One-instanton effective action

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"Adspecfts of Conformal Field Theories"
$\Theta\varepsilon\sigma\sigma\alpha\lambda o\nu\iota'\kappa\eta$

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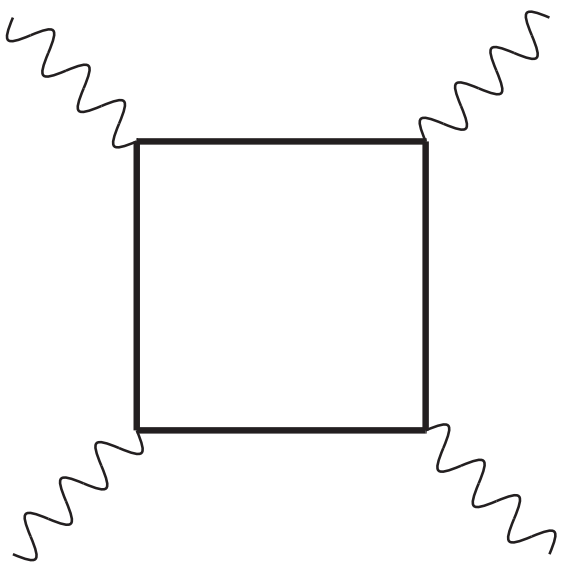
Outline

- Introduction
- $\mathcal{N} = 4$ on-shell super-fields
- One-instanton effective action in $Sp(2N)$, $Sp(2)$ vs $U(2)$
- Non-perturbative F^n and D^4F^n terms in the Coulomb branch
- Perturbative checks: D^4F^4 and D^nF^4 at one- and two-loops, F^n at one-loop and HEA
- Multi-instantons and $SL(2, Z)$ completion
- Conclusions and remarks

Introduction

- $\mathcal{N} = 4$ SYM in $D = 4$: the "harmonic oscillator" of QFT
- No UV divergences $\beta = 0$ (yet $\gamma_{\mathrm{non-BPS}} \neq 0$), exact SCFT
- IR divergences (governed by γ_{cusp} etc), non-local effective action for $\langle \varphi \rangle = 0$ 'unbroken phase'
- In the Coulomb branch $\langle \varphi \rangle = v$, 'local' effective action
- Integrating out massive modes with $m = g_{\mathrm{YM}} v$ produces higher-derivative terms: F^n, $D^m F^n$, ... E.g. at one-loop



Perturbative results

- Leading irrelevant operators, such as F^4, are known to be one-loop exact in $\mathcal{N} = 4$ SYM [Dine, Seiberg (1997)].
- *Mutatis mutandis i.e.* setting $\alpha' = 1/m^2$, coefficient coincides with that in effective action of D3-brane in $\mathrm{AdS}_5 \times \mathrm{S}^5$,

$$-\frac{g_{\mathrm{YM}}^4 N}{2(4\pi)^2 m^4} F^4 \,.$$

- Operator F^6, more precisely $F_-^2 F_+^4$ (MHV!) considered in [Buchbinder, Petrov, Tseytlin (2001)][Kuzenko (2004)]: generated at two loops

$$4\left(-\frac{g_{\mathrm{YM}}^4 N}{2(4\pi)^2 m^4}\right)^2 F^6 \,,$$

  which again agrees precisely with that of DBI.
- Leads to conjecture a non-renormalization theorem: F^6 is two-loop exact! [Buchbinder, Petrov, Tseytlin (2001)]

Non-renormalization theorems

- Due to $\mathcal{N} = 4$ SUSY constraints, the non-renormalization was proven recently [Chen, Huang, Wen (2015)].
- Furthermore, there is a non-renormalization theorem for general MHV operators

$$4^{\ell-1}\left(-\frac{g_{\mathrm{YM}}^4 N}{2(4\pi)^2 m^4}\right)^{\ell}(F_-)^2(F_+)^{2\ell}\,,$$

  which is ℓ-loop exact, coefficient coincides with DBI!
- One-loop contribution to general $(F_-)^{2p}(F_+)^{2q}$ was computed in [Buchbinder, Petrov, Tseytlin (2001)], and the result is different from that of DBI (except for $(F_-)^2(F_+)^2$).
- Operators $D^m F^n$ were not discussed in the literature.

Highly Effective Action (HEA)

- It was proposed recently by Schwarz that the DBI action of a single D3-brane in $\mathrm{AdS}_5 \times S^5$ with one unit of flux is the EHA for $\mathcal{N} = 4$ SYM with U(2) gauge group in the Coulomb branch [Schwarz (2013)].
- MHV sector OK, $SL(2, Z)$ OK
- Yet, disagreement with previous results for non-MHV operators $(F_+)^{2p}(F_-)^{2q}$
- NO higher derivative terms $D^m F^n$ in HEA!!
- As we will see, $(F_+)^{2p}(F_-)^{2q}$ and $D^m F^n$ can receive contribution at any loop (perturbative) as well as from instantons (non-perturbative).

Instantons in $\mathcal{N} = 4$ SYM

Depending on the number of SUSY, instantons can generate different effects

- In $\mathcal{N} = 1$ SYM, non-perturbative superpotentials $\mathcal{W}$, chiral condensates, ... dynamical SUSY breaking [Witten; Affleck, Dine, Seiberg; Novikov, Shifman, Vainstein, Zakharov; Amati, Konishi, Meurice, Rossi, Veneziano; Yankielowicz;...]
- In $\mathcal{N} = 2$ SYM, non-perturbative pre-potentials $\mathcal{F}$, S-W curves, ... localization [Seiberg, Witten; Nekrasov; ...]
- In $\mathcal{N} = 4$ SYM, in the super-conformal phase $\langle \varphi \rangle = 0$ corrections to correlation functions $\mathcal{G}_n$ of CPO's and to anomalous dimensions of non-BPS operators [Witten; Banks, Green; MB, Green, Kovacs, Rossi; Stanev; Dorey, Hollowood, Khoze, Mattis, Vandoren; ...], yet NO corrections to scattering amplitudes and thus to the effective action [MB, Brandhuber, Travaglini, Wen] ... Coulomb branch?

Non-perturbative adspecfts of AdS/CFT

- $\mathcal{N} = 4$ SYM instanton = Type IIB D-instanton a.k.a. D(-1)-brane or E(-1)-brane
- One-instanton moduli space coincides with AdS_5: extra coordinate = size ρ, S^5 from large N (at fixed g^2_{YM})
- Instanton profile coincides with bulk-to-boundary propagator

$$F^2(x) = \frac{\rho^4}{[(x - x_o)^2 + \rho^2]^4} = K_{\Delta=4}(x; z)$$

- Exact fermionic zero-modes $\zeta^A(z) = \rho\eta^A + (x - x_o)\bar{\xi}^A$
- No leading correction to $\gamma_{\mathrm{Konishi}}$ nor to γ_{cusp}

[Witten; Banks, Green; MB, Green, Kovacs, Rossi; ... MB, Brandhuber, Travaglini, Wen]

Instantons from D(-1)-D3 system

- The system of k D(-1)'s and N D3's describes $\mathcal{N} = 4$ SYM with k instantons and (S)U(N) gauge group.

  [Witten (1995)][Douglas (1996)] [Billó,Frau,Pesando,Fucito,Lerda,Liccardo (2002)]

- Lowest modes of open string with at least one end on D(-1)'s describe instanton super-moduli.

- D(-1)-D3 action coincides with the Yang-Mills action in the instanton background.

- Computing string amplitudes on disks with mixed boundary conditions (D3 and D(-1)), one can derive the exact couplings of the super-moduli to the physical on-shell super-fields.

$\mathcal{N} = 4$ on-shell super-fields

No manifestly $\mathcal{N} = 4$ SUSY off-shell
Yet, manifestly $\mathcal{N} = 4$ SUSY on-shell super-fields. Schematically

$$\begin{aligned}
\Phi^{AB} &= \varphi^{AB} + \Theta^A \lambda^B + \Theta^A \Theta^B F^- + \epsilon^{ABCD}[\bar{\Theta}_A \bar{\lambda}_B + \bar{\Theta}_A \bar{\Theta}_B F^+] + \dots \\
\bar{\Lambda}_A &= \bar{\lambda}_A + \Theta^B \partial \bar{\varphi}_{AB} + \epsilon_{ABCD} \Theta^B \Theta^C \partial \lambda^D + \epsilon_{ABCD} \Theta^B \Theta^C \Theta^D \partial F^- \\
\mathcal{F} &= F^+ + \Theta \partial \bar{\lambda} + \Theta^A \Theta^B \partial \partial \bar{\varphi}_{AB} + \varepsilon_{ABCD} \Theta^A \Theta^B \Theta^C \partial \partial \lambda^D \\
&\quad + \varepsilon_{ABCD} \Theta^A \Theta^B \Theta^C \Theta^D \partial \partial F^-
\end{aligned}$$

Set $k_{\alpha\dot{\alpha}} = u_\alpha \bar{u}_{\dot{\alpha}}$ and introduce auxiliary Grassman variables η_A

$$\begin{aligned}
\mathcal{F}_{\dot{\alpha}\dot{\beta}} &= [\bar{u}_{\dot{\alpha}} \bar{u}_{\dot{\beta}}] + \Theta^A_\alpha u^\alpha \bar{u}_{\dot{\alpha}} [\bar{u}_{\dot{\beta}} \eta_A] + \Theta^A_\alpha \Theta^B_\beta u^\alpha \bar{u}_{\dot{\alpha}} u^\beta \bar{u}_{\dot{\beta}} [\eta_A \eta_B] \\
&\quad + \Theta^A_\alpha \Theta^B_\beta \Theta^C_\gamma u^\alpha \bar{u}_{\dot{\alpha}} u^\beta \bar{u}_{\dot{\beta}} [u^\gamma \eta_A \eta_B \eta_C] \\
&\quad + \Theta^A_\alpha \Theta^B_\beta \Theta^C_\gamma \Theta^D_\delta u^\alpha \bar{u}_{\dot{\alpha}} u^\beta \bar{u}_{\dot{\beta}} [u^\gamma u^\delta \eta_A \eta_B \eta_C \eta_D]
\end{aligned}$$

Instanton super-moduli action

- Instanton moduli interactions among themselves and with external fields

$$S_{\rm inst} = \left(\chi_{AB}\,\chi^{AB}\,\delta^v_u + \Phi^{v'}_{ABu}\Phi^{ABv}_{v'}\right)\bar{w}^{u\dot\alpha}w_{v\dot\alpha} + \left(\chi_{AB}\,\delta^v_u + \Phi^v_{ABu}(a,\Theta)\right)\bar{\mu}^{Au}\mu^B_v$$
$$+ \left(D_{\dot\alpha\dot\beta}\,\delta^v_u + \mathcal{F}^v_{\dot\alpha\dot\beta u}(a,\Theta)\right)\bar{w}^{u\dot\alpha}w^{\dot\beta}_v + \left(\tilde\Theta^{\dot\alpha}_A\,\delta^v_u + \Lambda^{\dot\alpha v}_{Au}(a,\Theta)\right)\left(\bar{\mu}^{Au}w_{\dot\alpha v} + \bar{w}^u_{\dot\alpha}\mu^A_v\right)$$

  Φ_{AB}, $\Lambda^{\dot\alpha}_A$ and $\mathcal{F}_{\dot\alpha\dot\beta}$ $\mathcal{N}=4$ on-shell super-fields, rest are moduli and auxiliary fields.
- The effective action generated by one D-instanton is given by the integral over the instanton super-moduli space

$$S^{k=1}_{\rm eff} = c_N\,e^{2\pi i\tau}\int d^4a\,d^8\Theta\,d^6\chi\,d^3D\,d^8\tilde\Theta\,d^{2N}w\,d^{2N}\bar{w}\,d^{4N}\mu\,d^{4N}\bar{\mu}\,e^{-S_{\rm inst}}\,.$$

- Integration over $D_{\dot\alpha\dot\beta}$, $\tilde\Theta^{\dot\alpha}_A$ leads to super ADHM constraints.

One-instanton in the Sp(2N) theory

- Include orientifold planes $\Omega 3^+$ to D(-1)-D3 system: project out $\tilde{\Theta}_{\dot{\alpha}A}, \chi_{AB}$ and $D_{\dot{\alpha}\dot{\beta}}$ (for $k = 1$), identify D3-D(-1) strings with D(-1)-D3 strings

$$\bar{w}^u_{\dot{\alpha}} = w^u_{\dot{\alpha}}\,, \qquad \bar{\mu}^{uA} = \mu^{uA}$$

- The instanton action simplifies greatly

$$S^{k=1}_{\rm eff} = c_N\, e^{2\pi i\tau} \int d^4a\, d^8\Theta\, d^{2N}w\, d^{4N}\mu\, e^{-S_{\rm inst}}\,.$$

with

$$S_{\rm inst} = w^{\dot{\alpha}}\Phi_{AB}\Phi^{AB}\, w_{\dot{\alpha}} + \mu^A\Phi_{AB}\mu^B + w^{\dot{\alpha}}\mathcal{F}_{\dot{\alpha}\dot{\beta}}w^{\dot{\beta}} + \mu^A\,\Lambda^{\dot{\alpha}}_A w_{\dot{\alpha}}\,.$$

One-instanton in the Sp(2N) theory

- Integration over μ and w is Gaussian, and it leads to the final and – let me say – beautiful result

$$S^{k=1,\mathrm{inst}}_{\mathrm{eff},\mathcal{N}=4} = \int \frac{c_N\, e^{2\pi i\tau}\, d^4a\, d^8\Theta\, \mathrm{Pfaff}_{4N\times 4N}\left(\Phi_{Au,Bv}\right)}{\sqrt{\det_{2N\times 2N}\left(\Phi^{AB}\Phi_{AB} + \mathcal{F} + \bar{\Lambda}_A(\Phi^{-1})^{AB}\bar{\Lambda}_B\right)_{\dot{\alpha}u,\dot{\beta}v}}}$$

- Expand the integrand to order Θ^8, get instanton corrections to such higher-dimensional operators as F^n and $D^m F^n$.

Operator F^n

- For the operator F^n, without derivatives, we take

$$\bar{\Lambda} = 0 \qquad \mathcal{F}_{\dot{\alpha}\dot{\beta}} = F^{+}_{\dot{\alpha}\dot{\beta}} \qquad \Phi_{AB} = v_{AB} + \delta\Phi_{AB}$$

with

$$\delta\Phi_{AB} = \frac{1}{2} F^{-}_{\alpha\beta} \Theta^{\alpha}_{A} \Theta^{\beta}_{B}$$

- We find that all the MHV (and anti-MHV) operators $(F_-)^2(F_+)^{2n}$ $((F_+)^2(F_-)^{2n})$ have vanishing coefficients!!! Consistent with non-renormalization theorem.

- Operators $(F_-)^4(F_+)^{2n}$ receive instanton correction, as expected.

Operator D^4F^n

- For the operators D^mF^n, in particular the MHV $D^4(F_-)^2(F_+)^{2p}$ and its SUSY completion, take

$$\bar{\Lambda} = 0 \qquad \Phi_{AB} = v_{AB}$$

($\bar{\Lambda} \neq 0$, $\Phi_{AB} \neq v_{AB}$ contribute to non-MHV!) and

$$\mathcal{F}_{\dot{\alpha}\dot{\beta}} = F^+_{\dot{\alpha}\dot{\beta}} + \Theta^{A\alpha}\partial_{\alpha\dot{\alpha}}\bar{\lambda}_{\dot{\beta}} + \Theta^{A\alpha}\Theta^{B\beta}\partial_{\alpha\dot{\alpha}}\partial_{\beta\dot{\beta}}\bar{\varphi}_{AB}$$
$$+\varepsilon_{ABCD}\Theta^{A\alpha}\Theta^{B\beta}\Theta^{C\gamma}\partial_{\alpha\dot{\alpha}}\partial_{\beta\dot{\beta}}\left[\lambda^D_\gamma + \Theta^{D\delta}F^-_{\gamma\delta}\right]$$

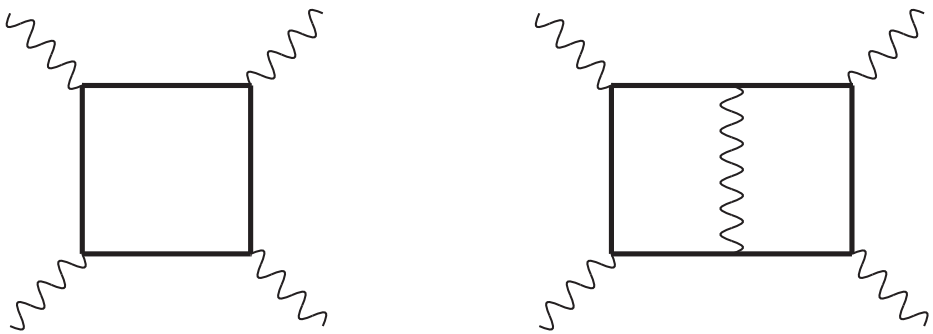
- The one-instanton MHV effective action

$$\begin{aligned} S_{\mathrm{MHV}} &= e^{2\pi i\tau}\int d^8\Theta\,\frac{\mathcal{F}^{2n}}{v^{4n}} \\ &= e^{2\pi i\tau}\,\delta^{(8)}\left(\sum_j \eta^A_j\,u^\alpha_j\right)\sum_{\mathrm{perm}}\frac{\prod_{i=1}^n[2i-1,2i]^2}{v^{4n}} \end{aligned}$$

contains all $D^4(F_-)^2(F_+)^{2p}$ operators

Perturbative results for $D^n F^4$

- One- and two-loop contributions to the operator $D^n F^4$ from



$D^2 F^4 = 0$ due to $s + t + u = 0$. For $D^4 F^4$

$$f_{\rm pert}^{D^4 F^4}(\tau, \bar{\tau}) = -\frac{N}{240(4\pi)^4 v^8} \left(\tau_2^2 + \frac{5N}{2\pi} \tau_2 + \dots \right) ,$$

Unique structure for $D^6 F^4$: $(1/1512, 7/720)$, for $D^8 F^4$: $(1/7560, 47/20160)$, for $D^{10} F^4$: $(1/39600, 7319/13608000)$ and for $D^{14} F^4$. Different structures for $D^{12} F^4$ and on.
... useful data for $\mathrm{SL}(2, Z)$ completion.

Perturbative checks: F^n at one-loop and HEA

One-loop effective lagrangian for 'constant' $F_\pm$ [Buchbinder, Petrov, Tseytlin]

$$\mathcal{L}_{1-\mathrm{loop}} = \frac{N}{(4\pi)^2}\left(\frac{F_+^2 F_-^2}{2v^4} + \frac{F_+^4 F_-^4}{16 g^4 v^{12}} - 5\frac{F_+^6 F_-^4 + F_+^4 F_-^6}{96 g^6 v^{16}} + \dots\right)$$

Only MHV operators are correctly reproduced by DBI-HEA proposed by Schwarz

$$\mathcal{L}_{\mathrm{HEA}} = -\frac{1}{8}(F_+^2 + F_-^2) + \frac{\kappa^2}{2} F_+^2 F_-^2 - \kappa^4 (F_+^2 F_-^4 + F_+^4 F_-^2)$$

$$+2\kappa^6 (F_+^2 F_-^6 + 3 F_+^4 F_-^4 + F_+^6 F_-^2) + \dots$$

where

$$\kappa = \frac{N}{(4\pi)^2 v^2}$$

Multi-instantons and $SL(2,Z)$ completion

$SL(2,Z)$ images $\tau_2^s \to \frac{Z_s(\tau,\bar\tau)}{\zeta(2s)}$ with

$$Z_s(\tau,\bar\tau) = \sum_{n,m\neq 0} \frac{\tau_2^s}{|m-n\tau|^{2s}} = 2\zeta(2s)\tau_2^s + 2\sqrt{\pi}\zeta(2s-1)\frac{\Gamma(s-\frac{1}{2})}{\Gamma(s)}\tau_2^{1-s}$$

$$+\frac{2\pi^s}{\Gamma(s)}\sum_{k=1}^{\infty}\sum_{d|k}\frac{k^{s-1}}{d^{2s-1}}(e^{2\pi i k\tau}+e^{-2\pi i k\bar\tau})\left(1+\frac{s(s-1)}{4\pi k\tau_2}+\dots\right)$$

non-holomorphic modular functions, such that $D^2 Z_s = s(s-1)Z_s$. Then 'predict'

$$f^{D^4F^4}(\tau,\bar\tau) = -\frac{N}{240(4\pi)^4 v^8}\left(\frac{Z_2}{2\zeta(4)}+\frac{5N}{2\pi}\frac{Z_1}{2\zeta(2)}+\dots\right)$$

OK 1-loop and 2-loop (by construction) and one-instanton (g's and π's). Peculiar $\log\tau_2$ (from Z_1, ??) and τ_2 (from Z_2, 3-loops?) ... multi-instantons

Further $SL(2,Z)$ 'predictions'

For $F_+^4 F_-^4$ terms

$$f_{\rm pert}^{F_+^4 F_-^4} = \frac{N}{(4\pi)^4 v^{12}} \tau_2 + \ldots \to f^{F_+^4 F_-^4} = \frac{N}{(4\pi)^4 v^{12}} \frac{Z_2}{2\zeta(4)} + \ldots$$

agreement with one-instanton g^0/π^6
For $F_+^6 F_-^4$ terms NOT modular invariant!! $F_+^6 F_-^4 \to \frac{c\tau+d}{c\bar{\tau}+d} F_+^6 F_-^4$

$$f_{\rm pert}^{F_+^6 F_-^4} = -\frac{5N}{96(4\pi)^5 v^{16}} \tau_2^3 + \ldots \to f^{F_+^6 F_-^4} = \frac{5N}{96(4\pi)^4 v^{16}} \frac{i\tau_2 \partial_\tau Z_3}{3\zeta(6)} + \ldots$$

agreement with one-instanton $1/g^2\pi^6$
Similarly for higher F^n terms 1-instanton OK!
Multi-instantons ... stay tuned

Conclusions and remarks

- We compute one-instanton effective action for $\mathcal{N} = 4$ SYM.
- By expanding the supersymmetric effective action in components, we obtain one-instanton contributions to operators such as F^n and $D^4 F^n$.
- We have also calculated $D^4 F^4$ (in fact $D^m F^4$) at one and two loops, combining with the result of one-instanton, $\mathrm{SL}(2, Z)$ symmetry may determine the full answer.
- We would like to investigate multi-instanton as well as higher-loop contributions.
- Finally, possible non-renormalization theorems for general operators?