_m A_n - \tan\theta\,(h_{mn} - g_{mn}\phi)$







conformal diagram of

1. translational compactification
2. rotational compactification
3. Janus Γ

$\Gamma =$ Janus Wedge

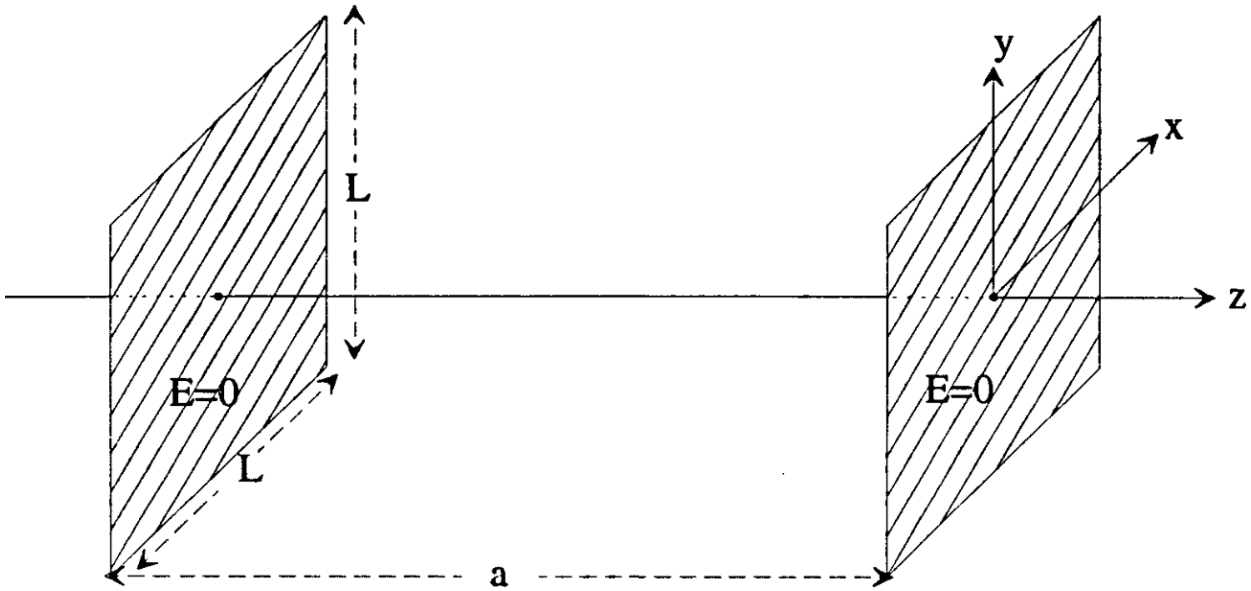
- Introduce "compactification" scale
- $\Gamma =$ Janus interval: $\theta = [-\pi/2, \pi/2] \quad \rightarrow \quad [-\alpha, +\alpha]$
- 2 parameters: R_{AdS}, α
- boundary conditions at $\theta = \pm\alpha$
- KK mass tower operator

$$\boxed{L_m = \partial_\theta + m \tan\theta}, \qquad m = \mathbb{Z}$$

Casimir as Toy Example

free Maxwell theory on a capacitor (x^m, z)

$$\partial^M F_{Mn} = \partial^m F_{mn} + \partial_z^2 A_n - \partial_z \partial_n \phi = 0$$
$$\partial^M F_{Mz} = \partial^2 \phi - \partial^m \partial_z A_m = 0$$



boundary conditions

- Neumann A_m + Dirichlet ϕ

$$\sum_{k=0}^{\infty} \cos(k\pi z/L) \left(\partial^m F_{mn}^{(k)} - \frac{k^2\pi^2}{L^2} A_n^{(k)} - \frac{k\pi}{L} \partial_n \phi \right) = 0$$

$$\sum_{k=1}^{\infty} \sin(k\pi z/L) \left(\partial^m \partial_m \phi + \frac{k\pi}{L} \partial^m A_m \right) = 0$$

  $k = 0$: massless spin-1
  $k \neq 0$: massive Stuckelberg spin-1
  $L \to 0$: massless spin-1 (no massless spin-0)

- Dirichlet A_m + Neumann ϕ
  $L \to 0$: massless spin-0 (no massless spin-1)

- Dirichet/Neumann A_m and ϕ
  $L \to 0$: no massless mode survives

D3 ending on D5/NS5

- a D3 brane ending on D5-brane or NS5-brane
- preserves 1/2 of supersymmetry
- bulk = 4d N=4 vector multiplet = 3d N=4 vector + N=4 hyper
- D5 boundary: 3d N=4 hyper as massless modes (D for vector, N for hyper)
- NS5 boundary: 3d N=4 vector as massless modes (D for hyper, N for vector)
- fits well with IIB $SL(2, Z)$ S-duality and 3d mirror symmetry exchanging vector and hyper

AdS Compactification of Spin-1

- equations of motion

$$\nabla^m F_{mn} - L_{d-3} L_0 A_m + L_{d-3} \partial_m \phi = 0$$
$$-\Box \phi + L_0 \nabla^m A_m = 0$$

- mode functions for A_m, ϕ are related each other:

$$L_{d-3} \Theta_n^\phi = \sum_m c_{nm}^{\phi A} \Theta_m^A; \qquad L_0 \Theta_n^A = \sum_m c_{nm}^{A\phi} \Theta_m^\phi$$

- locality: $c_{mn} = 0$ for $m \neq n$

$$L_{d-3} \Theta_n^\phi = c_n^{\phi A} \Theta_n^A, \qquad L_0 \Theta_n^A = c_n^{A\phi} \Theta_n^\phi$$

- eigenvalue equations for mode functions

$$\boxed{L_{d-3} L_0 \Theta_n^A = (c_n^{A\phi} c_n^{\phi A}) \Theta_n^A, \quad L_0 L_{d-3} \Theta_n^\phi = (c_n^{A\phi} c_n^{\phi A}) \Theta_n^\phi}$$

Spin-1 Cont'd

- equations of motion

$$\sum_k (\nabla^m F^k_{mn} + c^{A\phi}_k c^{\phi A}_k A^k_n - c^{\phi A}_k \partial_n \phi^k)\Theta^A_k = 0$$

$$\sum_k (\Box \phi^k - c^{A\phi}_k \nabla^m A^k_m)\Theta^\phi_k = 0$$

- Are these compatible with gauge invariance?

$$\begin{aligned} & \delta A_m = \partial_m \Lambda, \qquad \delta\phi = L_0 \Lambda \\ \rightarrow \quad & \delta A^k_m = \partial_m \Lambda^k, \qquad \delta\phi^k = c^{A\phi}_k \Lambda^k \end{aligned}$$

- Stuckelberg massive spin-1 fields with identification

$$\boxed{c^{A\phi}_k = -c^{\phi A}_k = M_k \quad \text{(to be determined)}}$$

Spin-1 Boundary Conditions

- consistent boundary conditions subject to relations between Θ^A, Θ^ϕ
- KK towers $\to \infty$ as $\alpha \to 0$
- light (massless and AdS massive) towers finite as $\alpha \to 0$
- higher-∂ boundary conditions for only Stuckelberg modes (beware)
- consistent boundary conditions

$$
\begin{aligned}
1: &\quad L_0 A_m(\pm\alpha) = 0, \qquad \phi(\pm\alpha) = 0 \\
2: &\quad A_m(\pm\alpha) = 0, \qquad L_{d-3}\phi(\pm\alpha) = 0 \\
3^*: &\quad L_{d-2} A_m(\pm\alpha) = 0, \qquad L_{d-2} L_{d-3}\phi(\pm\alpha) = 0
\end{aligned}
$$

- BC1: spin-1 = massless, spin-0 = none
- BC2: spin-1 = none, spin-0 = massless
- BC3: spin-1 = massive, spin-0 = Stueckelberg

$$M_0^2 = d - 2$$

AdS Compactification of Spin-2

equations of motion

$$K_{mn}(h) - (d-1)(2h_{mn} - g_{mn}h) - L_{d-3}L_{-2}(h_{mn} - g_{mn}h)$$
$$-L_{d-3}(\nabla_m A_n + \nabla_n A_m - 2g_{mn}\nabla \cdot A) + \frac{d-1}{d-2} g_{mn} L_{d-3} L_{d-4}\phi = 0$$
$$\nabla^m F_{mn} - 2(d-1)A_n - L_{-2}(\nabla^m h_{mn} - \nabla_n h) - \frac{d-1}{d-2} L_{d-4}\nabla_n \phi = 0$$
$$\Box\phi - d(\frac{1}{d-2} L{-}1 L_{d-4} + 1)\phi - 2L_{-1}\nabla^m A_m + L_{-1}L_{-2}h = 0$$

$\Theta_k^{\ell|s}$ = k-th mode function for spin-ℓ from spin-s obeys relations

$$L_{-2}\Theta^{2|2} = c^{21}\Theta^{1|2}$$
$$L_{d-3}\Theta^{1|2} = c^{12}\Theta^{2|2}, \quad L_{-1}\Theta^{1|2} = c^{10}\Theta^{0|2}$$
$$L_{d-4}\Theta^{0|2} = c^{01}\Theta^{1|2}$$

Spectrum

- KK mass spectrum

$$
\begin{aligned}
M^2_{k,3|2} &= M^2_k - d \\
M^2_{k,2|2} &= M^2_k \\
M^2_{k,1|2} &= M^2_k + d - 2 \\
M^2_{k,0|2} &= M^2_k + 2(d-3)
\end{aligned}
$$

- M_n = spin-2 mass, which depends on KK levels
- both KK and finite mass spectrum depends on the choice of boundary conditions
- different boundary conditions are classified by possible relations among $\Theta^{0|2}, \Theta^{1|2}, \Theta^{2|2}$ mode functions

Spin-2 Gauge Transformation

- spin-2 gauge transformation

$$\delta h_{mn} = \nabla_m \xi_n + \tan\theta g_{mn} \xi)_\theta$$
$$\delta A_m = \frac{1}{2}\partial_m \xi_\theta + \frac{1}{2} L_{-2} \xi_m$$
$$\delta \phi = L_{-1} \xi_\theta$$

- from mode expansion

$$\delta^k_{mn} = \nabla_m \xi^k_n + \frac{c^{12}_k}{d-2} g_{mn} \xi^k$$
$$\delta A^k_m = \frac{1}{2}\partial_m \xi^k + \frac{c^{21}_k}{d-2} \xi^k_m$$
$$\delta \phi^k = c^{10}_k \xi^k$$

- possible gauge symmetries are classified by all possible boundary conditions of mode functions

Higher-∂ Boundary Conditions

- mode functions of different spins are related one another, recall

$$L_{-2}\Theta^{2|2} = c^{21}\Theta^{1|2}$$
$$L_{d-3}\Theta^{1|2} = c^{12}\Theta^{2|2}, \quad L_{-1}\Theta^{1|2} = c^{10}\Theta^{0|2}$$
$$L_{d-4}\Theta^{0|2} = c^{01}\Theta^{1|2}$$

- elimination of mode functions to s independent modes lead to higher-∂ boundary conditions
- higher-∂ leads in general to nonvanishing current flows across the boundaries
- for spin-0, absent
- for spin-1, only for Stueckelberg field (gauge mode)
- for spin-s $s \geq 2$, this leads to partially massless modes

After dust settles.....

- equation of motion + gauge transformation yield Stuckelberg massive spin-2 systems

$$\delta h_{mn} = (\nabla_m \xi_n - \frac{M}{d-2}\xi)\theta_h$$
$$\delta A_m = \frac{1}{2}(\partial_m \xi + M\xi_m)\Theta_A$$
$$\delta \phi = \xi \Theta_\phi$$

- details of spectra and gauge symmetry depend on the choice of boundary conditions
- possible boundary conditions of mode functions the same for gauge parameters
- 1 BC with massive spin-2, 1 BC with massive spin-1, 1 BC with massless spin-2, 1 BC with negative mass spin-2
- $s-1$ many new boundary conditions with partially massless spin-2

Spin-2 Boundary Conditions

- BC1: spin-2 = massive, spin-1 = Stueckelberg, spin-0 = Stuckelberg

$$M_2^2 = d, \qquad \Theta^{2|2} = \cos^{d-2}\theta$$

- BC2: spin-2 = none, spin-1 = massive, spin-0 = Stueckelberg

$$M_1^2 = 2(d-1), \qquad \Theta^{1|2} = \cos^{d-2}\theta$$

- BC3: spin-2 = massless, spin-1 = none, spin-0 = massive

$$M_0^2 = d, \qquad \Theta^{2|2} = \sec^2\theta, \;\; \Theta^{0|2} = \cos^{d-4}\theta$$

- BC4: spin-2 = partially massless, spin-1 = Stueckelberg, spin-0 = none

$$M_2^2 = -d+2, \qquad \Theta^{1|2} = \sec\theta$$

- BC5: spin-2 = massive, spin-1 = Stueckelberg, spin-0 = Stueckelberg

$$M_2^2 = -2(d-3), \qquad \Theta^{0|2} = 1$$

AdS Compactification of Spin-s

- precisely the same logic and procedure extends to HS fields
- quadratic part of action is completely fixed by gauge transformations even for massive HS fields
  previous detailed study by Zinoviev
- KK tower states and finite states form different multiplets
- $2s + 1$ many possible boundary conditions for spin-s mode functions
- $s - 1$ many AdS-intrinsic boundary conditions
  $\rightarrow$ partially massless HS fields
- finite states = spin-s partially mass field $\oplus$ massive spin $k - 1$ $(2 \leq k \leq s)$

New Boundary Conditions and Spectrum

- spin-s = partially massless

$$M^2 = -(s-k-1)(s+k+d-4), \qquad (k = 1, \cdots, s-1)$$

- spin $k+1, k+2, \cdots, s-1$ = Stueckelberg of spin-s
- spin-k = none
- spin-$(k-1)$ = massive

$$M^2 = (s-k+2)(s+k+d-5)$$

- BC with massless specctrum:
  massless spin-s plus massive spin-$(s-2)$ of
  $M^2 = 3[2(s-2)+d-2]$
- massive spin-$(s-2)$: decoupled Stueckelberg state as a massive spin-s field becomes partially massless field
  2nd partially massless: $D(sd-3, s)$
  massive spin-$(s-2)$: $D((s-2)+d+1, s-2)$

Summary

- rich possibilities of boundary conditions and spectra
- KK approach accommodates all known massive HS states by different boundary conditions
- $s-1$ many partially massless boundary conditions for spin-s HS field
- quadratic action completely determined by boundary conditions and residual gauge symmetries
- most boundary conditions include partially massless spin-s fields
  dual to Lifshitz $O(N)$ model (Bekaert, Grigoriev)
- computation of Casimir energy for $AdS_4 \to AdS_3 \times \mathbb{S}^1_+/\Gamma$ shows agreement with CFT dual, at least for small α
- KK compactifiation of cubic coupling in progress
- extension to $\mathbb{S}^{D-d}_+/\Gamma$ underway

String Theory Applications

- non-Hermitian AdS/CFT (disordered, dissipative, open)
- ds/CFT correspondence (dS foliation of AdS)
- interpolating supergravity solutions
- Gaiotto-Tomasiello geometry (NS5-D6 intersections)
- AdS wrapped branes and new CFT defect sectors
- Janus geometry
- AdS black holes

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Tassos, You were terrific! Thanks a lot!